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Conventional Necessity and the Contingency of Convention

by Neil Tennant *

Summary

I defend a conventionalist view of logical and (some) mathematical truths against the criticisms of Quine and Stroud. Conventionalism is best formulated by appealing to sense-conferring rules governing important logical and mathematical expressions. Conventional necessity can be understood as arising from these rules in a way that is immune to Quine's and Stroud's criticisms of the earlier formulation of conventionalism, in which stress was incorrectly laid on axiomatic systems of logic.

Résumé

Je soutiens, en dépit des critiques de Quine et de Stroud, une conception conventionnaliste des vérités logiques et de (certaines) vérités mathématiques. La meilleure façon de formuler le conventionnalisme est de faire appel à des règles conférant un sens à d'importantes expressions logiques et mathématiques. La nécessité conventionnelle peut être comprise comme provenant de ces règles par des voies qui ne donnent pas prise aux critiques de Quine et de Stroud à l'égard de l'ancienne formulation du conventionnalisme, dans laquelle l'accent était mis, à tort, sur les systèmes axiomatiques de la logique.

Zusammenfassung

Gegen Quines und Strouds Kritiken verteidige ich eine konventionalistische Auffassung der logischen und (gewisser) mathematischer Wahrheiten. Der Konventionalismus wird am besten durch Berufung auf bedeutungstiftende Regeln, die den Gebrauch wichtiger logischer und mathematischer Ausdrücke bestimmen, formuliert. Konventionelle Notwendigkeit kann als von diesen Regeln hervorgebracht verstanden werden — und das zwar auf eine Art, die sie vor Quines und Strouds Anfechtungen schützen. In der von diesen kritisierten, früheren Fassung wurde zu unrecht das Gewicht auf axiomatische Systeme der Logik gelegt.

1. Just over fifty years ago Quine published his classic paper 'Truth by Convention'. It registered his influential doubts about the tenability of the positivist doctrine, given its most forceful expression by Carnap, that logical (and mathematical) truths were true by convention; that they were somehow grounded in language. The same view has also been called the linguistic doctrine of logical truth. The doctrine was devised to take care of logical and mathematical truths within the overall positivist framework. For in this framework there was to be no room for Kantian synthetic *a priori*'s. What could

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not be reduced to the obtaining of suitable successions of sensory experiences was to be subsumed to the delicate inter-connectedness of language. Since mathematics had to be absorbed in this way, logic too found itself swept in the same direction. Its already analytic status merely found reaffirmation within the linguistic doctrine: 'true by virtue of the meanings involved' gives way to 'true by virtue of the operation of rules governing the expressions involved'.

But here we have a hint that there is more to the conventionalist thesis than that all logical and mathematical truths are analytic *a priori*. Even if there were a successful logicist reduction of mathematics to a more basic theory deserving the title 'logic', one could regard the mathematics thus reduced, and the reducing theory itself, as analytic *a priori* and yet deny that the ground of its truth lies in language in the austere sense intended by the conventionalist. For one might wish to explain the analyticity by appeal to a semantic ontology (individual concepts, general concepts, propositions) within which it was the special inter-relatedness of various items that gave rise to the truth of those statements regarded as logically true. And in holding such a view one would not be of very different philosophical temper from the Platonist arithmetician who likewise holds that arithmetical truth arises by virtue of the special inter-relatedness of abstract objects — numbers — subsisting in some abstract realm into which great mathematicians had better insight than their less gifted colleagues.

It would appear, then, that there is a special ingredient in the linguistic doctrine, the conventionalist view, that cannot be ignored when assessing its claim to be an adequate account of the status of logical and mathematical truths and our knowledge of them. The ingredient is an ontological one; or, rather, an anti-ontological one of sorts. There is an underlying *nominalism* to the conventionalist account. When, according to it, a statement is true by virtue of the conventions governing the use of its component expressions, one is cautioned further not to regard the ground of that truth — the state of affairs by virtue of which the statement is true — as involving anything more than bare physical facts. The bareness of these facts is not a matter of their atomicity or their localization — for, after all, the conventions governing the use of an expression could supervene upon, or arise from, a very extensive body of ordinary facts about people, their utterances and their interactions — but is rather a matter of their humdrum ontological composition. People and physical events such as the buffeting of eardrums by compressed air could figure in the ground of conventionalist truth. Perhaps even events under intentional descriptions, such as assertions, could so feature — since they are still physical events. But what is to be avoided is any recourse to a realm of abstract entities and internal relations among them.

