

turn many of the English words I use, such as ‘accomplishment’, into terms of art before they become satisfactory names for these values. The project might then go on to establish the facts about what agents are like and how societies work, some of which beliefs will also be fairly reliable. These reliable beliefs will give us what the critical exercise needs: *a realistic conception of what shapes, and must shape, ethical norms and how norms work in society*. Only with this picture have we got the resources for effective criticism of our ethical beliefs. Every suggestion of a way to reflect critically on ethics employs some such picture, consciously or not. Price’s own modest ‘internal and localised’ modes of criticism do. My hunch is that Price thinks that norms arise from certain sensitivities that combine both understanding and sentiment, and that our norms are put under critical pressure when we refine these sensibilities. There is very much in that suggestion with which I sympathise. But most versions of it seem to me to exaggerate the role that sensitivity to features of situations can play. These versions seem to me yet another way of oversimplifying ethical life; they underplay, for instance, the social legislative role of moral norms, the messy practical considerations that enter to give them determinate shape. But that is not the important point. What is important is that we cannot know what the best critical procedure in ethics is—for example, what role sensitivity to features of situations play, indeed how much part purely ethical considerations play and what part social compromise and concessions to human shortcomings must also play—until we know fairly fully what forces shape moral norms. We have to know what ethics can do and is doing before we can usefully assess how well it is doing it now.

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## CRITICAL NOTICE

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### BEYOND THE LIMITS OF THOUGHT

#### *1. Summary and general evaluation*

Graham Priest has written a book that is clear and contentious, artful and arresting (*Beyond the Limits of Thought*. Cambridge University Press, 1994. xv + 274 pp. £35.00). It presents an account of contradictions that appear to arise at various ‘limits’: the limits of expression, the limits of iteration, the limits of cognition, and the limits of conception. It ventures a “precise structural characterisation of the limits in question, in the shape of the Inclosure Schema. . . The thesis of the book is that such limits are dialethic; that is, they are the subject, or locus, of true contradictions” (p. 3).

Priest is well-known as a champion of dialetheism, the view that there are true contradictions—and that logic accordingly has to be revamped (somewhat!). It would be a mistake, however, to look to the present book for a detailed defence of dialetheism, for that is not what is on offer. The reader whose hackles would rise at the merest suggestion that the principle of non-contradiction might be wrong will nevertheless benefit from, and follow easily, Priest's entire discussion of historical arguments and their logical analyses. For this is an attempt to distil the gravamen of various important arguments throughout the history of philosophy. It is a work of synoptic taxonomy, or logical cladistics; the dialetheism can easily be backgrounded as one reads along, if one disagrees with it. The occasional invocation of dialetheic doctrine functions, at worst, as a 'here endeth the present lesson'; at best, as a spur to discerning, if one will, a different lesson for the case in hand than the one advanced by the author.

Priest does not state the Inclosure Schema right away, but does so only after several examples of arguments that supposedly conform to it. Because of that order of presentation, however, it was not clear to me that every one of the arguments could, with profit, be seen as exhibiting all the details of the Inclosure Schema as eventually stated. This general misgiving will be illustrated in § 2.2 below with reference to the Liar.

The examples are drawn with considerable historical sweep, and Priest is to be commended for his lucid presentations of just enough historical and textual details in each case. (Indeed, some of the summaries of doctrine, even when *en passant*—such as his coverage of Kant's categories, and the four Antinomies; or Hegel's obscure theory of the true infinite; or Derrida's even more obscure theory of presence and *différance*—are models of exposition within the self-imposed limits of space.) Priest takes each example and deftly works it into his master pattern—a sub-argument for what he calls Transcendence, and a sub-argument for its contradictory, which he calls Closure. By the end of such a survey, with its clarion refrains, even a beginning student would have internalised the philosopher's standard technique of self-reflection in pursuit of contradiction. Formal details are introduced only as needed, and in appropriate measure. The author provides helpful tables to summarise the identifying features of the paradoxes and to enable their comparison according to the general scheme he proposes.

Priest organises his various 'paradoxes of the limit' into four main groups, corresponding to the respective limits of expression, iteration, cognition and conception. Sometimes it is a little unclear to which group a given paradox exclusively belongs; and Priest also shows in some cases how failed attempts to avoid a paradox from one of these groups will transmute it into a paradox from one of the others. Only the paradoxes from pre-Kantian philosophy are expressly assigned, in Part I, to any of the four groups. Kant's antinomies (treated in Part II), and the paradoxes of set-theoretic infinities and of self-reference (treated in Part III), are handled separately. Some of the latter, it would appear, could be assigned to one, perhaps more, of the four main groups. For example, the Cantor–Burali-Forti paradox of the collection of all ordinals strikes me not only as a paradox of the limit of iteration, but also

as a paradox of the limit of conception, or of expression. Likewise, Weyl's 'heterological' paradox could be viewed as a paradox of the limit of cognition, or of conception, or of expression. It would be useful to have some way of determining the 'primary' group membership in such cases.

Explicitly included by the author under the heading of paradoxes of the limits of *expression* are: Cratylus's strong version of the doctrine of flux, Aristotle's doctrine of prime matter, and Cusanus on the incomprehensibility of god. Among paradoxes of the limits of *iteration* he includes Aristotle's denial of the completed infinite and Leibniz's repair of Aquinas's Cosmological Argument. (I would also include here Priest's 'Fifth Antinomy'.) Among the paradoxes of the limits of *cognition* he counts Pyrrhonian scepticism, Socrates's refutation of Protagorean relativism, and the Liar and Knower Paradoxes. Paradoxes of the limits of *conception* include Anselm on the inconceivability of God; Berkeley's master argument for idealism; Koenig's paradox of the collection of all definable ordinals; Berry's paradox of the set of all natural numbers definable in fewer than  $n$  words; Richard's paradox of the set of definable real numbers in  $[0,1]$ ; and (I would think) Kant's difficulties in making noumena clear.

