Math 2177 recitation: Differential equations 2

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(You can find all my recitation handouts and their solutions on my homepage http://u.osu.edu/yuzhang/teaching/)

In this recitation we consider second order linear homogeneous equation with constant coefficients:

$$ay'' + by' + cy = 0$$

To solve this type of equations, we need to consider the corresponding characteristic equation

$$ar^2 + br + c = 0$$

1 Case 1: $b^2 - 4ac > 0$

When $b^2 - 4ac > 0$, $ar^2 + br + c = 0$ has two distinct real roots

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \; r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Then the general solution for $ay'' + by' + cy = 0$ is $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$, c_1, c_2 are arbitrary constants.

(This case has been discussed in last recitation.)

2 Case 2: $b^2 - 4ac = 0$

When $b^2 - 4ac = 0$, $ar^2 + br + c = 0$ has repeated real roots

$$r_1 = r_2 = r = -\frac{b}{2a}$$

Then the general solution for $ay'' + by' + cy = 0$ is $y = c_1 e^{rt} + c_2 t e^{rt}$, c_1, c_2 are arbitrary constants.

Exercise 1. Find the particular solution to

$$y'' - 8y' + 16y = 0, \; y(0) = 3, \; y'(0) = 10$$

Then decide $\lim_{t\to\infty} y(t)$

3 Case 3: $b^2 - 4ac < 0$

When $b^2 - 4ac < 0$, $ar^2 + br + c = 0$ has two complex roots of the form

$$r_{1,2} = \lambda \pm i\mu, \quad (r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a})$$

Then the general solution for $ay'' + by' + cy = 0$ is $y = c_1 e^{\lambda t}\cos(\mu t) + c_2 e^{\lambda t}\sin(\mu t)$, c_1, c_2 are arbitrary constants.

Exercise 2. Find the particular solution to initial value problem

$$y'' - 8y' + 17y = 0, \; y(0) = -4, \; y'(0) = -1$$

Exercise 3. Find the particular solution to initial value problem

$$y'' + 16y = 0, \; y(\frac{\pi}{2}) = -10, \; y'(\frac{\pi}{2}) = 3$$

4 More exercises

Exercise 4. Construct a second order homogeneous linear equation with constant coefficients, such that $y_1 = \frac{1}{\pi}e^{-t}$ and $y_2 = -\pi e^{3t}$ are two of its solutions.

Exercise 5. Let $y(t)$ be the solution of the initial value problem

$$y'' + 3y' - 10y = 0,\ y(0) = \alpha,\ y'(0) = -2$$

Suppose $\lim_{t\to\infty} y(t) = 0$, find the value of α.

5 Bonus questions

Exercise 6. (1) Show that the boundary-value problem

$$y'' + \lambda y = 0,\ y(0) = 0,\ y(1) = 0$$

has only the trivial solution $y = 0$ for the cases $\lambda = 0$ and $\lambda < 0$.

(2) For the case $\lambda > 0$, find the values of λ for which this problem has a nontrivial solution and give the corresponding solution.

Exercise 7. Suppose a, b, and c are all positive constants. Let $y(t)$ be a solution of the differential equation

$$ay'' + by' + cy = 0$$

Show that $\lim_{t \to \infty} y(t) = 0$.