Victor Vanquished

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Heck, R. 1997a. *Language, Thought and Logic: Essays in Honour of Michael Dummett.* Oxford: Oxford University Press.
Hodes, H. 1984. Logicism and the ontological commitments of arithmetic. *Journal of Philosophy* 81: 123–49.
Lowe, E. J. 1998. *The Possibility of Metaphysics: Substance, Identity and Time.* Oxford: Clarendon Press.
MacBride, F. 2000. On finite Hume. *Philosophia Mathematica* 8: 150–59.
MacBride, F. Forthcoming. Speaking with shadows: a study of neo-Fregeanism. *British Journal for the Philosophy of Science.*
Quine, W. V. 1976. Carnap and logical truth. In his *Ways of Paradox and other essays*, 107–32. Cambridge, Mass.: Harvard University Press.
Wright, C. 1983. *Frege's Conception of Numbers as Objects.* Aberdeen: Aberdeen University Press.
Wright C. 1997. On the philosophical significance of Frege's theorem. In Heck 1997a: 201–44. Repr. in Hale and Wright 2001: 272–306.
Wright C. 1998. On the harmless impredicativity of $N^{=}$ ('Hume's Principle'). In *The Philosophy of Mathematics Today*, ed. M. Schirn, 339–68. Oxford: Clarendon Press. Repr. in Hale and Wright 2001: 229–55.
Wright, C. 2000. Is Hume's principle analytic? *Notre Dame Journal of Formal Logic* 40. Repr. in Hale and Wright 2001: 307–22.

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NEIL TENNANT

The naive anti-realist holds the following principle:

($\Diamond$K) All truths are knowable.

This unrestricted generalization ($\Diamond$K), as is now well known, falls prey to Fitch's Paradox (Fitch 1963: 38, Theorem 1). It can be used as the only suspect principle, alongside others that cannot be impugned, to prove quite generally, *and constructively*, that the set $\{p, \neg Kp\}$ is inconsistent (Tennant 1997: 261). From this it would follow, intuitionistically, that any proposition that is never actually known to be true (by anyone, at any time) is false; and it would follow, classically, that every truth is known (by someone, at some time).

Michael Dummett (2001) has offered a new diagnosis of the avoidable error involved in the unrestricted principle. He responds to the Fitch paradox by abandoning the unrestricted generalization, and confining the knowability claim to *basic*, or grammatically primitive, statements:

ANALYSIS 62.2, April 2002, pp. 135–42.

(◊KB) All *basic* truths are knowable.

He therefore follows a new version of what I called the 'restriction strategy' (1987: 246). The restriction strategy is the strategy that I favour, too – but by restricting the generalization so that it reads:

(◊KC) All *Cartesian* truths are knowable.

A Cartesian proposition is a proposition p – of any syntactic complexity – such that Kp is consistent. This restriction has had its critics (Hand and Kvanvig 1999; Williamson 2000), but is still, in my view, sustainable (Tennant 2001a, 2001b). Nevertheless, any alternative restriction strategy that might vouchsafe a justification for the most basic kind of anti-realism – that of intuitionism in arithmetic – is of intrinsic interest.

The anti-realist's overarching argument is as follows. There is no way that one could (in principle) know a sentence to be true except on the basis of a warrant recognizable as such – a finitary truth-maker that is in principle surveyable. This epistemic constraint on truth requires us, in the face of (intuitionistically provable) undecidability and incompleteness phenomena, to refuse to grant the principle of bivalence. If the only kind of truth is that which is recognizable (in principle) as such, then bivalence can be sustained only at the cost of 'Gödelian Optimism'. In the absence of any convincing argument for such optimism, truth-as-knowable leads one to anti-realism. This is the stable, reflective equilibrium in which one seeks to have all one's forms of 'logical' inference justified as analytic, not as metaphysical, principles. (For a more detailed development of this view, see Tennant 1996.)

Arithmetic is indeed a special case in the context of this discussion, because the class of basic sentences is decidable (as to truth-value). Yet Dummett does not explain or justify principle (◊KB) by appeal to the *decidability of truth-value* of basic sentences; rather, one is led to believe that it is merely their being *logico-grammatically primitive* that makes them knowable, if true. The possibility that Dummett leaves open – and one that is arguably realized – is that certain basic sentences might be undecidable as to truth-value. The possibility is realized in the form of grammatically primitive sentences involving highly theoretical predicates and referring terms. These predicates and terms can acquire their cognitive significance from the way in which they are introduced into scientific discourse in order to forge logical connections that would not obtain in their absence. (See Tennant 1997, ch. 11, 'Cognitive significance regained'.) Such provenance for theoretical primitives can make even the simplest sentences involving them effectively undecidable. *If true*, such a sentence would nevertheless (according to (◊KB), hence also (◊KC)) still be *recognizably* true (that is, knowable). And that makes perfectly good doctrinal sense for an anti-realist. The only puzzle, then – to be pressed below – is why the requirement that a sentence be *recognizably* true, if true at all, is not maintained for (some appropriately qualified) class of *complex* sentences. My own principle (◊KC) sought to formulate that qualification correctly.