However one wishes to classify the remaining set-theoretic paradoxes, the ones that Priest covers and that I have not mentioned so far are the Mirimanoff paradox of the collection of all well-founded sets; Cantor's paradox of the power-set of the universe of all pure sets; and Russell's paradox of the set of all non-self-membered sets.

This is an impressive and formidable list, to say the least, and Priest is to be commended for his attempt to bring abstract order into the variety. More of that in due course. (Interestingly, Priest sets the Curry paradoxes aside in cases where negation cannot be equated with implication of absurdity. They do not fit into his scheme. See 11.8.)

The discussion of Hegel in Chapter 7 does not reveal any new arguments displaying the master pattern, but is devoted instead to showing how Hegel explicitly reconciled himself to the idea that certain objects—especially the true infinite—were inherently contradictory. "Since there are perfectly sound (according to Hegel) arguments to the effect that the World (that is, the infinity generated in each Antinomy) has contradictory properties, it does have contradictory properties" (p. 114). "Hegel thinks that the Kantian Antinomies are but the tip of an iceberg. *All* our concepts, and not just the generated infinities of the Antinomies, are embroiled in antinomic arguments. These are the arguments which drive forward our thinking from one category to the next, and so generate the dialectical progression of categories in the *Logic*" (p. 115).

Let us now take a more detailed look at Priest's rendering of some of the historical arguments. My selective quotations will, of course, continue to cut longer stories much shorter. I choose one example from each of Priest's four groups: Aristotle's doctrine of prime matter (expression); Leibniz's repair of Aquinas's Cosmological Argument (iteration); Socrates's refutation of Protagorean relativism (cognition); and Berkeley's master argument for idealism (conception). This last example will receive the longest description.

*Aristotle on prime matter*

[P]rime matter . . . is the ultimate bearer of all properties. . . . [P]rime matter, residing at the lowest level, is not itself enformed matter of a lower level. It therefore has no form, only accidents. . . . It can be anything, but is, in itself, nothing. . . . [T]o say what a substance *is* (as opposed to citing some of its properties) is to give its form or essence. Thus, since [prime matter] has no form, one cannot say what it is. . . . Prime matter is, therefore, precisely something beyond the limit of the expressible (Transcendence). But we can happily give its essence; we have already done so in describing it (Closure). (pp. 19–20).

*Leibniz's repair of Aquinas' Cosmological Argument*

One formulation of Leibniz's Principle of Sufficient Reason is 'there is no truth for which a reason does not subsist.' A novel phenomenon, says Priest, then emerges:

Consider the sequence generated by repeated applications of the generator 'cause of', and let this sequence be  $\sigma$ . . . (It can have no last member. . .) By the PSR, we can apply the operator to this to produce a reason for  $\sigma$ . Assuming that nothing can be a reason for itself, [fin] this reason cannot be a member of  $\sigma$  (Transcendence). But the cause of  $\sigma$  is exactly one of the things generated by applying the operator in the prescribed fashion. Hence it is in  $\sigma$  (Closure); and hence we have a contradiction at the limit of the iterable. (p. 41)

*Socrates's refutation of Protagorean relativism*

Protagorean relativism is committed to the claim that nothing is (objectively) true, i.e., the instance of the Cognition Scheme  $\forall x(x \in \Sigma \rightarrow \neg Tx)$ , where  $\Sigma$  is the set of all statements and  $T$  is a non-relative truth-predicate. Let this sentence be  $\phi$ . Then, by the usual argument, it follows that  $\neg T\langle\phi\rangle$ . Hence, Protagoras' own views are not true (Transcendence). But Protagoras has given arguments for his view. If these arguments worked, they would establish that  $\phi$  is true (Closure). (p. 56)

*Berkeley's master argument for idealism*

[A]ll things (except, possibly, minds themselves) are *essentially* thought. It is not just that there *is* nothing that is not thought; there *could be* nothing. . . . I will call the [weaker] claim that everything is thought of, 'Berkeley's Thesis'. . . . [O]ne argument . . . on which, he says, he is prepared to rest everything . . . is given about three quarters of the way through the first of the *Three Dialogues Between Hylas and Philonous*. (pp. 65–6)

Priest proceeds (pp. 66–7) to give a lengthy analysis of the logical structure of this argument. The essentials are as follows. Where  $c$  is an (intentional) object, let  $\tau(c)$  mean ‘ $c$  is conceived’; where  $\phi$  is a proposition or possible state of affairs, let  $T(\phi)$  mean ‘it is conceived that  $\phi$ ’. (I shall be restoring some parentheses that Priest suppresses.) Berkeley’s Thesis is  $\neg\exists x\neg\tau(x)$ . Hylas claims that he conceives that there is something that is not conceived, whence:

$$T(\exists x\neg\tau(x))$$

Philonous attempts to reduce this to absurdity, showing that his opponents cannot even conceive their own thesis.

Now we can help ourselves to what Priest calls the Conception Schema. He states it as an implication, but I shall state it as a rule of inference:

$$\frac{T(\phi(c))}{\tau(c)} \text{ (CS)}$$

We also have Hilbert’s (first)  $\epsilon$ -axiom, which is expressible as the rule of inference (H):

$$\frac{\exists x\phi(x)}{\phi(\epsilon x\phi(x))} \text{ (H)}$$

Priest assumes also that propositional conception ‘prefixes’ logical consequence; which can be stated as the following rule (PC) of natural deduction:

$$\frac{\begin{array}{c} \text{---(i)} \\ \alpha \\ \vdots \\ T(\alpha) \end{array}}{T(\beta)} \beta_{(i)} \text{ (PC)} \quad \text{where } \alpha \text{ is the only assumption on which } \beta \text{ depends}$$