Historically, then, conventionalism came out as the residue of logicism modulo nominalism:

conventionalism =
the doctrine of analyticity of logic and (via appropriate reduction to logic)
of mathematics
MINUS
any abstract semantic ontology of meanings

2. For a paper of its vintage, Quine's 'Truth by Convention' has aged remarkably well. It was written immediately in the wake of Gödel's discovery of his famous incompleteness theorems, which were even by then being perceived as fatal to the logicist programme of reducing mathematics to logic. Its central complaint — the Lewis Carroll point about the infinite regress we have to embark upon as we appeal to more rules to shore up the application of other rules — is one that has found widespread acceptance. It would be material enough for one paper to take issue with this orthodoxy. I shall do so in this paper; but I shall also try, a little more ambitiously, to sketch what I take to be the essential surviving truth in conventionalism, and integrate it into a general theory of meaning which is designed to pay to logic and mathematics the austere respect intended by the original linguistic doctrine of logical and mathematical truth. One more historical observation will prepare my reader for the line I intend to pursue; and then I shall get into the details of the philosophical argument that I want to advance. This is the observation that even as Quine was writing his own paper, and appealing to Lukasiewicz's axioms for propositional logic in setting up his regress argument, Gentzen was (presumably) busy writing his classic paper 'Untersuchungen über das logische Schliessen'. It was a paper that ought, to my mind, have effected a much more radical re-orientation to matters logical than it has even unto the present time. It ought to have shaken the Quinean orthodoxy that logic deals in logical truth; and that reasoning gets going with axiomatic starting points, even if they be empty of any content. For in Gentzen's paper is a two-pronged response to this orthodoxy. First, it is inference that comes to the fore, rather than theoremhood; and the rules of inference themselves are framed in such a way as to focus on the peculiar compositional contribution made by each logical operator in turn. Exactly how this bears significantly on the reply to Quine's attack on the conventionalist thesis will emerge below.

3. What is the problem Quine poses the conventionalist? 'In a word,' he says (p. 104)

the difficulty is that if logic is to proceed *mediately* from conventions, logic is needed for inferring logic from the conventions

At least one other influential contemporary writer sees the problem in precisely the same way. Thus Stroud (1981, p. 243):

Even among logical truths alone there are an infinite number we now accept, and so we could not have conventionally endowed each of them singly with the truth we now attribute to them. Our explicit conventions would have to have been general ones, from which the truth of individual logical truths follows. But if one truth follows logically from another then the conditional truth with the latter as antecedent and the former as consequent is itself a logical truth, and so in attempting to render true by general convention some individual logical truth we make essential use of some other logical truth whose truth is so far unaccounted for.

There are two crucial features in the problematic thus posed. First, there is the recognition that logical truths are infinitely numerous; so that some manner of generating them has to be found. Secondly, there is the (more contentious) claim that such conventions as could be appealed to on the linguistic doctrine must be explicit and definite. Quine himself says

It may be held that we can adopt conventions through behaviour, without first announcing them in words . . . On the other hand, it is not clear wherein an adoption of conventions, antecedently to their formulation, consists . . .

I shall have to take as granted, and not in need of detailed argumentative support within the scope of this paper, the claim that David Lewis's theory of convention 'without convening' adequately allays the misgivings Quine expresses. Lewis has given an admirable account of how a convention can be established as a behavioural regularity in a group by virtue of being a salient solution to a recurring co-ordination problem. I have used the leading ideas of this account to argue (in 'The aetiology of entrenchment', *Anti-Realism and Logic*) that a growing language could acquire logical operators governed by Gentzen's introduction and elimination rules because there would be selective forces for the retention of any operator that obeys rules displaying the special sort of harmony that characterizes Gentzen's rules. In this way the rules themselves need not be explicitly formulated by the speakers among whom the rules nevertheless govern linguistic transactions. Nevertheless, there is a clear and justifiable sense in which the rules codify the conventions according to which the logical operators are used. So in my critique of the central argument of 'Truth by Convention', I shall concentrate on the 'nothing comes from nothing' strand: the strand alleging the essential presupposition of logic in order to get logic.

