The Umpire's Dilemma and the Ashes of Realism

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Abstract Radford [1985] poses a *prima-facie* problem for the anti-realist or intuitionist who holds that all truths are knowable yet who refuses to assert, or even denies, that all declarative sentences have determinate truth-values—values that might be independent of our means for determining what they are. This study sets out the Umpire's Dilemma and explores the prospect for an anti-realist solution of the problem that it poses. The problem should appeal to our honoree Alan, given that he has very little truck for 'in principle possibilities'.

1 On a personal note

Alan was so kind as to write as follows, in his Preface to *Proof through Truth*:

> To start fairly near the beginning, my earliest inspiration in turning to philosophy of mathematics was Neil Tennant, who, as a young lecturer, turned my head away from Bradley towards Quine (though later reflection discerns more in common between the two than I initially would have suspected); towards logic (implanting a firm bias in favour of the Gentzen/Prawitz approach); and towards philosophy of mathematics. I hope he does not count his work with me as a failure because I remained, then and now, immune to the attractions of intuitionism and relevance. Following a widespread tradition, I bite the hand which fed me in Chapter 6, where I criticize the kind of appeal to idealization which Neil makes in accounting for our mathematical knowledge. (Weir [2010], p. vi)

It gives me unadulterated joy, as a happy victim of philosophical patricide, to show (or at least try to, in due course) that the old codger might have had some philosophical kevlar under his vest, which Alan's viewfinder might not have revealed when he had me in his sights. The reader might be expecting from me a stern critical study of Chapter 6, designed to persuade Alan to abandon his puzzling combination of a Byzantine mereology of physical tokens, a refusal to countenance abstract types,

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and a happy embrace of completed infinities, *including* infinite *proofs*. But that would strike an altogether discordant note in a *Festschrift*. We are gathered here between these covers, as it were, to celebrate Alan's wonderful example of how to philosophize, regardless of the doctrinal outcome.

Let me begin with some reminiscences of my own about those earlier days with Alan. We all know him to be a brave and kind man of fantastic good humor. Nowhere was his character on better display than back in the early summer of 1976, when Alan was one of four Edinburgh students whom I was accompanying to Dartmouth on our annual exchange between the two Philosophy Departments. We flew into Newark late, and the group opted for a night in a local motel—except for Alan. He was so anxious to get to Dartmouth that he struck out on his own, taking a taxi to the downtown Greyhound station in NYC. There his taxi driver relieved him, at knifepoint, of his wallet and traveller's cheques. Alan's subsequent conversation with a ticket clerk inside the station was overheard by a fellow Scot. 'Dinnae worry, lad!' the stranger said. He paid for Alan's ride to Hanover, and gave Alan his address in New Orleans for repayment. The first thing I had to do for Alan on arrival in Hanover was help him send the money to this kind compatriot. It was his overriding concern; he did not succumb to any post-traumatic stress at having his life threatened while being mugged. During our visit at Dartmouth, he was also the group's uproarious entertainer. It was Alan who chivvied us to gather to watch Startrek each week; it was Alan who regaled us with his discoveries in the vast aisles of the meat and poultry department of the local supermarket. To this day I remember the Renfrewshire brogue of his exclamation 'Breasts! Thighs!' in feigned disbelief at the upfront coarseness of American advertising.

2 Introducing an underappreciated problem for the anti-realist

Cricket, even if Wittgenstein might not have appreciated the fact, is not only the most important game, but also the most important form of life. Dummettian anti-realism is inspired by Wittgensteinian doctrines about meaning as use, manifestationism and rule-following. So Dummettian anti-realism should be able to resolve the dilemma to be set out below. Not to be able to do so would, philosophically speaking, just not be cricket.

2.1 A little lesson on some rules of cricket (for American, and perhaps even certain Scottish, readers)

Two ways in which a batsman (i.e., striker) can be dismissed (or *Out*) are to be *Caught* or to be *Leg Before Wicket (LBW)*.