Now take  $\neg\tau(x)$  for  $\phi(x)$ . Consider the following proof of absurdity using the rules set out so far, where in the application of (CS) the term  $\epsilon x\neg\tau(x)$  is substituted for the place-holder  $c$ :

$$\frac{\frac{\frac{T(\exists x\neg\tau(x))}{T(\neg\tau(\epsilon x\neg\tau(x)))} \text{ (CS)} \quad \frac{\frac{\frac{\frac{\exists x\neg\tau(x)}{\neg\tau(\epsilon x\neg\tau(x))} \text{ (H)} \quad \frac{\exists x\neg\tau(x)}{\neg\tau(\epsilon x\neg\tau(x))} \text{ (H)}}{\neg\tau(\epsilon x\neg\tau(x))} \text{ (PC)} \quad \frac{\exists x\neg\tau(x)}{\neg\tau(\epsilon x\neg\tau(x))} \text{ (H)}}{\tau(\epsilon x\neg\tau(x))} \text{ (CS)} \quad \frac{\exists x\neg\tau(x)}{\neg\tau(\epsilon x\neg\tau(x))} \text{ (H)}}{\tau(\epsilon x\neg\tau(x))} \text{ (CS)} \quad \frac{\exists x\neg\tau(x)}{\neg\tau(\epsilon x\neg\tau(x))} \text{ (H)}}{\perp}$$

This “demonstrates that a contradiction follows from the claim that something

is not conceived  $[\exists x \neg \tau(x)]$  (and that this itself is conceived  $[T(\exists x \neg \tau(x))]$ ” (p. 71). Priest thinks that Berkeley has not yet won the day, however; and his criticism, in the natural deduction context, is that not all the inferences of the *reductio* argument just given are *dialetheically* sound. First, Priest gives various interpretations under which both premises are true. Then, on p. 73, he argues as follows, resting his case clearly on dialethic doctrine:

Having established the truth of the premisses, let us turn to the relevance of this fact. This is that a *reductio* is illegitimate in the context of the argument. If the premisses of the argument are true, so is the conclusion; but *if the contradiction entailed by certain assumptions is true, this entailment cannot be used to reduce those assumptions to absurdity.* (p. 73—my emphasis). . . premisses that are true entail a contradiction, which is also, therefore, true. (p. 74)

In § 4.8, Priest proceeds to dispose of all objections to the effect that one of the extra axioms (or extra inferences, in my presentation) must be defective. So clearly Priest himself trusts these extra principles.

. . . Berkeley’s *reductio*, despite his attempts to defuse its conclusion, establishes a contradiction. In fact, as should be clear, it is a contradiction typical of a limit of thought. As one half of the (repaired) argument demonstrates, . . .  $\exists x \neg \tau(x)$  . . . is not in the class of conceived (conceivable) things (Transcendence); but, as the other half shows, it *is* in the class of conceived (conceivable) things (Closure). (p. 76)

Priest boils the *reductio* down even further, to what he calls the *Core Argument*, whose modal interpretation he calls *Berkeley’s Paradox*. In the inferential version the Core Argument is as follows:

$$\frac{\tau(\exists x \neg \tau(x)) \quad \frac{\exists x \neg \tau(x)}{\neg \tau(\exists x \neg \tau(x))} (H)}{\perp}$$

Priest observes (p. 76): “The Core. . . does not employ the Conception Schema, Prefixing or even the operator T. Moreover, the extra premiss is true, or at least can be made true. One *can* conceive of an object that is not conceived (inconceivable), in the appropriate sense; I do, and so can you.”

I shall postpone to § 2.3 below my criticisms of this analysis of Berkeley’s Paradox. This ends my presentation of Priest’s examples from each of his four main groups. There are several other illuminating discussions which my own non-paradoxical limits of space prevent me from sampling.

Part 4 of the book, ‘Language and its limits’, contains some deft exposition and discussion of the doctrines about language from Frege, Wittgenstein, Quine, Davidson and Derrida. The treatment is much more informal, but contains attempts to fit their various quandaries into the Transcendence–Closure pattern that is now familiar. More on that in § 2.5 below.

## 2. Criticism

### 2.1 *The Inclosure Schema, and its associated reductio proof*

The Inclosure Schema (pp. 147, 172) involves a set  $\Omega$ , two properties  $\phi$  and  $\psi$ , and a function  $\delta$ , satisfying the following conditions:

- |  |               |
|--|---------------|
| (1) $\Omega = \{y   \phi(y)\}$ exists and $\psi(\Omega)$ | Existence     |
| (2) if $x \subseteq \Omega$ and $\psi(x)$ then:          |               |
| (a) $\delta(x) \notin x$                                 | Transcendence |
| (b) $\delta(x) \in \Omega$                               | Closure       |

$\delta$  is the diagonaliser, or bound-breaker (Transcendence): “the notion of diagonalisation provided the corner-stone of an adequate understanding of boundary-transcendence” (p. 140).  $\Omega$  is the totality that we obtain from our totalising tendency in thinking (Existence), which Priest dignifies (and rather dogmatically endorses) as The Domain Principle: for every potential infinity there is a corresponding actual infinity (p. 137). The trouble is that  $\Omega$  is so totalising that it catches all images under  $\delta$  (Closure). The crunch comes when one substitutes  $\Omega$  for  $x$  in (2). The statement that is both proved and refuted for the contradiction is  $\delta(\Omega) \in \Omega$ . The proof part is Closure; the refutation part, Transcendence.

But for there to be proof of  $\delta(\Omega) \in \Omega$ , the object  $\delta(\Omega)$  must exist. That it does indeed exist is presumably guaranteed by the existence of  $\Omega$  itself. But this needs to be made explicit. Thus, at the very least, we need the additional principle ‘If  $\Omega$  exists, then  $\delta(\Omega)$  exists’.

The Inclosure Schema already trafficks in set membership, so for any reasoning involving its constituents we need at the very least some principles from what I call the existentially non-committal ‘pure logic of sets’.<sup>1</sup> This is a free logic, in which we do not commit ourselves to the existence of denotations for each and every term. Set-terms can be formed from formulae with  $x$  free by means of the variable-binding operator  $\{x | . . . x . . . \}$ . The principles from the pure logic of sets to which we would have to appeal are the following:

- (1) Any member of the set of all  $\phi$ s has property  $\phi$ ; and
- (2) If the set of all  $\phi$ s exists and  $u$  exists and has property  $\phi$ , then  $u$  is a member of the set of all  $\phi$ s.