Dummett interprets Kp as 'someone, at some time, knows that p', which is the same interpretation employed in the sources cited above. Embedded within this definiens is the construction 'x knows at time t that p'. Dummett does not tell the reader whether Victor takes this latter expression as basic. (I owe this observation to Joe Salerno.) It seems, intuitively, that with complex sentences p one would have no reason to expect the relation 'x knows at time t that p' to be decidable (as to truth-value). Still, if it is true that x knows at time t that p, there ought to be some way for both x and others to recognize that this is the case. Fortunately we do not have to reach a considered conclusion on the question whether 'x knows at time t that p' should be taken as basic (and hence within the scope of Victor's very restricted knowability principle). That is for Victor, not for us, to decide; for it will turn out that the main criticism to be levelled below does not depend on our knowing Victor's answer to this question.

Dummett also commits his anti-realist to the inference

p; *ergo*, p is true,

or, in symbols,

$$\frac{p}{Tr(p)}$$

Let us call this the rule of semantic ascent, or *Tr*-introduction.[1] (Its converse would be the rule of disquotation, or *Tr*-elimination.) Interestingly, Dummett seems to postulate this principle, rather than seek to derive it – as Tarski did – from a materially adequate and formally correct definition of truth.

The question now arises whether the anti-realist should also maintain (across the board)

p is true; *ergo*, it is possible (in principle) to know that p

– in symbols,

$$\frac{Tr(p)}{\Diamond K(p)}$$

Dummett wants his theorist Victor to commit himself to such an inference (and, indeed to its converse) only when p is an *atomic* or basic sentence of the language, such as is treated by a basis clause in an inductive definition of truth and satisfaction. Thus Victor has the following as his basis clause in his theory of truth:

(i) $Tr(A) \leftrightarrow \Diamond K(A)$, if A is a basic statement.

[1] We omit the corner quotes that should, strictly speaking, be used in connection with the predicate '*Tr*'. No confusion will result.

Dummett has Victor leave the clauses for the standard connectives and quantifiers just as they would be in the usual (intuitionistic or classical) Tarskian inductive definition of truth and satisfaction. These clauses are biconditionals that 'distribute truth' (or satisfaction) from a complex sentence on the left-hand side to its constituents on the right-hand side. According to Dummett, this will allow Victor to handle the Fitch 'counter-example' of an unknown truth – a proposition B such that

$$B \,\&\, \neg K(B)$$

– without contradiction. For, from the displayed sentence, he says,

> Victor will still be committed by his inductive characterization of truth to inferring ... both that it could have been or could later be known that B and that in fact it never has been and never will be known that B ... ; [and] that was precisely the type of situation he wished to envisage. (Dummett 2001: 2)

Now the claim that *it never has been and never will be known that B* is simply ¬K(B). So the right-hand conjunct of Victor's inferred conclusion is the right-hand conjunct of his premiss B & ¬K(B). How, precisely, does Victor carry out the inference from B & ¬K(B) to the conclusion ◊K(B) & ¬K(B)? The non-trivial part is deducing ◊K(B) from B & ¬K(B). Given Dummett's postulated rule of semantic ascent remarked on above, one imagines Dummett would have Victor employ a proof of the following form:

$$\cfrac{\cfrac{\cfrac{B \,\&\, \phi}{Tr(B \,\&\, \phi)}}{Tr(B)}}{\Diamond K(B)}\ \text{by clause (i)}$$

Note that the internal structure of the second conjunct ϕ is irrelevant for this deduction. It is a mere accident, from the point of view of any reasoner following the foregoing proof, that ϕ happens, in Victor's example, to be the claim that it is not known that B.

The same conclusion can be reached by using a weaker rule of semantic ascent – one that holds only for basic statements, such as B. For consider the proof

$$\cfrac{\cfrac{\cfrac{B \,\&\, \phi}{B}}{Tr(B)}\ \text{weaker rule of smantic ascent}}{\Diamond K(B)}\ \text{by clause (i)}$$

Victor is therefore able to infer ◊K(B) & ϕ from B & ϕ. Hence, by substitution, Victor can infer ◊K(B) & ¬K(B) from B & ¬K(B). Moreover, according to Dummett, the entertained proposition B & ¬K(B) does not lead to contradiction. By contrast, if we still had the knowability principle in the naive form (◊K), then B & ¬K(B) *would* lead to contradiction. Problem solved, it would seem.

