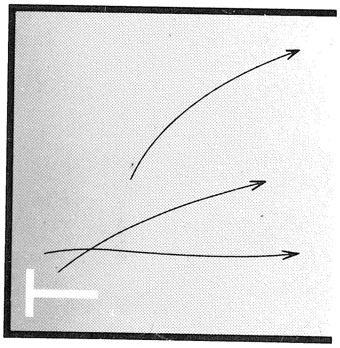
5 INTERNATIONAL CONGRESS of logic, methodology and philosophy of science

CONGRÈS INTERNATIONAL de logique, méthodologie et philosophie des sciences

Contributed Papers
— o —
Mémoires soumis



LONDON, ONTARIO, CANADA

August 27 août —— September 2 septembre

Natural Deduction for Restricted Quantification, Identity and Descriptions.

Neil Tennant, University of Edinburgh.

A universally free logic is one which discards the existence assumptions (I) the universe is non-empty, and (II) every term denotes. I set forth below a system of first order logic which is universally free. It treats all terms (i.e. names, descriptive terms and functional terms) as genuine expressions of the language, and not contextually defined. It also treats restricted quantification. It is a natural deduction system without any axioms, even for identity.

The system results in solution to the problem

(*) Combine an analysis of the logical form of predication according to which terms are genuine expressions of the language and clearly complete predicates to yield sentences, with the basic semantical assumption

(A) A term's failure to denote renders any atomic predication involving it false.

The problem is basically that of providing for scopes of terms within sentences: e.g. "t is not F" versus "It is not the case that t is F". If t does not denote, then by (A) the former is false and the latter true.

The solution offered is this: term-insertion, like quantification, is a variable-binding operation. Just as we use bound variables to show which argument places of predicates are sealed off by which completions of quantifiers, so also we may use them to show which places are sealed off by which completions by terms. Thus "t is F" becomes $t_x Fx$, to be read as "t (exists and) is F". We can now distinguish between $t_x{-}Fx$ and $-t_x Fx$, etc.

As a special case of (A), we have that t=t is false if t does not denote. Thus t=t is tantamount to "t exists". I therefore take x=x as the logical translation of "x is a thing".

For any quantifier Q, "Q F's are G's" has logical form $Qx[Fx]Gx$. Thus "Every F is G" becomes $\forall x[Fx]Gx$ and "Every thing is G" becomes $\forall x[x{=}x]Gx$, thereby restoring to logic an obvious analogy. But, for fluency in deduction, we treat "the" as a term-forming operator on single predicates, rather than as a sentence-forming operator on pairs of predicates. Thus "The F is G" becomes $(\iota xFx)_y Gy$.

The law of self-identity, $\forall x[x{=}x]x{=}x$, is as trivially provable in our system as $p \supset p$. Substitutivity of identicals is the only deductive rule needed to prove the other laws of identity; so first order logic with identity needs no axioms.

Syntax. Terms and wffs are defined by the usual simultaneous recursion; but only variables may occupy argument places after predicate and function letters. Let us say that this definition is of terms and wffs in the primitive sense. If the definition is altered to allow terms in general to occupy argument places, we have a definition of terms and wffs in the wider sense. Henceforth by "term" ("wff") I shall mean term (wff) in the wider sense, unless otherwise indicated.

If a wff has an occurrence of a term t in an argument place, we say the wff is of theform A(t). We define an abbreviation function on terms and wffs thus:

$$@(f(t_1\ldots t_n)) = f(@(t_1)\ldots@(t_n))\ ;\quad @(\iota xA) = \iota x@(A)\ ;$$

$$@(F(t_1\ldots t_n)) = F(@(t_1)\ldots@(t_n))\ ;\quad @(t_xA) = @(A^x_{@(t)})\ ;$$

$$@(A\&B) = @(A)\&@(B)\ ;\quad @(\exists x[A]B) = \exists x[@(A)]@(B)\ ;$$

elsewhere @ is the identity mapping. Thus @(A) is of the form B(@(t)) if and only if some occurrence of t in A is not within the scope of $-$, $\vee$, $\supset$, $\equiv$ or $\forall$ in A. For A in the primitive sense, @(A) is of the form B(@(t)) only if t must denote for A to be true.