4. Let us run through Quine's argument in a little more detail. He writes (p. 88, pp. 90-91):

. . . if we are to construe logic . . . as true by convention, we must rest logic ultimately upon some manner of convention other than definitions: for . . . definitions are available only for transforming truths, not founding them . . .

Now suppose . . . that we abstract from existing usage of the locutions 'if-then', 'not' (or '-'), and the rest of our logical primitives, so that for the time being these become meaningless marks, and the erstwhile statements containing them lose their status as statements and become likewise meaningless, neither true nor false; and suppose we run through all those erstwhile statements, or as many of them as we like, segregating various of them arbitrarily as true. To whatever extent we may carry this process, we to that extent determine meaning for the initially meaningless marks 'if', 'then', '-', and the rest. Such contexts as we render true are true by convention. . . if, in (this) process of arbitrarily segregating statements (sic — but surely Quine must mean here *erstwhile* statements, that is, sentence forms?) as true by way of endowing our logical primitives with meaning, we assign truth to those statements which, according to ordinary usage, are true and involve only our primitives essentially, then not only will the latter statements be true by convention, but so will all statements which are true under ordinary usage and involve only logical expressions essentially. Since . . . every logical truth involves . . . only logical expressions essentially, the described scheme of assigning truth makes all logic true by convention . . . The . . . assignment of truth thus goes far toward fixing all logical expressions in point of meaning, and fixing them in conformity with usage.

Note the very holistic procedure Quine thinks has to be adopted if we are to render logical truths true by convention. We have somehow to generate all logical truths in schematic form, so that every schematic form generated is indeed logically true by ordinary lights. Quine is in effect saying that a finitary sound and complete axiomatisation of a logic renders it true by convention. The 'convention' is a global one: *these* forms are to count as logically true; whence, *these* special expressions (the logical ones) are to have such-and-such meanings (fixed by the overall generation of precisely *those* forms as the logically true ones). In the same holistic spirit Quine considers rendering truths in various branches of mathematics true by convention. Setting aside problems to do with non-finitely axiomatisable theories (of which he takes no note), he writes (pp. 99-100):

In (any . . . branch of mathematics which may resist definitional reduction to logic) we merely set up a conjunction of postulates for that branch by fiat, as a conventional circumscription of the meanings of the constituent primitives, and all the theorems of the branch thereby become true by convention . . . In this way mathematics becomes conventionally true, not by becoming a definitional transcription of logic, but by proceeding from linguistic conventions in the same way as logic does.

Remember that Quine is not here actually advancing the conventionalist thesis by thus formulating it for logic and for mathematics. Rather, he is giving the conventionalist rope with which to hang himself. Thus the objection that has to be considered against Quine here is that he perhaps restricts unduly the materials and methods of which a more sophisticated conventionalist might avail himself. I shall be sustaining that objection below. But let us resume the outline of Quine's argument.

He proceeds to use the Łukasiewicz axiomatisation of propositional logic to fix as true the sentence 'If time is money then time is money'. Since some of the instances of Łukasiewiczian axioms using 'Time is money' as an atomic propositions are amusingly, if not grotesquely, prolix, I shall take the liberty of abbreviating it to 'p' throughout; and shall not mind unduly my p's and q's in the statement of the axioms. 'p' will feature both as a placeholder within axioms, and as a particular proposition substituted for placeholders. The derivation within the Łukasiewicz calculus is as follows, using gappy underlining for substitution, and unbroken underlining for application of the sole rule of the calculus, namely *modus ponens*:

$$\frac{\frac{\frac{p \supseteq (\neg p \supset q)}{p \supset (\neg p \supset p)} \quad \frac{p \supset (\neg p \supset p)}{p \supset (\neg p \supset p)} \supset ((\neg p \supset p) \supset p) \supset (p \supset p)}{p \supset p} \quad \frac{p \supset (\neg p \supset p)}{p \supset p} \supset ((q \supset r) \supset (p \supset r))}{p \supset p}$$

Now comes Quine's central objection. He identifies as the conventions 'whereby logic itself is set up' the general claims that all instances of the axioms are true, and that applications of *modus ponens* are truth-preserving. But (p. 103)

derivation of the truth of any specific statement from the general convention thus requires a logical inference, and this involves us in an infinite regress

For (and here I faithfully paraphrase his argument) in deriving a statement *q* from statements *p* and $(p \supset q)$ on the authority of the claim that applications of *modus ponens* are truth-preserving, we *infer* from the latter claim and the specific premise that the statements *p* and $(p \supset q)$ are true, the conclusion that the statement *q* is true.