But, one may ask, how satisfactorily? The solution has to be assessed within the wider context of the theory of truth, and the normal philosophical demands we place on it. The most important of those demands is that all instances of the *T*-schema should be non-trivially derivable as theorems within any adequate theory of truth that employs more fundamental axiomatic principles than those instances themselves. Dummett does not have Victor able thus to derive all such instances, within his reformed inductive theory of truth with its epistemically flavoured basis clause (i). It is easy to see that his theory of truth *cannot* afford deductive passage to B from *Tr*(B) even for atomic B, unless further rules of inference are supplied governing ◊ and K (or just ◊K) in the metalanguage. And, even after such rules have been supplied, Victor's theory of truth affords no such deductive passage from (possibly complex) B to *Tr*(B). Dummett seeks instead to have Victor postulate this as a further principle, since he is unable to justify it the way one does in the usual demonstration of material adequacy. Victor does not have semantic ascent and disquotation *drop out* of his theory of truth. Rather, he has them grafted onto it, as a necessary afterthought.

One sympathetic to Dummett's strategy on Victor's behalf might retort at this point that Victor may as well be given the theoretical inference from B to *Tr*(B) (for atomic B), since that is but half of the usual basis clause of the unreformed truth-theory anyway. Very well; let us grant Victor that much. Assuming further that the metalanguage furnishes also the 'factive' inference, either basic or derived, from ◊K(B) to B, we shall have, in effect, a theory that can be re-axiomatized as follows:

> take the normal Tarskian theory of truth, without any modal epistemic modification of its basis clause; and graft onto it the further metalinguistic principle B ↔ ◊K(B) (for atomic B).[2]

We shall now be able to *derive* – in an informative and non-trivial fashion – all instances of

$$\phi \leftrightarrow Tr(\phi),$$

for both atomic and complex ϕ, provided only that ϕ contains no occurrences of K or of ◊. For we have not yet been told by Dummett what the truth-theoretic clause would look like for the operator K or for the operator ◊ *in the object language*. Yet one would *need* such clauses in order to be able to meet Tarski's adequacy condition for a theory of truth for an object language containing those expression-forming operators.

[2] What we really need to say here is: for such metalinguistic B as translate basic sentences in Victor's object language.

Bearing in mind that K(p) is here being understood as short for $\exists x \exists t(x\mathrm{K}_t p)$, what we are really after is a clause for open sentences of the form $x\mathrm{K}_t p$. I can only suggest the following:

the assignment f of individuals to free variables in $u\mathrm{K}_t p$ satisfies it if and only if the denotation of u relative to f knows, at (the instant of time given by) the denotation of t relative to f, that f satisfies p.

This suggestion comes from straightforwardly applying the usual Tarskian method for 'distributing' the semantic predicate (here, satisfaction by the assignment f) across syntactic constituents. Let us set aside the problem – by no means insignificant – that we here seem to be requiring, of any individual knower, that he have attitudes toward contents that have embedded within them such theoretical notions as Tarskian satisfaction.[3]

We shall also need a clause for the operator 'possibly'. The most straightforward one would simply 'translate up' the object-linguistic $\Diamond$ as the correspondingly metalinguistic modality $\underline{\Diamond}$:

f satisfies $\Diamond\phi$ if and only if $\underline{\Diamond}$ (f satisfies ϕ).

We shall set aside the question whether such a straightforward treatment of the possibility operator is really appropriate for the particular kind of context that we are dealing with here – that of know*ability* of a proposition. It could be argued that this modal suffix, in this sort of context, should be treated as something other than an alethic modal operator admitting of a possible-worlds interpretation. For, very importantly, one has to secure the 'factive' nature of the complex $\Diamond$K: one can always infer ϕ from $\Diamond \mathrm{K}\phi$.

Let us generously assume that Victor's newly adopted clauses for K and for $\Diamond$, whatever they are, allow him to meet Tarski's adequacy condition on the resulting theory of truth for the object language containing those operators in addition to the usual logical operators and whatever extralogical vocabulary might be under consideration. The important point is that Dummett allows Victor the further (metalinguistic) postulate

(ii) $\mathrm{B} \leftrightarrow \Diamond\mathrm{K}(\mathrm{B})$, for atomic B.

Let us concentrate now on the language of arithmetic, augmented with K and $\Diamond$, and permitting variables to range over thinkers as well as numbers. Call this the language of epistemic arithmetic. We may now ask: what objection can Dummett raise against an epistemic arithmetician who

[3] The obvious complaint will be that of course there will be knowers entirely innocent of such notions. The obvious reply to this complaint will be to point out that the converse of satisfaction, namely 'holding true of', is one that it would be reasonable to expect to be grasped by any thinker. One would just have to finesse matters so that the thinker could enjoy the fruits of the theory of satisfaction and truth without having to think set-theoretically about (finite) assignments of individuals to variables. We shall not explore this aspect of the dialectic any further here.

shares with Victor all that has been postulated thus far, but who also insists on developing his truth theory (in the metalanguage) by using the full power of classical logic?