The proof that precipitates absurdity from the Inclosure Schema will use all the principles mentioned. It tells any normal philosopher-logician that something is wrong with one of the axioms or rules used. Either one of the axioms is false, or an application of one of the rules is unsound. The quietist

1. See Ch. 7 of N. Tennant, *Natural Logic* (Edinburgh University Press, 1978; 2nd, revised edn. 1990).

response—that of the dialetheist—is to stand by all the axioms and rules and to accept the absurdity with equanimity. My own view—one probably shared by many—is that this is unsatisfactory. For we could avoid absurdity by demurring at *any one* of the axioms or rule applications (or, for overkill, any combination of more than one of them).

Of all the possible objections, by far the most telling is that we have no right, in general, to assume that  $\Omega$  exists. If the axiomatic status of this existential assumption is denied, and the claim in question is turned into an assumption for the sake of argument, then of course the *reductio* proof resulting from the Inclosure Schema (and the ancillary principles we have supplied) establishes the non-existence of  $\Omega$ . This is such a time-honoured response to the paradoxes of the absolute infinite in the early history of set theory that it is remarkable to find no sustained argument from Priest for why this response really will not do. (To be fair, he does refer the reader elsewhere for his case for naive set theory—to *In Contradiction* (Kluwer, 1987).)

Priest does not anticipate (nor, therefore, dispose of) the following objection. Casting all paradoxes into a form requiring the existence of some totality  $\Omega$  is dialethic cajolery, a thinly-disguised attempt to make the notion of ‘contradictory objects’ more acceptable. Many of the paradoxes can be presented in their ‘synthetic’ form, exploiting only the predicates that Priest would have one use, illicitly, to form the dubious ‘totalities’. Thus the most valuable analysis still eludes one: an analysis that would reveal the essence of the arguments in the ‘synthetic’ cases, without any detour through this ontological excrement.

It is not enough to give an honorific title like ‘The Domain Principle’ to a tendentious piece of naiveté. Right at the outset Priest announces that “there is a totality (of all things expressible, describable, etc.) and an appropriate operation that generates an object that is both within and without the totality. I will call these situations *Closure* and *Transcendence*, respectively” (p. 4). It is at this very point that the question of the existence of the totality should be raised.

The lessons of the various paradoxes are, surely, that

- (1) some ‘totalities’ cannot exist—in particular nothing contains everything;
- (2) no language can express every possible proposition;
- (3) no theory can contain every truth.

Nowhere does Priest give these non-dialethic responses the attention they deserve. He does, to be sure, have some very instructive things to say by way of criticism of Russell’s (9.6–9.8) and Ramsey’s (10.5–10.7) responses to the logical and set-theoretic paradoxes; but even these two writers had not seen their way fully to the attitude of chastened resignation registered by (i)–(iii).

## 2.2 *Objections to Priest’s diagnosis of the paradoxes*

An important part of Priest’s treatment is the new classification he offers (in Ch. 10) of all the paradoxes under review. He criticises Ramsey’s well-known

and widely-adopted distinction between logico-mathematical paradoxes (Ramsey's Group A) and linguistic or semantic paradoxes (Ramsey's Group B):

Ramsey's criterion has the flimsiest basis. For a start, if one wants to draw a fundamental distinction, this ought to be done in terms of the *structure* of the different paradoxes. Ramsey's distinction depends on the relatively superficial fact of what vocabulary is used in the paradoxes, and, in particular, whether this belongs to mathematics properly so called. But worse, this is a notoriously shifting boundary. (p. 156)

Priest complains that there would seem to be no place, within either one of Ramsey's two groups, for what Priest calls the 'Fifth Antinomy' or for Berkeley's Paradox. For this complaint to stick, of course, one would need to be convinced that (a) the 'Fifth Antinomy' was not simply the Burali-Forti paradox in another guise; and (b) that Berkeley's Paradox survives the criticisms of it to be given in § 2.3 below. Ramsey was, after all, concerned only with what he described as "these contradictions [the paradoxes of *self-reference*]" [my emphasis], and one could question whether either the Fifth Antinomy or Berkeley's Paradox is genuinely self-referential. In his description of the Fifth Antinomy Priest expressly says "we never produce a self-referential thought in this process" (p. 111), the process being that of iterating the operator 'the thought of', applied initially to any object. What makes it virtually isomorphic to the Burali-Forti paradox is that "when we have an unbounded sequence of thoughts we next produce the thought of all of them, and keep going" (*loc. cit.*), which is just like the formation of a limit ordinal. And whereas a successor ordinal is the set of all preceding ordinals, a successor thought, in Priest's sequence, is just the thought of the immediate predecessor.

The criticism of Ramsey's two groups as not collectively exhaustive is therefore not absolutely conclusive. What, then, of the criticism that, if distinctions are to be drawn at all, then Ramsey has drawn too few, and indeed drawn his single distinction in the wrong place? Ramsey's Group A survives in Priest's re-classification as the group of paradoxes of the limit of iteration (p. 157). Ramsey's Group B, Priest thinks, can usefully be subdivided into two sub-groups: paradoxes of the limit of conception (definition); and paradoxes of the limit of cognition.

Now for the Liar (which is one of the paradoxes of the limit of cognition) to manifest the structure called for in the Inclosure Schema, certain conceptual extensions have to be prosecuted, which one might wish to challenge along the line already indicated above. For the Inclosure Schema, remember, deals with the set-theoretic notion of a totality ( $\Omega$ ) and membership thereof. Yet the usual presentations of the Liar are utterly innocent of any such set-theoretic notions. One can appreciate its paradoxicality without ever having to invoke or commit oneself to any such thing as 'the set Tr of all true sentences'. Priest parries this objection (p. 158) by claiming that

there is an analytic connection between satisfying a condition and being in a set: something satisfies a condition iff it is in the set of things satisfying

that condition. [fn] Hence, facts about totalities are implicit in facts about conditions, and vice versa. Using this observation, we can manipulate the paradoxes of [the limit of cognition] into a form in which they fit the Inclosure Schema quite legitimately.