This, however, is to regard *giving proofs* within the calculus as co-terminous with the meta-commentaries that would establish their soundness, and would consequently justify detaching from the truth of their premisses to the truth of their conclusion. On Quine's account no proof would ever constitute a *suasive* argument: the threat of regress would keep one grounded, unable to make any epistemic advance, unable to confer truth on conclusions of proofs whose premisses are already known to be true. The epistemic role of deduction is surely this: one starts with premisses accepted as true; one provides a proof of a conclusion from those premisses; and then one detaches to the truth of the conclusion. Or, better: one asserts the premisses on the basis of various warrants; one provides a proof of a conclusion from those premisses; and then asserts the conclusion. What one does *not* do is re-capitulate the proof step-by-step with the corresponding truth-attributions to the sentences involved at each inference within it.

This account of deduction comes into its own only with a natural deduction framework such as the one set up by Gentzen. It is the rules of indirect proof, in particular (such as *reductio ad absurdum*, or the rule of conditional proof) which resist Quinean meta-commentary as the most natural interpretation of what is happening when they find application within a proof. But then we must remind ourselves of the highly artificial generative system within which Quine gave his example and against the background of which he unfortunately set up his general objection to conventionalism. His very own derivation of $(p \supset p)$ would be replaced in a Gentzen system by the following proof:

$$\frac{p}{p \supset p} (1)$$

Or, if one needs a little more stylistic detail:

Assume p for the sake of argument

So, if *p* then *p* (and we no longer need the assumption)

Here, in an obvious sense, the final proof (degenerate though it may be) has no sentential starting points, in the sense of undischarged premisses whose truth is required in order for truth to be transmitted to, or conferred upon, the conclusion. In a natural deduction system the practice of making assumptions for the sake of argument, and then discharging them, is one which enables us to construct proofs that end with conclusions resting on no assumptions whatever. All the assumptions in such a proof will have been discharged in the course of the argument, by applications of rules such as *reductio ad absurdum* and conditional proof — precisely those indirect rules to which I adverted above.

Rather, therefore, than quibble with Quine over the appropriate way to take or read the proofs in his own Łukasiewiczian system, let me develop a comprehensive alternative account of what the conventions actually are that comprise the source of logical (and mathematical) truth, and how they go to work, or are employed in order to generate those truths as such. I have, indeed, already embarked on the account I have in mind.

5. Stroud poses the following challenge to the conventionalist:

Precisely what relation is supposed to hold between our thinking or speaking in certain ways and the necessary truth of any of those things we now regard as necessarily true? If our thinking or speaking in those ways is in some sense 'responsible' for the necessary truth of the things we now accept as necessary, exactly how is that notion of «responsibility» to be understood?

Consider something that we believe to be necessarily true, for example, 'If all men are mortal and Socrates is a man then Socrates is mortal'. Does conventionalism, or indeed any view according to which necessary truth is in some sense our own 'creation',

imply that the truth of that statement is due solely to our present ways of thinking or speaking in the sense that if we had thought or spoken in certain other ways, or had adopted relevantly different “conventions” then it would not have been true that if all men are mortal and Socrates is a man then Socrates is mortal?

To which one must reply that the answer is negative. For if we had thought or spoken in certain other ways it would not have altered the necessary truth of what we mean in English (as governed by present logical conventions) by “If all men are mortal and Socrates is a man then Socrates is mortal”.