The batsman is out *Caught* if the ball, upon being properly bowled, makes above-ground contact with his bat or gloves, and is thereafter caught by a fielder without the ball making prior or simultaneous contact with the ground. A good example of this would be a case where the ball, only ever so slightly deflected after nicking the edge of the bat, is caught by the wicketkeeper, who stands behind the wicket. Afficionados call this being *Caught behind*.[1]

The batsman is out *LBW* if two conditions are met: (i) the ball strikes any part of his[2] body (*without* making prior or simultaneous contact with his bat) on a line between the two wickets,[3] or on the off-side;[4] and (ii) if the batsman had not been in the way, and the ball had been allowed to complete its trajectory, then it would have struck the batsman's wicket.[5]

Usually the epistemic situation with *LBW* cases is very unclear for the batsman himself, both with respect to condition (i) and with respect to condition (ii). He is not well-positioned to judge, even in certain quite clear cases, that he is indeed out *LBW*. The bowling-and-fielding team does not rely on any presumed confluence of honour and relevant omniscience on the part of the batsman. Instead, they appeal to the umpire to deliver the verdict of *LBW*.[6]

Umpires always act under the dictate 'If in doubt, give *Not Out*.'

2.2 The Umpire's Dilemma

The ground has now been prepared for the statement of our dilemma. Here (in our own words) is the deeply problematic situation that Radford asked his reader to consider.

> The umpire is certain that the ball strikes the batsman's leg pad in line with the stumps, and would not hesitate to give him out *LBW* if that were all. He is also certain that the ball went from the pad straight into the wicket-keeper's gloves. What he is not certain about is whether the ball nicked the bat before hitting the pad. Since he is not certain that it did, he

[1] If the batsman does not immediately 'walk' (as he would be honour-bound to do if he himself knew that he had indeed been *Caught behind*), the bowling-and-fielding team has to appeal to the umpire if they wish to have a verdict of *Caught behind*. This they do by screaming 'Huzzat?', which the Quinean radical interpreter would be within a thick edge of rendering as 'How's that?'. It is an indeterminacy not yet fully resolved whether the apostrophe-'s' is short for 'is' or 'was' or the timeless 'is'. Even the totality of all possible behavioural evidence on the part of cricketers and umpires might underdetermine the metaphysics of time within that community. This might explain why cricket can appear to be so tedious, to the uninitiated radical interpreter.

[2] The use of the masculine pronoun is justified by appeal to the ease of use of 'batsman' as opposed to 'bats*person*'.

[3] Each wicket consists of three parallel and upright *stumps*, topped off with two *bails*.

[4] This is the side to which the batsman's toes point when he assumes his stance to face the bowler.

[5] Batsmen hardly ever 'give themselves' *Out* in the case of a possible *LBW* dismissal. A rare case where the batsman might be honour-bound to walk would be where he had stepped right back to the wicket, and had taken a full-toss delivery on his pads right in front of the wicket.

[6] This is done with a theatrical and histrionic yell of *'Huzzat!'* (radically interpretable as *'How's that?'*), usually from high in mid-air and with arms flung upwards. The umpire, if he agrees, raises a single finger at the batsman, who must then walk. This raised finger is *not* radically interpretable as any kind of insult. There is absolutely nothing American about the act.

cannot give the batsman out *Caught*. But since he is not certain that it did not, he cannot give the batsman out *LBW*. So although he knows that the batsman has broken one of those two laws, he cannot give him out as the laws stand (there is no 'disjunctive' way of being out, *Caught*-or-*LBW*). Video replays offer no help to the 3rd umpire—they do not reveal whether or not the ball nicked the bat, no matter how closely he looks.

Problem for the anti-realist or intuitionist: this is a case of the umpire knowing that the batsman has broken either the *LBW*-law or the *Caught*-law [$K(p \vee q)$], without either knowing p or knowing q [$\neg K p,\ \neg K q$].

3 First anti-realist response

The anti-realist might respond as follows.

The question as to whether or not
N: the ball nicked the bat
has a discoverable answer. And that gives the anti-realist a way out. The umpire already knows
Δ: the ball went straight into the wicket-keeper's gloves
and knows that
$\Delta, N \vdash$ *Caught (behind)*; whence, obviously,
$\Delta, N \vdash$ *LBW* $\vee$ *Caught (behind)*. The umpire also knows that
Γ: the ball struck the batsman's pads in line with the stumps,
and would otherwise have hit the wicket
and knows that
$\Gamma, \neg N \vdash$ *LBW*; whence, obviously,
$\Gamma, \neg N \vdash$ *LBW* $\vee$ *Caught (behind)*.
So by Dilemma on the decidable proposition N, the anti-realist concludes that

$$\Delta, \Gamma \ \vdash \ \mathit{LBW} \ \vee \ \mathit{Caught\ (behind)}.$$

Realist rejoinder: That is not satisfying. The answer (*LBW*, or *Caught*, as the case may be) is not discoverable by the umpire. Yet he does know that *LBW* $\vee$ *Caught*.