All well and good, provided only that one buys into that easy equivalence between satisfying a property and being in ‘the set’ defined by that property, with no qualifications whatsoever about set existence! Priest relegates to a footnote on the same page the concession that his observation “appeals to the naive notion of set, and would be challenged, for example, by those who subscribe to ZF or some similar form of set theory”. But since one of the major issues raised by the set-theoretic paradoxes is precisely the legitimacy of the naive notion of set, it is questionable, to say the least, to base a systematic classification of *all* the paradoxes on a theoretical presumption to the effect that a deeper investigation of the set-theoretic ones (yet to be carried out) will yield both a certain diagnosis of their origins and a long-suffering resignation to the incontinence of dialetheism for *all* the paradoxes.

Certainly, one can massage the reasoning underlying the Liar into the form of the Inclosure Schema, by (unnecessarily) invoking the existence of the set  $Tr$  of all true sentences; and one might even concede that (since one is dealing with *sentences* of some well-defined, albeit semantically closed, language) the totality  $Tr$  would not be subject to the same existential qualms as, say, the Russell class  $\{x|x \notin x\}$  or the universal class  $V = \{x|x = x\}$ . But the derivation of absurdity in the case of the Liar certainly doesn’t *have* to proceed in that fashion. Priest’s commendable insight that the classification “ought to be done in terms of the *structure* of the different paradoxes” should be focused more on what reasoning is *actually* carried out, and what reasoning will suffice in a *minimal way* for the precipitation of absurdity.

Priest says (p. 149): “We want not just any old pattern, but the essential pattern.” But to intrude the set  $Tr$  of all truths into the scene is to render the logical landscape as a Capability Brown might have done. It introduces an inessential element, a set-theoretic folly, which then plays a role in yielding an appearance of *too much* structure: the structure, that is, of Priest’s Inclosure Schema.

The semi-formal reasoning that Priest sets out on pp. 158–9 to generate the contradiction involved in the Liar paradox turns out to be much more involved than it need be. He uses a parameter  $a$  in order to establish Transcendence in the form  $\sigma(a) \notin a$ ; and Closure in the form of  $\sigma(a) \in Tr$ . Only upon substituting  $Tr$  for the parameter  $a$  do we obtain the contradiction between  $\sigma(Tr) \notin Tr$  and  $\sigma(Tr) \in Tr$ . If, however,  $Tr$  were to be substituted for the parameter  $a$  at the outset, considerable simplification of the proof would result. (The reader will have to take this claim of mine on trust, because the formal displays would take up too much space.) *And this is how the reasoning with the Liar most obviously proceeds*—albeit usually with a truth-predicate instead of membership in  $Tr$ .

The quintessential proof that one obtains by not bothering with the unnecessary parametrisation via  $a$  will tell us a lot more about the nature of

the paradox than will its artificial padding-out with strictly unnecessary devices. Priest indulges in the parametrisation via  $a$ , so it seems, with the sole purpose of making the logical reasoning appear to comport with his Inclosure Schema.

The *real* structure, which, to my mind, is potentially revealing of the genuine nature of the paradoxes, lies elsewhere: it is to be located in the overall shape of the (most direct) *proofs* involved in the precipitation of absurdity. Indeed, the matter is even a little more abstract than that: the structure in question is to be located in the overall shape of the *non-terminating reduction sequences* that result from the attempt to turn such proofs, *per impossibile*, into normal form.<sup>1</sup> Now, what is remarkable about the proof that one would construct from the Inclosure Schema is that it *appears* to be in normal form. But this is because *too much has been schematised*. All the detailed work is masked by the 'rules' of Transcendence and Closure, which actually need to be derived in the case of each paradox, rather than simply assumed (as is clear from Priest's own illustration of the reasoning in the Liar, on p. 158). When the required derivation-work is interpolated so that the steps in the *reductio* proof are honestly *derived*, the abnormality of the resulting proof finally becomes evident. The proof of absurdity from the Inclosure Schema, as it stands, sweeps all the interesting logical detail under the carpet, in pursuit of an overall structure that abstracts too much from the argumentative details, and also interpolates too much that is unnecessary.

The advantage of this proof-theoretic diagnosis is that it provides a criterion also for when we have *solved* and *avoided* a paradox. In a naive system of set theory, for example, the proof of Russell's paradox cannot be normalised. But in the more carefully thought out system of free logic for pure sets, the same reasoning becomes a proof of non-existence, and this proof is in normal form.

The misleading appearance of normality of the proof for the Inclosure Schema thwarts the very insight about paradoxical proof-structure that I am saying goes to the heart of the matter. It is only when one looks at the intrinsic pattern of reasoning in, say, the Liar paradox (and not its massaged version conforming to the Inclosure Schema) that it becomes clear that the reasoning involved is *not* in normal form, and indeed *cannot be brought into* normal form. This makes the 'contradiction' involved in the Liar quite unlike that involved in straightforwardly inconsistent sets of assumptions in a semantically open language. For the proofs of absurdity in the latter case can be brought into normal form.