It behooves us only to say what these conventions are, and how they are responsible for the logical truth of this English sentence. Unlike Quine’s conventionalist target, I shall not give my conventions as general axioms concerning the logical truth of sentences of particular forms, or concerning the transmission of truth from premisses of particular rules to their conclusions. Rather, my conventions will be much more molecular than that. They will focus on the characteristic semantic contribution made by each logical construction occurring in the English sentence concerned. This semantic contribution will not need to be spelled out in the terminology of truth-attribution. Rather, it will consist in a schematic representation of argumentative role. The logical operators “all” and “if . . . then . . .” and “and” form complex statements whose role in proofs can be described in terms of simpler statements and their roles in proofs:

$$\begin{array}{l}
 \frac{}{Aa} \text{---}(i) \\
 \cdot \\
 \cdot \\
 \cdot \\
 \frac{Ba}{\forall x(Ax)Bx} \text{---}(i) \quad \text{where } a \text{ does not occur in any assumption,} \\
 \quad \text{other than } Aa, \text{ on which } Ba \text{ depends} \\
 \frac{A \quad B}{A\&B} \\
 \frac{}{A} \text{---}(i) \\
 \cdot \\
 \cdot \\
 \cdot \\
 \frac{B}{A\supset B} \text{---}(i)
 \end{array}$$

The corresponding elimination rules are in harmony with these, allowing us to ‘unpack’ from a complex statement only what we must have been able to ‘put in’:

$$\begin{array}{l}
 \frac{A \quad \forall x(Ax)Bx}{B} \\
 \frac{A\&B \quad A}{A} \\
 \frac{A \quad A\supset B}{B}
 \end{array}$$

The elimination rule $\frac{A\&B}{A}$ is justified because a warranted

assertion of $(A\&B)$ requires a warrant for the assertion of A . Likewise the elimination rule

$$\frac{A \quad A\supset B}{B}$$

is justified because the warrant for the assertion of $A \supset B$ is a derivation of B from A , which turns warrants for the assertion of A into warrants for the assertion of B . Finally the elimination rule

$$\frac{At \quad \forall x(Ax)Bx}{Bt}$$

is justified because in order to assert $\forall x(Ax)Bx$ one must have a schematic warrant

$$\begin{array}{l}
 Aa \\
 \cdot \\
 \cdot \\
 \cdot \\
 Ba
 \end{array}$$

which, upon substitution of t for a produces a warrant for the assertion of Bt from any warrant for the assertion of At (cf. Prawitz 1974).

Putting our rules into effect, we can produce an outright warrant for the assertion of the sentence “If all men are mortal and Socrates is a man then Socrates is mortal”:

- (1) $\frac{\forall x(x \text{ is a man}) \rightarrow x \text{ is mortal} \ \& \ S \text{ is a man}}{S \text{ is a man}}$ (1)
 $\forall x(x \text{ is a man}) \rightarrow x \text{ is mortal} \ \& \ S \text{ is a man}$
 $\forall x(x \text{ is a man}) \rightarrow x \text{ is mortal}$ (1)
 If $(\forall x(x \text{ is a man}) \rightarrow x \text{ is mortal} \ \& \ S \text{ is a man})$ then $S \text{ is mortal}$ (1)

What emerges clearly is that our conventions are responsible for the logical truth of the conclusion *without being antecedents of a logically true conditional with that conclusion as consequent*. We therefore see the Quinean problem, as posed most explicitly by Stroud in the quotation at the beginning of section 3 above, as spurious. For it is now clear that the sense in which the truth of individual logical truths ‘follows from’ conventions is *not* the same as that in which the consequent of a logically true conditional follows from the antecedent. It would be better to speak of logical truth (and indeed: consequence in general) as *arising from* or *flowing* from the conventions in question. For we are interested in truth (and now, more generally: consequence) *by* convention, not in what follows from the truth *of* conventions. It may be true that a certain convention holds: but we cannot hold that a certain convention *is true and logically implies* individual logical truths. Conventions are not truth bearers. Rather, they regulate truth bearers: they are the means whereby truth bearers assume, shift and deposit their loads. There is no circularity, and no uncovered case, in an account of conventions as the source of all logical truth. We must distinguish between the way in which

- a) evidence concerning our linguistic behavior bears on the claim that an explicitly formulated convention (such as one of the rules framed above) holds, from the way in which
 b) once given the convention, we work out that certain individual truths are logical truths.