Anti-realist's rebuttal: It is not important, in so far as this is to be taken as a *philosophical* problem rather than a mere problem for umpiring procedures, that it be the *umpire himself* who can get himself into a position to know which of *LBW*, or *Caught*, is true. The umpire's decision methods, given his responsibility to make timely decisions, need to be highly feasible. They cannot be expected to match the power or range of methods that would guarantee a result *in principle*, with no limits on the time or energy that might have to be expended in order to arrive at a decision. To repeat: we are seeking a resolution of the *philosophical* problem. We are not seeking a resolution of the problem, for the umpire, of how best to arrive at a timely judgement. So all that is required, philosophically, is that it be possible, in principle, for someone suitably placed and with the totality of evidence that would be available to her from being so placed, to discover, *eventually*, exactly which one of the propositions *LBW*, or *Caught*, is true. *In this sense*, there is a way one could discover which of *LBW*, *Caught*, is true. And that assurance is all that the umpire would need in order to be able, with a clear conscience, and on the evidence currently available to him, to give the batsman *Out*. The umpire's reasoning would be as follows:

$$\dfrac{\dfrac{\dfrac{\Delta \qquad \overline{N}^{\,(1)}}{\mathit{Caught}}}{\mathit{Out}} \qquad \dfrac{\dfrac{\Gamma \qquad \overline{\neg N}^{\,(1)}}{\mathit{LBW}}}{\mathit{Out}}}{\mathit{Out}}\;(1)$$

The step of constructive dilemma (labelled (1)) is licit because *N* ('the ball nicked the bat') is a claim that is decidable in principle. Given that the umpire knows both Δ and Γ, his verdict of *Out* (courtesy of the reasoning just regimented) is subject to no doubt. He cannot be 'in doubt'; so he does not need to give 'Not Out'.

Further realist objection: This proposal entails a new (and impermissible) category of dismissal, which would have to be recorded in the scorebook. The new category is *LBW or Caught*. But the rules of cricket do not allow for such disjunctive categories. The umpire is required to have arrived at a non-disjunctive determination—an *atomic* judgement, if you will—of *the exact way in which the batsman has been dismissed*.

4 Second anti-realist response

So be it. You are saying that the rules implicitly require dismissals to be recorded not just as

Out, LBW

or

Out, Caught

but rather as

The umpire *knows that* the batsman is *Out, LBW*

or

The umpire *knows that* the batsman is *Out, Caught*.

As alternatives to 'knows that' here, one might consider also 'has determined that', or 'is morally certain that', or 'believes beyond any reasonable doubt that'. What is important is only that the attitudinal phrase be one that does *not* apply to either of the two disjuncts in the circumstances described in the Umpire's Dilemma.

Let us use K for the kind of operator under consideration here. Let u be the umpire. The suggestion is that the rules implicitly require, for licit dismissal, that the dismissal of the batsman as described in the Dilemma be recordable either in the form uK(*LBW*) or in the form uK(*Caught*). And, given the circumstances described, this fails to be the case even though uK(*LBW*∨*Caught*) (so the realist would have it, as well as the anti-realist who is prepared to treat as bivalent statements that are decidable in-principle).

Renewed realist objection: So now the anti-realist faces a problem that takes the following general form.

One wishes to claim the consistency of the set

$$\{\neg xKp, \neg xKq, xK(p \vee q)\}.$$

(The umpire does *not* know that the batsman is *LBW*; does *not* know that the batsman is *Caught*; but *does* know that the batsman is *either LBW or Caught*.) But, on the anti-realist construal of knowledge of a disjunction, this is impossible. Anyone who knows that $p \vee q$ should know *which* of p, q is true. That is, the following is a principle of (anti-realist) epistemic logic:

$$\frac{xK(p \vee q)}{xKp \vee xKq}.$$

One will then have a proof of the *in*consistency of the aforementioned set:

$$\dfrac{\dfrac{xK(p \vee q)}{xKp \vee xKq} \qquad \dfrac{\neg xKp \quad \overline{xKp}^{\,(1)}}{\bot} \qquad \dfrac{\neg xKq \quad \overline{xKq}^{\,(1)}}{\bot}}{\bot}{\scriptstyle(1)}$$