### 2.3 Is Berkeley's Paradox really a paradox?

Consider the proof given above for Berkeley's Paradox. It would have to be the application of (H) (or, in Priest's version, his use of the Hilbert axiom  $\exists x \neg \tau(x) \rightarrow \neg \tau(c)$ ), or the assumed truth of  $\tau(\exists x \neg \tau(x))$ , for which the

1. See my papers 'Proof and Paradox', *Dialectica*, vol. xxxvi (1982), pp. 265–296; and 'On Paradox without Self-Reference', *Analysis*, vol. lv (1995), pp. 199–207.

objector would have to raise problems if he has no stomach for the paradox. Surely there is something sophisticated in saying, as Priest does, that one can conceive of an object that is inconceivable? My own view is that it is not enough simply to form the  $\epsilon$ -term ‘ $\epsilon x(x \text{ is not conceivable})$ ’ and then to claim that this term refers to something—which, hence, is conceivable (presumably as (that?) something-or-other that is not conceivable). In the weak sense of conceivability that Priest had earlier essayed in response to objections to the effect that the CS fails if understood *de dicto*, he had written (p. 75), “To conceive of something . . . it is necessary [and sufficient?—N.T.] only to bring before the mind and understand a noun-phrase (or other representation) which, *as a matter of fact*, refers to (or represents) *the object in question*” [my emphases]. He goes on to claim that ‘ $\epsilon x \neg \tau x$ ’ certainly refers to something, hence that he is conceiving of the thing.

All this strikes me as extremely tenuous and dubious. I would draw the line after definite descriptions, and disallow the indefinitely descriptive epsilon terms. I would be prepared to agree that

to conceive of something it is sufficient to bring before the mind and understand a *definitely denoting* phrase which, as a matter of fact, refers to the object in question

but I would *not* agree that

to conceive of something it is sufficient to bring before the mind and understand an *indefinitely* ‘denoting’ phrase which ‘refers’ to the object in question

precisely because the notion of reference here would be so tendentious. There *would be no* ‘matter of fact’ as to whether an epsilon term  $\epsilon xF(x)$  referred to *this* F or to *that* F, in cases where there were more than one F. Nor is there any way for Priest to insist that F could be so chosen as to have a unique satisfier, in order to avoid this objection. For, if there were an inconceivable thing, then there would be at least one other such thing—for example, the set that contains that inconceivable thing as its sole member. Hilbert’s epsilon terms are just too fishy, semantically, to support such weighty logical and metaphysical conclusions as Priest would have us draw. I would sooner say that I had *failed* to conceive of any inconceivable thing via my understanding of the legitimate logical use to be made of the epsilon term ‘ $\epsilon x(x \text{ is inconceivable})$ ’, than say that I *had* thereby conceived of an inconceivable thing, but at Priest’s price of buying into dialethic logic and accepting the truth of the ensuing contradiction.

#### 2.4 *Objections to dialetheism*

What about Priest’s dialetheism? He maintains that “orthodox logic, however well entrenched, is just a *theory* of how logical particles, like negation, work; and there is no guarantee that it is correct” (p. 4). Just a ‘theory’, with no

special commendation of its epistemic status? Or ‘just’ a theory—as though, simply by virtue of being a theory, a set of statements cannot be a priori, or even merely true? An appropriate *tu quoque* here would be to look more closely at the motivating intuition to which the reader is treated for dialetheism:

Given two states of affairs, there are, in general, four possibilities: one but not the other holds, vice versa, both or neither. In particular, given the states of affairs  $\alpha$  is true and  $\alpha$  is false, there are, without begging any questions, those four possibilities. The standard logical theory just *assumes* that only two of them should be allowed for. . . Suppose, then, that we allow sentences to take as semantic values one of the corresponding four *subsets* of the set  $\{\text{true}, \text{false}\}$ . (pp. 4–5)

I shall come back to this in a moment. Priest goes on to say

it is worth asking *why* dialetheism should be so outrageous to the sensibility of modern philosophers. The answer is, I am afraid, sociological rather than rational. . . . I know of no historical defence of the law of non-contradiction since Aristotle worth mentioning. Is this because Aristotle’s arguments were conclusive? Hardly. Arguably they do not work at all. (p. 5)

I for one do find dialetheism outrageous, and am grateful for an opportunity to say why. Consider first the casual initial claim

Given two states of affairs, there are, in general, four possibilities: one but not the other holds, vice versa, both or neither.

This is simply not true. Consider the following two states of affairs concerning the present moment:

- (R) my field of vision is red all over
- (G) my field of vision is green all over.

There are only three possibilities:

- (1) my field of vision is red all over (hence not green all over);
- (2) my field of vision is green all over (hence not red all over);
- (3) it is neither the case that my field of vision is red all over nor the case that my field of vision is green all over.

(3) could arise by virtue of my field of vision containing some colour other than red or green. (Indeed, (3) is almost always the case.)

Why are there not four possibilities, as Priest maintains? Why is there no possibility that he calls ‘both’: my field of vision is red all over *and* my field of vision is green all over? The answer is obvious. (R) and (G) are *contraries*: that is, they cannot both be true. Other pairs similar to (R) and (G) can easily be found. For example, the pair

- (H) I am here beside you right now
- (T) I am way over there, far away from you, right now

would do just as well. The metaphysical impossibility of being in two places at the same time makes (H) and (T) contraries. Similarly for the mathematically mutually exclusive

- (S) This figure is a square
- (T) This figure is a triangle

or the antonymous pair

- (S1) He is short for an Armenian
- (T1) He is tall for an Armenian.

All these pairs allow for only three compound possibilities. But some pairs allow for *only* one compound possibility! Take, for example,

- (S2) If Socrates is mortal then Socrates is mortal
- (T2) If Tully is mortal then Tully is mortal.

There is only one compound possibility here: the one in which *both* hold. The reduced degree of freedom is owing to these propositions' necessity (or non-contingency).

What Priest *should* have said is

Given two *mutually independent and contingent* states of affairs, there are, in general, four possibilities: one but not the other holds, vice versa, both or neither.

