Some statistical models and corresponding SAS software code

<table>
<tr><td>Completely randomized (CR), one factor</td><td>$Y_{ij} = \mu + \alpha_i + e_{ij}$, $e_{ij} \sim N(0, \sigma^2)$</td><td><code>class alpha ;
model y = alpha;</code></td></tr>
<tr><td>Randomized complete block (RCB), one factor</td><td>$Y_{ij} = \mu + \alpha_i + b_j + e_{ij}$, $b_j \sim N(0,\sigma_b^2)$,
$e_{ij} \sim N(0, \sigma_e^2)$</td><td><code>class alpha block;
model y = alpha;
random block;</code></td></tr>
<tr><td>CR, two (crossed) factors</td><td>$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk}$
$e_{ijk} \sim N(0, \sigma_e^2)$</td><td><code>class alpha beta block;
model y = alpha|beta;</code></td></tr>
<tr><td>RCB, two factors</td><td>$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + b_k + e_{ijk}$
$b_k \sim N(0,\sigma_b^2)$, $e_{ijk} \sim N(0, \sigma_e^2)$</td><td><code>class alpha beta block;
model y = alpha|beta;
random block;</code></td></tr>
<tr><td>CR, split plot
(in designs in table without a block effect [as here], “block” in SAS code is a label for the replicate experimental unit)

Equivalence of two models holds when σ_d^2 is positive (same for the RCB, below)</td><td>$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + d_{ik} + e_{ijk}$
$d_{ik} \sim N(0, \sigma_d^2)$, $e_{ijk} \sim N(0, \sigma_e^2)$

$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk}$
$e_{ijk} \sim N(0, \boldsymbol{R})$
$\boldsymbol{R}$: var.-cov.matrix
(fixed corr. or covariance)</td><td><code>class alpha beta block;
model y = alpha|beta;
random block*alpha;</code>

--or, equivalently--
<code>model y = alpha|beta;
repeated /sub=block*alpha
type=cs;</code></td></tr>
<tr><td>RCB, split plot</td><td>$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + b_k + d_{ik} + e_{ijk}$
$b_k \sim N(0,\sigma_b^2)$,$d_{ik} \sim N(0, \sigma_d^2)$, $e_{ijk} \sim N(0,\sigma_e^2)$

$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + b_k + e_{ijk}$
$b_k \sim N(0,\sigma_b^2)$, $e_{ijk} \sim N(0, \boldsymbol{R})$
$\boldsymbol{R}$: var.-cov.matrix
(fixed corr. or covariance)</td><td><code>class alpha beta block;
model y = alpha|beta;
random block block*alpha;</code>

--or, equivalently--
<code>model y = alpha|beta;
random block;
repeated /sub=block*alpha
type=cs;</code></td></tr>
<tr><td>CR, <u>repeated measures</u> (time: β), with one experimental factor (α); Covariance: <code>CS</code>, <code>AR(1)</code>, <code>UN</code>, <code>CSH</code>, <code>ARH(1)</code>, etc., etc.</td><td>$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk}$
$e_{ijk} \sim N(0, \boldsymbol{R})$

$\boldsymbol{R}$: var.-cov.matrix
(many possibilities)</td><td><code>class alpha beta block;
model y = alpha|beta / ddfm=kr;
repeated /sub=block*alpha
type=■;</code></td></tr>
<tr><td>RCB, <u>repeated measures</u> (time: β), with one experimental factor (α); Covariance: <code>CS</code>, <code>AR(1)</code>, <code>UN</code>, <code>CSH</code>, <code>ARH(1)</code>, etc., etc.</td><td>$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + b_k + e_{ijk}$
$b_k \sim N(0,\sigma_b^2)$, $e_{ijk} \sim N(0, \boldsymbol{R})$

$\boldsymbol{R}$: var.-cov.matrix
(many possibilities)</td><td><code>class alpha beta block;
model y = alpha|beta / ddfm=kr;
random block;
repeated /sub=block*alpha
type=■;</code></td></tr>
<tr><td>RCB, <u>repeated measures</u> (time: β), with one experimental factor (α); Covariance: <code>CS+AR(1)</code> [special case]</td><td>$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + b_k + d_{ik} + e_{ijk}$
$b_k \sim N(0,\sigma_b^2)$, $d_{ik} \sim N(0,\sigma_d^2)$, $e_{ijk} \sim N(0,\boldsymbol{R})$

$\boldsymbol{R}$: var.-cov.matrix
(autoregressive error only)</td><td><code>class alpha beta block;
model y = alpha|beta / ddfm=kr;
random block block*alpha;
repeated /sub=block*alpha
type=ar(1);</code></td></tr>
<tr><td>RCB, <u>repeated measures</u> (time: β), with <u>two</u> other experimental factors (α and γ);
Covariance: <code>CS</code>, <code>AR(1)</code>, <code>UN</code>, <code>CSH</code>, <code>ARH(1)</code>, etc., etc.