Anti-realist rejoinder: There is an equivocation here on the force of the epistemic operator K. The sense of K in which the umpire does not know that the batsman is out *LBW*, and does not know that the batsman is out *Caught*, is the *occurrent* sense of ‘knows, right now, with conclusive justification, and with no further investigations of matters of fact, or drawings of inferences, being required’. Let us designate this as $\mathbf{K}$, and reserve the italic K for the ‘laxer’ kind of knowledge. This is the kind of knowledge (of a conclusion) that can be attained by taking steps of constructive dilemma on decidable propositions on which no decision has yet been reached. Note that, accordingly, it is much *harder* for a claim of the form $\neg xKp$ to be true. For this would require (on the anti-realist construal of negation) that one be able to demonstrate the absurdity, *modulo* what one already knows, of the hypothesis that xKp. The umpire surely cannot be represented as not knowing that the batsman is *LBW* (or as not knowing that the batsman is *Caught*) in *this*, weaker, sense of knowledge. For someone *might*, say, come forward with hugely magnified, slow-motion videography of the event, showing conclusively, one way or the other, whether the bat nicked the ball, and hence which exact category of dismissal applies. And to assert both $\neg xKp$ and $\neg xKq$ is (mistakenly) to rule out such a possibility.

The anti-realist would therefore reply to the realist's renewed objection above by pointing out that consistency is being claimed only for the set

$$\{\neg x\mathbf{K}p, \neg x\mathbf{K}q, xK(p \vee q)\}.$$

(The umpire does not *occurrently* know that the batsman is *LBW*; does not *occurrently* know that the batsman is *Caught*; but *does* know, in the weaker sense just described, that the batsman is *either LBW or Caught*.) Furthermore the anti-realist refuses to endorse the epistemic principle of inference

$$\frac{xK(p \vee q)}{xKp \vee xKq}.$$

What he *will* concede is only the weaker inferential principles

$$\frac{x\mathbf{K}(p \vee q)}{x\mathbf{K}p \vee x\mathbf{K}q}$$

and

$$\frac{xK(p \vee q)}{\Diamond\exists y\, yKp \vee \Diamond\exists y\, yKq}.$$

Of a disjunction known by x (in the laxer sense K) one can only say: either its first disjunct is *knowable*, or its second disjunct is. And what is meant here by a propositon's being *knowable* is just that it be possible for *someone or other* (not necessarily x himself) to know the proposition in question.

5 Different construals of doubt

'When in doubt, give *Not Out*.' Cricket is the nursery for criminal law. But what kind of doubt is at issue here? Presumably, if one thinks of credences or subjective probabilities, a proposition p is in doubt (for the thinker in question) if $\Pr(p)$ is not high enough. What is 'high enough"? Presumably a threshold such as .7 or .9 or .99, or . . . . Whatever it is, one can assume that a value of .5 is not high enough. This is the value that a rational agent would assign to any proposition on which he was no more confident that it was true rather than that it was false, or *vice versa*.

Let us assume that the umpire is in this state of mind with regard to the proposition N (that the bat nicked the ball). He *heard* no tell-tale 'click' of willow on leather; he *saw* no apparent deflection of the flight of the ball. *Yet* he could see that the ball was awfully close to the bat, and indeed could see no daylight between bat and ball. For the umpire, it could simply go either way.

This ensures that he is indeed in a state of doubt with regard to the claim *LBW*, and with regard to the alternative claim *Caught*. Yet he is in no state of doubt (see the proof above) with regard to the disjunctive claim *either LBW or Caught*.

Given all this, what should be his state of mind regarding the claim *Out* itself? In the proof above, we completed each case-proof by inferring the broad claim *Out* from the respective category-of-dismissal claims *LBW* and *Caught*. But perhaps one cannot infer *Out* so effortlessly in this way. Perhaps the relevant material (or Sellarsian) inference rules should not be

$$\frac{\mathit{LBW}}{\mathit{Out}} \qquad \text{and} \qquad \frac{\mathit{Caught}}{\mathit{Out}}$$

but rather the following, which require chapter-and-verse for any verdict of *Out*:

$$\frac{\mathit{LBW} \quad \textit{not Caught}}{\mathit{Out}} \qquad \text{and} \qquad \frac{\mathit{Caught} \quad \textit{not LBW}}{\mathit{Out}}$$

If *these* were the rules against the background of which doubt about *Out* were to be assessed, the umpire would be in a state of justifiable doubt about *Out*. This is because *not Caught* is *not* something he is in a position to affirm; nor is *not LBW*.