Denying the law of non-contradiction (the logical law  $\neg (P \ \& \ \neg P)$ ) seems to me a desperate, meaning-violating move. It is desperate because there are surely better responses to the paradoxes—ones that do not force us to reconcile ourselves to the existence of “truly contradictory objects” (p. 186). It is meaning-violating because of what negation means. One proves a negation  $\neg A$  by showing that the assumption of (the truth of)  $A$  is contrary to (the assumed truth of) whatever other premises  $X$  one wishes to hold on to:

$$\frac{X, A \quad \overline{\quad}^{(i)}}{\vdots} \quad \perp \quad \overline{\quad}^{(i)} \quad \neg A$$

Thus (I would argue)<sup>1</sup> the notion of negation actually rests on the more fundamental notion of *necessary contrariety*, or *impossibility of joint truth* of at least two different propositions. For the notion of negation to have entered the language, it would have sufficed, on this view, for the language to have *contrary* atomic propositions. Such propositions B, C would have admitted a primitive, atomic inference of the form

$$\frac{B \quad C}{\perp}$$

Pairs of propositions like this abound: colour exclusions, conflicting location reports, conflicting shape ascriptions, singular predications (concerning the same thing, stuff or situation) of survival-relevant antonyms such as tame/fierce; nourishing/poisonous; bright/dark; light/heavy; big/small; hot/cold; dry/wet; sharp/blunt; smooth/rough; hard/soft; deep/shallow; far/near; friend/foe; male/female; young/old; noisy/quiet; good/bad; clever/stupid. . . . These multiply *non-independent* atomic propositions would form the soil in which negation, hence explicit denial, could subsequently flourish: from B one could infer  $\neg C$ , and from C one could infer  $\neg B$ . It would then be analytic that it is impossible for both P and  $\neg P$  to be true.

Wittgenstein did a great disservice to the philosophy of language when, in the *Tractatus*, he insisted that all atomic propositions be logically independent of one another. This confused the use of atomic schematic letters for logical purposes (as mathematically independent variables over the set of truth-values) with the actual functioning of those sentences of a natural language that have minimal logico-grammatical structure. The latter are locked into all manner of antonymous oppositions, and indeed need to be; for their evolutionary function must have been to mark salient extremes that were survival-relevant.

Consider also the havoc that can be wrought by iterating Priest's assurance that the spectrum of compound possibilities is always replete. At the first stage, from two states of affairs, Priest happily generates  $2^2 = 4$  compound possibilities. He has not told us what a 'state of affairs' actually is; so why can't each one of these four possibilities be taken in its turn as a state of affair, for further compounding? On Priest's own recipe of unconstrained compounding, there would then be  $2^4 = 16$  compound possibilities: at the next stage, there would be  $2^{16}$  possibilities. In general, at the n-th stage there would be 2 to the 2 to the 2 to the . . . (n times) possibilities. We have heard already of fuzzy logic; this would be dizzy logic.

### 2.5 Frege and subsequent philosophers of language

I resume here the discussion, postponed from the end of § 1, of Priest's attempt to diagnose further occurrences of the Transcendence–Closure pattern

1. See my paper 'Negation, Absurdity and Contrariety', in D. Gabbay and H. Wansing, (eds.), *Negation* (Kluwer Academic Press), forthcoming.

in the thinking of Frege and subsequent major philosophers of language. Here is a sample summary passage, in which I underline the portions with which I would not concur:

Thoughts are articulated. To form them we must combine simpler building-blocks. But thoughts are not mere lists of their components. There must therefore be things which hold them together as unities. Let us call these (with apologies to modern physics) *gluons*. Gluons are not the same kind of thing as the components they glue, and hence not the kind of thing one can express claims about. But anything can be an object of thought; in particular, we can think about gluons. Thus they *are* the same kind of thing as other constituents, and we can express claims about them.

We have, here, a contradiction typical of the limit of thought. It is also not difficult to see that it fits the Inclosure Schema, albeit the limit case where  $\psi(x)$  is ' $x = \Omega$ '. Let  $\phi(y)$  be ' $y$  is expressible (in some language, say English)', so that  $\Omega$  is the set of all states of affairs expressible in English. Take any of the gluons that holds an English proposition together. And let  $\delta(\Omega)$  be a situation concerning that gluon, maybe that it is a gluon of a certain proposition. Then, by the arguments we have had,  $\delta(\Omega)$  both is (Closure) and is not (Transcendence) expressible. (p. 212)

One would have a paradox worth pondering here only if there were no appeal to false assumptions in the course of deriving it.

First of all, Frege would have balked at the invocation of gluons; for Frege, the unsaturatedness of certain expressions meant that they could be saturated by others to form compound expressions. There would be no need for gluons. A name completes, or saturates, a predicate to form a sentence; not the other way round. And the saturation does not call for any gluons.

Secondly, even if there were gluons, the second underlined claim is false for the following reason. (Let us avoid Priest's vacillation between talk of thought and talk of its expression in language; and try to conduct the discussion just in terms of thoughts and their components.) The correct contrast between the components of a thought, and whatever gluons might be involved in that thought, is that the components are things *by means of which* we think (or make judgements), whereas gluons are not such things. Rather, gluons are things *by means of which* we can make judgements *by means of* thought-components. Their intentional instrumentality, as it were, is at one remove. But that does not put them *beyond the reach of thought itself*. Thus I would complain that there is, in fact, no valid argument for Transcendence here, contrary to what Priest claims.

By way of final comment on this 'instance' of the Inclosure Schema, note how stretched is the attempt to force it into the supposed mould.

Let us turn now to Priest's summary of Davidson's predicament:

according to Davidson . . . the notion of truth in some language, L, is not expressible in L. In particular, 'true-in-English' is not expressible in English

(Transcendence). But, of course, it is expressed in English by ‘true-in-English’ (Closure). To say what cannot be expressed, one has to express the very thing. (p. 227)

This strikes me as fair comment; but it would have been useful to have had here a more extensive discussion of the truth-definitions for semantically closed languages available from the work of Woodruff and Martin, Kripke, Gupta and Herzberger. These treatments opt for Closure and defuse Transcendence, by offering analyses of the way in which paradoxical sentences cannot receive a stable assignment of truth-values. So the criticism is the same as in the preceding case: Closure, yes; Transcendence, no.

What about those theorists who maintain the indeterminacy or non-factuality of meaning? In the following quotations the underlinings are mine.

