Math 2177 recitation: Matrix and vectors

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1 Different forms of equations and matrix operations

Given a system of linear equations, for example:

$$2x_1 + 3x_2 - x_3 = 4$$
$$5x_1 - 3x_2 - 4x_3 = 1$$
$$2x_1 + x_2 + 7x_3 = 10$$

There's three other different ways of presenting this system:

By augmented matrix: $\begin{bmatrix} 2 & 3 & -1 & 4 \\ 5 & -3 & -4 & 1 \\ 2 & 1 & 7 & 10 \end{bmatrix}$

By vector equation: $x_1 \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -4 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 10 \end{bmatrix}$

By matrix equation: $\begin{bmatrix} 2 & 3 & -1 \\ 5 & -3 & -4 \\ 2 & 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 10 \end{bmatrix}$, here on left hand side we are taking product of two matrices.

Recall for matrices **A** and **B**. The (i, j) entry of their product **AB** (if defined) is the dot product (ith row of **A**) $\cdot$ (jth column of **B**)

Example :
$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 0*4+1*6 & 0*5+1*7 \\ 2*4+3*6 & 2*5+3*7 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 26 & 31 \end{bmatrix}$$

Exercise1 : Compute the following matrix operations. Write "undefined" for expressions that are undefined.

(1) $3 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} 10 & 20 & 30 \end{bmatrix}$ - $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

(3) $\begin{bmatrix} 3 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -4 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & -3 \\ -4 & -1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -3 & -4 \end{bmatrix}$

(5) $\begin{bmatrix} 3 & 2 \\ 2 & 1 \\ 3 & 4 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} -2 & 6 \\ 0 & 0 \end{bmatrix}$ (6) $\begin{bmatrix} 4 & -5 & 4 & 0 \\ 6 & 3 & 0 & -3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -6 & -1 \end{bmatrix}$

2 Connection between homogeneous and non homogeneous equations

Consider matrix equation $\mathbf{A}\overline{x} = \overline{b}$, where $\mathbf{A}$ is a matrix, $\overline{x}, \overline{b}$ are vectors. If we already know a solution $\overline{p}$, i.e. $\mathbf{A}\overline{p} = \overline{b}$, then any solution of $\mathbf{A}\overline{x} = \overline{b}$ can be write as $\overline{x} = \overline{p} + \overline{h}$, where $\overline{h}$ is solution of corresponding homogeneous equation $\mathbf{A}\overline{x} = 0$. This is because $\mathbf{A}(\overline{x} - \overline{p}) = \mathbf{A}\overline{x} - \mathbf{A}\overline{p} = \overline{b} - \overline{b} = 0$.

Exercise2 : Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 6 \end{bmatrix}$ and let $\mathbf{A} \cdot \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \overline{b}$. Find the general solution to $\mathbf{A}\overline{x} = \overline{b}$.

3 Linear combination and linearly dependence

Say vector $\overline{u}$ is a linear combination of vectors $\overline{v_1}, \overline{v_2}, \cdots, \overline{v_k}$ if the equation

$$x_1\overline{v_1} + x_2\overline{v_2} + \cdots + x_k\overline{v_k} = \overline{u}$$

has solutions.

Say vectors $\overline{v_1}, \overline{v_2}, \cdots, \overline{v_k}$ are linearly dependent if the equation

$$x_1\overline{v_1} + x_2\overline{v_2} + \cdots + x_k\overline{v_k} = 0$$

has **nonzero** solutions. Otherwise $\overline{v_1}, \overline{v_2}, \cdots, \overline{v_k}$ are called linearly independent

Exercise3 : Determine which of the following lists of vectors are linearly independent:

(1) $\overline{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \\ 5 \end{bmatrix}, \overline{v_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \overline{v_3} = \begin{bmatrix} 16 \\ 6 \\ 2 \\ -7 \\ 0 \end{bmatrix},$

(2) $\overline{v_1} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

(3) $\overline{v_1} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \overline{v_2} = \begin{bmatrix} -2 \\ 2 \\ 5 \end{bmatrix}, \overline{v_3} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}, \overline{v_4} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$

(4) $\overline{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \overline{v_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \overline{v_3} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \overline{v_4} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(5) $\overline{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \overline{v_2} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \overline{v_3} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$