Neither of these passages has the character of a logical derivation of a conclusion from premisses, as a Quinean anticonventionalist would have one imagine. The first passage (a) is a complicated inductive inference to the best description of regularities of use. It would invoke Lewis’s analysis of convention, and be based on the appropriate behavioural evidence. The second (b) might involve deductions of conclusions from conditional assumptions *in accordance with* the conventions (here, rules of inference) *without* thereby using the conventions themselves as premisses *from* which the logical truth in question is inferred. The example about Socrates above made this clear. The Socratic conditional turned out a logical truth *not conditionally upon* some supposedly truth-bearing conventions as premisses, but rather by means of a deduction *in accordance with* the conventions (here, rules of inference) in

such a way as to render the logical truth in question independent of any assumptions at all.

I take this analysis to constitute an adequate response to the challenge to identify the kind of convention that can be shown to make logical truths true by convention, in such a way as not to presuppose logical truth in doing so. On this account, logical truth and logical consequence, and the necessity of both, arise as by-products of the harmony principle constraining both the evolutionary adoption of logical operators, and speakers’ acquisition of grasp of their meanings. When we adopt a new logical operator we should conservatively extend the class of assertable statements: no statements not involving that operator should become assertable simply because we have some extra rules governing the new operator. Likewise we should conservatively extend the consequence relation: no new deducibilities or consequences should obtain among statements not involving that operator. Finally, there is that aspect of harmony by which, as it were, the introduction and elimination rules for an operator mutually determine each other: the introduction rule specifies what grounds we must have for asserting a statement with its operator dominant, and the elimination rule spells out the commitments made by one who makes such an assertion. The elimination rule must wring from the statement *all* that could be behind its assertion. But it can also wring from it only what may legitimately be required of an assertor before he be justified in making the assertion. His new way of putting things must not put new things the listener’s way. This final aspect of harmony has been beautifully explicated in Prawitz’s account of the normalization theorem as a soundness theorem (cf. Prawitz 1974; and the discussion of conservative extension in my *Anti-Realism and Logic*). Our idea of necessity arises because we enter fixed points in our practice, in order not to overstretch our communicative net. But the sense in which we enter those fixed points (our rules of inference) is not that explicit, deliberative sense which Quine seemed to believe was required of any convention worth the label. Rather, that the logical operators of an evolved and graspable language do obey rules displaying this sort of harmony is the sort of fact that one would expect, on naturalistic or evolutionary grounds, when confronted with the very fact that the language serves the function of facilitating information exchange among its speakers. Just as the heart must expand and contract in order to pump blood around, so too must we be able to infer conclusions, *and* reason away from premisses, in order to circulate information.

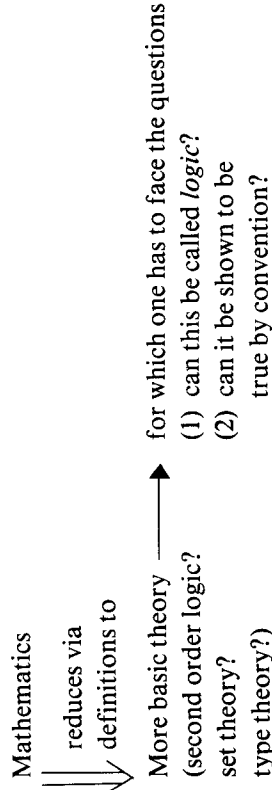
6. From the perspective we have attained, I want now to say a little more about mathematical truth. My remarks about the historical context of ‘Truth by Convention’ might have been a little misleading; for indeed there is no

explicit recognition in Quine's paper of the re-thinking that Gödel's theorem was to occasion. On p. 87 indeed, we find Quine saying that
 ... the thesis that all mathematics is logic is ... substantiated by *Principia* to a degree satisfactory to most of us.

However difficult it may be today to uphold this logicist claim, we cannot but agree with Quine when he says on the same page that

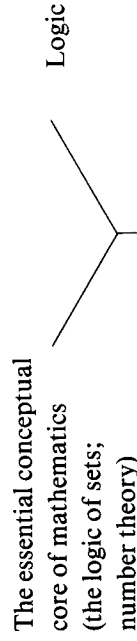
... we cannot regard mathematics as true purely by convention unless all those logical principles to which mathematics is supposed to reduce are likewise true by convention.