On Quine, Priest has this to say:

even to claim that one cannot refer determinately to objects, presupposes that we can refer to those objects (and not to undetached object parts) to say what it is that we cannot refer to! [fn] Thus we have a contradiction at the limits of expression again. The Indeterminacy of Reference cannot be expressed (Transcendence); yet Quine expresses it (Closure). (p. 221)

Priest’s remarks on Kripke’s Wittgenstein are as follows:

the conclusion that results from the skeptical arguments, and that Wittgenstein wishes us to grasp, is beyond expression (Transcendence). Yet it is possible to express it; I have just done so and so does Kripke (Closure). Hence we have a contradiction at the limit of expression. (p. 235)

On Derrida:

A text, then expresses no intrinsic meaning, but may be taken to mean indefinitely many things. Now apply this observation to Derrida’s own text. We take Derrida to be advocating a certain view, namely, arguing against presence, the determinacy of sense. Yet, if he is right he is not advocating anything with stable and determinate sense at all. What, then, are we supposed to make of what he says if there is nothing *as such* that he says? Or, to put it the other way, given that he does express certain views (those that I have summarised), he is expressing something (Closure) that, if he is right, cannot be expressed (Transcendence). (p. 240)

The right response, in my view, is to reject out of hand the three underlined claims of Transcendence on behalf of Quine, Kripkenstein and Derrida. The arguments for *impossibility of expression* require, as sub-arguments, arguments for the *truth* of the doctrines in question. But the latter arguments are all defective, in that they are not even truth-preserving from their premises to

their conclusions.<sup>1</sup> (Note also that in order to be non-defective it would not be enough for the arguments just to be truth-preserving; their premises would have to be true as well.) And, if they were not inferentially fallacious, then the resulting ‘paradox’ would simply show that one of their ultimate premises must be false! Those who maintain the indeterminacy or non-factuality of meaning would, on Priest’s analysis, surely refute themselves. Hence Priest cannot correctly claim that they have established Transcendence with respect to expressibility.

It would appear, then, despite surface similarities indicating otherwise, that ‘the paradoxes at the limit of expressibility’ discussed in Part 4 do not cause the degree of consternation that the standard ones do.

## 2.6 Concluding remarks

I do not know of any paradox that is strictly classical, in the sense that the associated natural deduction deriving absurdity requires the use of one of the strictly classical laws of negation (classical *reductio ad absurdum*, the law of double negation elimination, the law of excluded middle, or the rule of dilemma). Nor do I know of any paradox (apart from the so-called ‘paradoxes of implication’!) that arises only because of the use, in one’s logic, of *ex falso quodlibet*. Where there is paradox, it confronts both the classicist and the relevant intuitionist alike. I take this as a clue to the fact that the ‘limit’ paradoxes derive from a deeper confusion over certain parts of our ideology or ontology. The burden of Priest’s discussion is to locate the source of these paradoxes in some sort of combination of logic and ontology: in the ‘existence’ of ‘inconsistent objects’. But to my simple mind, things just are, and they bear whatever properties they do without bearing any contrary properties. If we cannot sort out the substance of the world so that this simple-minded attachment of properties is possible, then we had better sort things out differently, rather than opt for new-fangled attachments of contradictory properties to what we mistakenly take still to be out there.

Impressive as is Priest’s attempt to unify our view of paradoxical matters from a dialethic standpoint, I find too distorting the strange metaphysical lenses through which I am being asked to view what I believe is really a world devoid of contradictions (except at the level of thought and language). Just as projective geometry postulates its (unreal) points at infinity in order to achieve a certain overall simplicity and elegance, so too does Priest’s dialetheism postulate unreal and contradictory absolute infinities in order to fit matters paradoxical into a simple and elegant scheme. But, just as the mathematician can be asked to characterise real space (in which there are no points at infinity), so too can the metaphysician be asked to characterise real existence (in which there are no contradictory objects).

*Beyond the Limits of Thought* is a wonderful sermon to the converted; but is written also so as to titillate the heathen, exploring regions where it

1. For a sustained critique of the relevant arguments of Quine and Kripkenstein, see Chapter 4 of my book *The Taming of the True* (Oxford University Press, 1997).

would seem that consistent thought is no longer possible. As an alternative metaphysical travel guide it may even succeed, and rouse some from what Priest thinks is dogmatic slumber—only to enter a state of metaphysical titubation. I confess that I am still deep in the arms of some metaphysical Morpheus. After studying this logical alchemist's manual, I find myself still in a state of denial (which may, of course, be assertion!). Something inside me (perhaps my other self) is still privately challenging the author to show what is wrong with the following position, which incorporates a few of the logical deviations of Priesthood, but does not take the last step into the inner dialetheic sanctum: no proposition is both true and false; not every proposition is either true or false; not every contradiction entails every proposition; if a true disjunction has a false disjunct then its other disjunct is true; any proposition that entails a contradiction is false. On these logical tenets, all the usual paradoxes still arise. But the logical exercise refreshes the mind, enabling it to take a harder look at what really exists and what concepts it is legitimate to deploy. It gives one the strength to deny existence when the assumption of existence leads, logically, to contradiction; and likewise to eschew the use of concepts whose use leads to contradiction.

Despite my disagreement with Priest both on particular points and on overall philosophical outlook, I trust it is clear that I regard his book as a most worthwhile and thought-provoking contribution to metaphysics and the theory of meaning. He packs into it much more rigorous analysis and enlightening exposition than space permits me to discuss.

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## BOOK REVIEWS

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### GENERAL PHILOSOPHY

*Reason, Truth and Self: The post-modern reconditioned*

By MICHAEL LUNTLEY

Routledge, 1995. xvi + 248 pp. £37.50 cloth, £10.99 paper

To those philosophers who cringe at the mere mention of 'postmodernism' this book should come as a pleasant surprise. Michael Luntley proves that it is possible to discuss rationally and critically something called 'postmodernism'. How close Luntley's views of what constitutes postmodernism are to the trendier uses of the term is debatable, the important point is that potentially sceptical readers should not be put off by the title of this book.

The main question the book addresses is how to sustain our belief in the possibility of objectivity, truth and rationality in face of the challenges posed