[take out “random block” if CR]</td><td>$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{il} +$
$(\beta\gamma)_{jl} + (\alpha\beta\gamma)_{ijl} + b_k + e_{ijkl}$,
$b_k \sim N(0,\sigma_b^2)$, $e_{ikjl} \sim N(0,\boldsymbol{R})$

$\boldsymbol{R}$: var.-cov.matrix
(many possibilities)</td><td><code>class alpha beta gamma block;
model y = alpha|beta|gamma /
ddfm=kr;
random block;
repeated /sub=block*alpha*gamma
type=■;</code></td></tr>
</table>

<table>
<tr>
<td>Same, but with gamma as fixed location effect (and randomized blocks within locations—note the two subscripts on b)</td>
<td>$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{il} + (\beta\gamma)_{jl} + (\alpha\beta\gamma)_{ijl} + b_{kl} + e_{ijkl}$,

$b_{kl} \sim N(0,\sigma_b^2)$, $e_{ikjl} \sim N(0,\boldsymbol{R})$

R: var.-cov.matrix
(many possibilities)</td>
<td><code>class alpha beta gamma block;</code>
<code>model y = alpha|beta|gamma / ddfm=kr;</code>
<code>random block(gamma);</code>
<code>repeated /sub=block*alpha*gamma type=■;</code></td>
</tr>
<tr>
<td>Same, but with gamma as random location effect (and randomized blocks within locations)

[a random location effect is very different from previous, and here we only consider some random-effect interactions]</td>
<td>$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{il} + (\beta\gamma)_{jl} + b_{kl} + e_{ijkl}$,
$\gamma_l \sim N(0,\sigma_\gamma^2)$, $(\alpha\gamma)_{il} \sim N(0,\sigma_{\alpha\gamma}^2)$, $(\beta\gamma)_{jl} \sim N(0,\sigma_{\beta\gamma}^2)$, $b_{kl} \sim N(0,\sigma_b^2)$, $e_{ikjl} \sim N(0,\boldsymbol{R})$</td>
<td><code>class alpha beta gamma block;</code>
<code>model y = alpha|beta / ddfm=kr;</code>
<code>random gamma block(gamma) gamma*alpha gamma*beta;</code>
<code>repeated /sub=block*alpha*gamma type=■;</code></td>
</tr>
<tr>
<td><u>RCB, split-split plot</u>
Whole-plot (α)
Sub-plot (β)
Sub-sub plot (γ)

[equivalence when variance terms are positive]</td>
<td>$Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{il} + (\beta\gamma)_{jl} + (\alpha\beta\gamma)_{ijl} + b_k + d_{ik} + f_{ijk} + e_{ijkl}$,

$b_k \sim N(0,\sigma_b^2)$, $d_{ik} \sim N(0, \sigma_d^2)$, $f_{ijk} \sim N(0, \sigma_f^2)$, $e_{ijkl} \sim N(0, \sigma_e^2)$

$Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{il} + (\beta\gamma)_{jl} + (\alpha\beta\gamma)_{ijl} + b_k + d_{ik} + e_{ijkl}$,

$b_k \sim N(0,\sigma_b^2)$, $d_{ik} \sim N(0, \sigma_d^2)$, $e_{ijkl} \sim N(0,\boldsymbol{R})$</td>
<td><code>class alpha beta gamma block;</code>
<code>model y = alpha|beta|gamma / ddfm=kr;</code>
<code>random block block*alpha block*alpha*beta;</code>

<code>--or, equivalently--</code>

<code>class alpha beta gamma block;</code>
<code>model y = alpha|beta|gamma / ddfm=kr;</code>
<code>random block block*alpha;</code>
<code>repeated / sub=block*alpha*beta type=cs;</code></td>
</tr>
<tr>
<td><u>RCB, split-plot+time</u>
Whole-plot (α)
Sub-plot (β)
Time (γ)</td>
<td>$Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{il} + (\beta\gamma)_{jl} + (\alpha\beta\gamma)_{ijl} + b_k + d_{ik} + e_{ijkl}$,

$b_k \sim N(0,\sigma_b^2)$, $d_{ik} \sim N(0, \sigma_d^2)$, $e_{ijkl} \sim N(0,\boldsymbol{R})$

R: var.-cov.matrix
(many possibilities)</td>
<td><code>class alpha beta gamma block;</code>
<code>model y = alpha|beta|gamma / ddfm=kr;</code>
<code>random block block*alpha;</code>
<code>repeated / sub=block*alpha*beta type=■;</code></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</table>

Unless indicated otherwise, Greek letters indicate fixed-effect terms (treatment, factor A [α], time or factor B [β], etc.), and *b* is the random block effect.

For all models <u>before</u> the RCB split-split-plot, the beta (β) term always refers to time, which is clustered within the subjects identified by block*alpha, or for more factors, block*alpha*gamma.