Such a 'classical epistemic arithmetician' exploits classical logical principles in his metalanguage in order to induce a classical logic for the object language. He also makes adroit use of the fact that all basic statements of arithmetic are effectively decidable – hence, if true, knowable. So, for all atomic B in the language of arithmetic, he has justification for (ii).

By having confined the knowability principle to atomic statements, it would appear that Dummett has foregone the most important principled way for the anti-realist to argue against the illicit application of strictly classical rules of inference. No longer is he requiring of *every* proposition of arithmetic that, if it is true, then it is knowable. The suggestion that Victor restrict the knowability requirement to just the atomic truths of arithmetic happens to fall on very attentive ears on the part of his classically-inclined interlocutor. There is no longer any principled ground on which the latter can be enjoined *not* to treat the logical operators $\neg$, $\vee$ and $\exists$ in the non-constructive way that he does. To be sure, the truth-theoretic clauses suffice to generate all instances of the *T*-schema by means of *intuitionistic* proofs (indeed, intuitionistic *relevant* proofs); but, says the classicist, he does not see why that should be all that there is to the notion of truth. He thinks truth is bivalent; for he is willing to use, say, the law of excluded middle in the metalanguage. And the result of so doing is that the concept of truth becomes bivalent *for the object language* as well.[4]

Another objection to Victor's restricted stance on knowability is that, if there is indeed any atomic truth B that has never been, and never will be, known by anyone – an atomic sentence B, that is, for which $B \,\&\, \neg K(B)$ holds – then *that* itself is an unknowable fact. And Dummett accepts as much, for he acknowledges the correctness of the Fitchian *reductio* of $K(B \,\&\, \neg K(B))$. Thus $B \,\&\, \neg K(B)$ would have to be acknowledged, by virtue of its peculiar logical structure, as a necessary exception to *any* attempted formulation of a principle of knowability.

Why not, then, simply generalize from this example in delineating, as economically as possible, the class of potential counterexamples to knowability? Let us not throw the knowable babies out with the unknowable bathwater. What chapter 8 of *The Taming of the True* proposed was that it was precisely the test of $K\phi$'s inconsistency that should be applied. The knowability principle should be restricted to those truths ϕ which were

[4] This is exactly how matters worked out in Tarski's own hands. He surreptitiously invoked the law of excluded middle in the metalanguage when proving, of every sentence of the object language, that either it or its negation would be true. See Tarski 1956, Theorem 2 and n. 2 on p. 197.

Cartesian, that is, for which $K\phi$ was *consistent*. The correct restricted knowability principle, therefore, should be ($\Diamond$KC). Note that if ϕ is true but $K\phi \vdash \bot$, then ϕ must be complex. But there are also complex truths ϕ such that not-($K\phi \vdash \bot$). Indeed, of every complex truth ϕ of arithmetic, the anti-realist would say, it is at least *consistent* to claim $K\phi$.

Thus Dummett's Victor is erring even further on the side of caution by being willing to maintain only of *basic* sentences that they are, if true, then knowable. What Dummett has not given us is any rationale for restricting the principle any further than ($\Diamond$KC). His own suggested restriction is much more drastic, and, arguably, leaves the anti-realist helpless against any classical innovations (on complex sentences) on the part of his fellow epistemic arithmetician. Until such time as real problems for ($\Diamond$KC) have been shown to arise, it strikes me as a more sensible restriction than Victor's ($\Diamond$KB).[5]

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References

Dummett, M. 2001. Victor's error. *Analysis* 61: 1–2.

Fitch, F. 1963. A logical analysis of some value concepts. *Journal of Symbolic Logic* 28: 135–42.

Hand, M. and J. Kvanvig. 1999. Tennant on knowability. *Australasian Journal of Philosophy* 77: 422–28.

Tarski, A. 1956. The concept of truth in formalized languages. In *Logic, Semantics, Metamathematics*, tr. & ed. J. Woodger, 152–278. Oxford: Clarendon Press.

Tennant, N. 1996. The law of excluded middle is synthetic a priori, if valid. *Philosophical Topics* 24: 205–29.

Tennant, N. 1997. *The Taming of the True*. Oxford: Clarendon Press.

Tennant, N. 2001a. Is every truth knowable? Reply to Hand and Kvanvig. *Australasian Journal of Philosophy* 79: 107–13.

Tennant, N. 2001b. Is every truth knowable? Reply to Williamson. *Ratio* 14: 263–80.

Williamson, T. 2000. Tennant on knowable truth. *Ratio* 13: 99–114.

[5] Thanks to Joe Salerno for helpful correspondence.