The overall scheme was this:



Quine appeared to be too ready to accord to the theory of types in *Principia* the status of logic; and too reluctant to accord even propositional logical truths the status of truth by convention. Against the Quine of 'Truth by Convention' I would be inclined to the prevalent view (which he himself expressed in later writings — cf. his 1953, pp. 122-3) that whatever basic theory mathematics can be reduced to, it fails to count as logic. But I would also be willing to claim that a *great deal of mathematics, and all of logic, can, in a perfectly adequate sense, be regarded as true by convention.*

The overall scheme by which I would replace the one above is therefore:



both *true by convention*, by virtue of being reduced to utterly basic meaning-constituting *rules* that are respectively *sui generis*

Since my account of truth by convention appeals to rules in the mould of those already used above, I shall not repeat the general defence already sketched there of my claim that the inferences generated, and in particular the

outright assertions warranted, are *thereby* true by convention. The task that remains is simply to formulate the rules, especially for core mathematics. In doing so, I shall concentrate on the logic of sets.

The scheme above represents a minimal claim: it says at least this much is true by convention. Here the conventions are truly sense-conferring. I take them to be rules (in introduction and elimination format) that serve to fix the sense of crucial mathematical notions: 'member of', 'the set of all ϕ 's', 'the number of ϕ 's', 'zero', 'successor', 'natural number'. The core of mathematics should be, as far as possible, ontologically non-committal; but I am not persuaded that no existence claims may enter into specifications of the sense of the mathematical notions mentioned. Suffice it to say, I shall be as austere as possible. My claim, in support of the conventionalist thesis, will be that the rules laid down do indeed regulate our understanding of the notions involved. It does not matter if no-one before now has explicitly formulated them in this fashion; for they constitute, I claim, a correct analysis of exactly what it is that one has to grasp in order to have mastered these mathematical notions. Or, better: exactly all that one need grasp in order to master them; I am not precluding an acquisition of mastery by a different route.

We work within a free logic at first order. We do not assume that all terms denote; and we do not assume that there is anything. We do assume, however (or stipulate) that in order for an atomic predication of the form $A(t)$ to be true, the term t should denote an object. Thus we have the inference rule

$$\frac{A(t)}{E!t}$$

where $E!t$ is short for $Ex(x=t)$. We also refuse to regard anything as describable as the value of a function unless the function applies to an existing object:

$$\frac{E!f(t)}{E!t}$$

Our quantifier rules accordingly undergo slight modification. Universal introduction, for example, becomes

$$\frac{\quad (i)}{E!a}$$

$$\frac{Aa}{\forall xAx} \quad (i)$$

and the corresponding elimination becomes

$$\frac{\forall xAx \quad E!t}{At}$$

For the modification of the rules for the existential, and a proof of the completeness of the resulting system, see my *Natural Logic*, ch. 7.

So much for the underlying free logic. Now what about, say, the theory of sets? In setting up the rules that I do below, I am taking issue with the holism in this passage from Quine's 'Carnap on logical truth' (p. 114):

... it is difficult to conceive how to be other than democratic towards the truths of set theory. In exposition we may select some of these truths as so-called postulates and deduce others from them, but this is subjective discrimination, variable at will, expostory and not set-theoretic... Given this democratic outlook, finally, the law of sufficiency reason leads us to look upon S (defined earlier as 'a select species which is so fundamental that (one) cannot dissent from them without betraying deviation in (one's usage or meaning of 'E')' as including *all* sentences ((he should say: theorems)) which contain only 'E' and the elementary logical particles.

And later on p. 117:

In set theory we discourse about certain immaterial entities, real or erroneously alleged, viz. sets, or classes. And it is in the effort to make up our minds about genuine truth and falsity of sentences about these objects that we find ourselves engaged in something very like convention in an ordinary non-metaphorical sense of the word. We find ourselves making deliberate choices and setting them forth *unaccompanied by any attempt at justification* other than in terms of elegance and convenience. These adoptions, called postulates, and their logical consequences (via elementary logic), are true until further notice.