So the maxim should perhaps be

> Give *not Out* if in doubt as to which exact category-of-dismissal applies.

Thus (so this line of thought continues) the umpire is enjoined to give *Not Out* even when possessed of the little proof above. The problem with that proof was that it invoked the wrong material rules of inference.

But the proof can be repaired. Consider the following:

$$\cfrac{\cfrac{\cfrac{\Delta \quad \overline{N}^{(1)}}{\mathit{Caught}} \quad \cfrac{\Delta \quad \overline{N}^{(1)}}{\neg \mathit{LBW}}}{\mathit{Out}} \qquad \cfrac{\cfrac{\Gamma \quad \overline{\neg N}^{(1)}}{\mathit{LBW}} \quad \cfrac{\Gamma \quad \overline{\neg N}^{(1)}}{\neg \mathit{Caught}}}{\mathit{Out}}}{\mathit{Out}}\,{\scriptstyle(1)}$$

The modified, 'category-specific' rules are employed here. The rub is that any nick of ball on bat rules out *LBW*; whereas the absence of contact between bat and ball rules out *Caught*! So the modified rules afford no progress with the original problem. That problem, as we now appreciate more clearly, is generated by a constructive dilemma whose two horns are statements of matters of fact, not statements about umpiring states of mind. The modified rules would have to be modified even further:

$$\frac{\mathit{LBW} \quad \text{Umpire knows: } \textit{not Caught}}{\mathit{Out}}; \qquad \text{and}$$

$$\frac{\mathit{Caught} \quad \text{Umpire knows: } \textit{not LBW}}{\mathit{Out}}$$

Now, from Δ and N one cannot infer that the umpire *knows that* $\neg LBW$. And from Γ and $\neg N$ one cannot infer that the umpire *knows that* $\neg Caught$.

But can umpiring states of mind enter constitutively into the determination of category-of-dismissal (the making-it-the-case-that the batsman was out this way, rather than that)?

This is really a matter for philosophizing cricketers and cricketing philosophers to decide.[7] We create our categories by means of our rules. There is no mind-independent category of dismissal 'out there' to be determined by the umpire-as-discoverer. Rather, the category of dismissal depends, at least in part, on norm-governed states of mind of the thinker whose job it is to apply the categories in question. The umpire does not make the rules (even though the umpire is *always right*).[8] Nor, however, does the umpire 'merely' apply them to a realm of mind-independent and fully determinable facts-of-the-matter. Rather, it would appear that one could make a strong case for the view that the rules require that umpiring states of mind enter constitutively into the determination of the category-of-dismissal to which a cricketing event is to be assigned.

This will be a verdict accepted only under protest by metaphysical realists, among whom are all Australian philosophers, as Devitt [1996], at p. x seems to imply:

> I have always been a realist about the external world. Such realism is common in Australia. Some say that Australian philosophers are born realists. I prefer to attribute our realism to nurture rather than nature. David Armstrong has suggested (lightly) that the strong sunlight and harsh brown landscape of Australia force reality upon us. In contrast, the mists and gentle green landscape of Europe weaken the grip on reality.

This is hard to reconcile with professional cricketers from downunder spending the antipodean winter in the gentler, greener landscapes of England and claiming to be Kanty cricketers. Perhaps *they* are reconciled to the anti-realist conclusion reached above, and Australian philosophers like Devitt need to bring their philosophizing about X in line with the actual practice of X-ers. The resolution of the Umpire's Dilemma, if the rules do indeed demand determinate categories of dismissal to be assigned, is to give *not Out*.[9]

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[7] With Paul Grice and Jack Smart having passed on, one wonders who might step up to follow that opening partnership in pursuit of such investigations.

[8] Another deep and dignified lesson for the bewildered American reader, who lives in the land of instant replays, John McEnroe tantrums, CNN moms and dads, referee-bashers, and constant readiness for other forms of revolutionary dissent.

[9] Thanks are owed to Simon Beck for raising my awareness of the Umpire's Dilemma.

References

Michael Devitt. *Realism and Truth, 2nd edition*. Princeton University Press, Princeton, NJ, 1996.

Colin Radford. The Umpire's Dilemma. *Analysis*, 45:109–111, 1985.

Alan Weir. *Truth through Proof: A Formalist Foundation for Mathematics*. Clarendon Press, Oxford, 2010.