Proofs are certain trees of closed wff occurrences. A graphic rule such as

$$\dfrac{A\vee B \quad \begin{matrix}\overline{A}^{(i)}\\ \vdots \\ C\end{matrix} \quad \begin{matrix}\overline{B}^{(i)}\\ \vdots \\ C\end{matrix}}{C}(i)$$

is really shorthand for the corresponding clause in the recursive definition of proof, which in this instance is:

If Π_1, Π_2 and Π_3 are proofs of AvB, of C and of C respectively, depending on Δ_1, Δ_2 and Δ_3 respectively, then $\dfrac{\Pi_1 \ \Pi_2 \ \Pi_3}{C}$ is a proof of C depending on $\Delta_1 \cup (\Delta_2 \setminus \{A\}) \cup (\Delta_3 \setminus \{B\})$.

The basis clause is:

Any occurrence of A is a proof of A depending on $\{A\}$.

The full set of rules is as follows:

$$\frac{A}{@(A)} \quad \frac{@(A)}{A} \quad \frac{A \quad B}{A\&B} \quad \frac{A\&B}{A} \quad \frac{A\&B}{B} \quad \frac{A}{A\vee B} \quad \frac{B}{A\vee B} \quad \frac{A \quad A\supset B}{B} \quad \frac{A \quad A\equiv B}{B}$$

$$\frac{B \quad A\equiv B}{A} \quad \frac{A \quad -A}{*} \quad \frac{*}{A} \quad \frac{A(t)}{t{=}t} \quad \frac{A^x_t \quad B^x_t}{\exists x[A]B} \quad \frac{A^x_t \quad \forall x[A]B}{B^x_t}$$

$$\frac{t{=}u \quad A}{B}\ , \text{ where } A^t_u = B^t_u$$

$$\frac{t{=}t \quad -t_xA}{-A^x_t}\ , \text{ where } t_xA \text{ is a wff in the primitive sense}$$

$$\dfrac{\begin{matrix}\overline{A}^{(i)}\\ \vdots \\ *\end{matrix}}{-B}(i) \quad \dfrac{\begin{matrix}\overline{-B}^{(i)}\\ \vdots \\ *\end{matrix}}{A}(i)\ , \text{ where } A{=}@(B),\ B \text{ primitive}$$

$$\dfrac{\begin{matrix}\overline{A}^{(i)}\\ \vdots \\ B\end{matrix}}{C\supset D}(i) \quad \dfrac{\begin{matrix}\overline{A}^{(i)}\\ \vdots \\ B\end{matrix} \quad \begin{matrix}\overline{B}^{(i)}\\ \vdots \\ A\end{matrix}}{C\equiv D}(i)\ , \text{ where } A{=}@(C),\ C \text{ primitive and } B{=}@(D),\ D \text{ primitive}$$

$$\dfrac{A\vee B \quad \begin{matrix}\overline{A}^{(i)}\\ \vdots \\ C\end{matrix} \quad \begin{matrix}\overline{B}^{(i)}\\ \vdots \\ C\end{matrix}}{C}(i)$$

$$\dfrac{\begin{matrix}\overline{A}^{(i)}\\ \vdots \\ B\end{matrix}}{\forall x[C^a_x]D^a_x}(i)\ , \text{ where } A{=}@(C),\ B{=}@(D),\ C^a_x \text{ and } D^a_x \text{ are primitive, and the name a does not occur in any assumption other than A on which B depends}$$

$$\dfrac{\exists x[A]B \qquad \begin{matrix} \overline{A^x_a}^{\,(i)} \quad \overline{B^x_a}^{\,(i)} \\ \vdots \\ C \end{matrix}}{C}\,(i)$$

, where the name a does not occur in $\exists x[A]B$, in C or in any assumption other than A^x_a and B^x_a on which C (at its upper occurrence) depends.

$$\dfrac{\iota x A = \iota x A}{A^x_{\iota x A}} \qquad\qquad \dfrac{\iota x A = \iota x A \quad A^x_t}{t = \iota x A}$$

$$\dfrac{A^a_t \qquad \begin{matrix} \overline{A}^{\,(i)} \\ \vdots \\ a=t \end{matrix}}{t = \iota x A^a_x}\,(i)$$

, where the name a does not occur in any assumption other than A on which a=t depends.

Let $\triangle \cup \{X\}$ be a set of closed wffs in the primitive sense. Then $\triangle \vdash X =_{df}$ there is a proof of X depending on some subset of $\triangle$.

A theory of truth for the object language above can be provided in a metalanguage obeying the logic above.

<u>Conjecture</u>: The system above is complete with respect to the resulting notion of logical consequence; and admits a generalization of Prawitz's normalization theorem.