I disagree that choice of fundamental rules can be such a whimsical and unjustifiable matter. Sets are entities, by Quine's own admission. And it was Quine who proclaimed elsewhere "no entity without identity"! What, then, are the identity conditions for the set of all F's? What, that is, are the conditions under which one would be justified in asserting any statement of the form $t = \{x/Fx\}$? They are as follows:

$$\frac{(t) \quad \overbrace{Fa, E!a}^{(t)} \quad aEt}{aEt \quad E!t \quad Fa \quad (t)} \quad t = \{x/Fx\}$$

This, the introduction rule for the set term-forming operator, is basically the principle of extensionality for sets. The corresponding elimination rules are the two halves of Church's conversion schema:

$$\frac{t = \{x/Fx\} \quad Fu \quad E!u}{u \ \& \ t} \quad \frac{t = \{x/Fx\} \quad u \ \& \ t}{Fu}$$

These rules comprise the logic of sets. They specify the senses of 'E' and of the set term-forming operator as definitively as the rules for logical operators such as & and \forall specify their senses. Using this logic, one can develop all of what Quine calls the virtual theory of sets. This way we obtain a conceptual or ideological skeleton, waiting to be fleshed out with particular ontological commitments. These come in the form of outright existence assumptions ("the empty set exists") and various conditional existence assumptions (such as "the power set of any set exists").

With these further existential commitments one obtains the full mathematical force of set theory. But the fact that set theory goes beyond logic in thus calling into being a rich ontology of abstract objects, should not blind us to the analytic fact that we managed, via the sense-conferring rules above, to render a significant core of the theory true by convention. The conventions here are rules *a refusal to abide by the workings of which* would be clinching evidence for deviation in usage. This, I think, is the most basic sense in which truths may be true by convention.

A less basic sense would then attach to such "conventional" theoretical options as choosing to assert, say, the negation of the axiom of choice; or the existence of a measurable cardinal; or deciding one way or another any of the many interesting undecidable conjectures within Zermelo-Fraenkel set theory. The obvious, Platonist way to understand such decisions is to regard them as attempts better to delineate the actual structure of a realm of abstract objects, existing antecedently to, and independently of, our investigation of them. But when a conjecture C is independent of ZF (and this has been proved in ZF itself) who is to say whether adopting C or adopting (\neg C) as an axiom is going to bring us into closer correspondence with that abstract realm? — is going to yield a 'truer' (because true and more complete) account of it? *There is no fact of the matter here*, I would contend. We may succeed — indeed, we have succeeded — in specifying a determinate sense for important constituent expressions (such as 'member of' and 'the set of all ϕ 's') and thereby succeed in conferring necessary truth upon various statements, and necessary logical entailments among statements in general, *without* — and this is crucial — being beholden to determine, even if only in principle, a definite truth value for each and every meaningful sentence of the language.

A similar situation obtains for elementary arithmetic. Consider just the language based on the constants 0, s(), N() and #xϕx. Rules can be laid down that confer on these symbols the senses required for 'zero', 'successor of', 'is a natural' and 'the number of ϕ's'. For example, the introduction rule for 0 is

$$\frac{(i) \quad \frac{}{Fa, E!a} (i)}{\#xFx = 0} \wedge$$

(We may, if we like, require an extra premiss of the form E!0, in order to maintain ontological neutrality throughout). The corresponding elimination rule is

$$\frac{\#xFx = 0 \quad Ft \quad E!t}{\wedge}$$

So 0 comes out as the number of any empty concept; and that is how it should be. I shall not repeat here all the rules for the remaining primitives; these are given in detail in *Anti-Realism and Logic*, Part II. But it is worth remarking that the sense-conferring rules permit the derivation, within the system of introduction and elimination rules for the logical operators, of all the Peano postulates. They also ensure every instance of the *disnumerical schema*

$$\#xFx = n \text{ if and only if there are exactly } n \text{ F's}$$

which pins down the role of natural numbers in counting. I therefore regard at least this much of basic mathematics as true by convention, in the sense of that theoretical phrase that I have developed above. I do not regard the analyses provided above as failing in the "attempt at justification" which Quine maintained is never made. Justification is indeed called for, and the philosopher of logic and mathematics must provide it. What his analyses reveal are the deep conventions governing our mathematical language and thought. Logical and mathematical truth are indeed true by convention. But one cannot demand of conventions that they settle everything.

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