

Sortal quantification

J. E. J. ALTHAM AND NEIL W. TENNANT

This paper falls into two parts, the first by Altham, the second by Tennant. The second part contains the formal work; it sets forth a syntactic system for representing sortally quantified sentences of English, and provides semantics for the sentences so represented. Illustrations of the usefulness of the ideas presented are drawn from among the trickier kinds of sentences that have appeared in the literature. The first part gives some indication of the background to some of the ideas in the second part. It also shows the range of expressions that have some claim to treatment by the methods that follow.

Part I

The standard logical quantifiers of classical predicate logic, $(\exists x)$, and $(\forall x)$ – the existential and universal quantifiers respectively – are familiar, as is also the idea that their workings in the system which is their home may serve as a source of syntactical and semantic insights into a class of English sentences containing such words as *some*, *none*, *every*, and *any*. The best known example of the use of this idea is perhaps the use of the logician's notion of the *scope* of a quantifier to explain why it is that in some contexts *any* may be replaced by *every* without change of truth value, whereas in other contexts the substitution must be made with *some*.

The English words *every*, *any*, and *none* can be qualified by certain adverbs, so that we may say, for instance, *nearly every*, *scarcely any*, and *almost none*. It seems an attractive idea to suppose that the effect of such adverbial modification is to form expressions representable by some further logical quantifiers. Once we are seized of this idea, the possible scope for quantificational representation quickly comes to seem quite broad. For instance, the sentence *Almost every man owns a car* is logically equivalent to *Few men do not own a car*, which in turn is equivalent to *Not many men do not own a car*. So *few*, and *many* seem ripe for quantificational representation. Moreover, since *He always comes to see me on Sundays* is equivalent to *He comes to see me every Sunday*, it looks promising to bring *always* into the group with the other terms for which a quantificational approach seems

promising. Where *always* is properly given a temporal reading, it means something like *at all times*, and can be represented by universal quantification over times. There is indeed a pretty close correspondence between two sets of words as follows:

always ever often seldom sometimes never
every any many few some none

The terms in the upper line interrelate in the same way as do those on the lower. For instance, just as *I have few books* is equivalent to *I do not have many books*, so *I seldom go to London* is equivalent to *I do not often go to London*.

In addition, the logician's eye discerns quantifiers in certain adjectives. Bertrand Russell pointed out long ago the invalidity of the syllogism:

Men are numerous
 Socrates is a man
 Ergo, Socrates is numerous.

The first premiss is equivalent to *There are many men*. Analogous transcriptions can be made where other adjectives are concerned. For example *Storms are rare in this part of the world*, for which we might read *There are not many storms in this part of the world*, or *Storms do not often happen in this part of the world*.

Yet further quantifiers are discernible in natural language, unless the logician's eye deceives him. As well as *few*, *many*, and *nearly all*, we have *very few*, *very many*, and *very nearly all*, and yet more result from reiterated prefixing of *very*.

In their representations in a formal syntax, all the foregoing expressions come out as what Mr Tennant calls (I, I) -quantifiers – quantifiers which bind one variable in one formula. A formal syntax, together with appropriate semantics, which gave an appropriate treatment to all these would already be a significant generalization of classical quantificational methods on the pattern of ordinary logic. The further generalization to (I, k) -quantifiers, binding one variable in an ordered k -triple of formulas, gives a further increase in power. Thus consider the sentence *There are exactly as many Apostles as there are days of Christmas*. The occurrence of *there are* puts one in mind of quantifiers, and the fact that *there are* occurs twice matches nicely in intuition with the idea that we have here a $(I, 2)$ -quantifier. It has been usual for logicians to represent sentences like this one in the language of set-theory, as asserting a one-one relation between the

class of Apostles and the class of days of Christmas. The device of the $(1,2)$ -quantifier avoids recourse to such higher-order logic in giving the syntax of such sentences.

It seems quite significant, in connection with these $(1,2)$ -quantifiers, that we can build up additional ones in much the same way as we can $(1,1)$ -quantifiers. For instance, from *more than* we can go to *many more than*, and *very many more than*. We have such expressions as *nearly as many as* and *almost as few as*, and so on.

We pass on now to matters of semantics. The simplest form of sentence involving what we call a plural quantification is one of the form *There are many Ps*. One thing that is clear about the truth conditions of this is that it is false if there is only one *P*. Next, it seems clear that in general how many *Ps* there need to be for there to be many *Ps* depends on the size of the envisaged domain of discourse. For instance, in *There are many Communists in this constituency*, the domain of discourse would probably be the electorate of the constituency in question. This domain is smaller than the one envisaged in *There are many Communists in Italy*, and consequently the number of Communists there have to be for there to be many Communists in this constituency is smaller than the number there have to be for there to be many Communists in Italy.

This very strongly suggests the use of a numerical method in providing appropriate semantics. This was done in Altham (1971) by selecting, for each application of the logic presented therein, a number n which is the least number of things there have to be with a certain property for there to be many things with that property. For the purposes of the logic it was unnecessary to specify exactly what number should be chosen, nor indeed can this realistically be done. The important points are that n is constrained to be >1 , it varies with the domain of discourse, and its value relative to the numbers associated with other quantifiers should be correct. Thus the quantifier *a few* is given semantics in a way similar to those for *many*, in terms of the least number of things that must have some property if there are to be a few things with that property. This number again varies with the domain of discourse. If such numbers are termed threshold-numbers, then the essential condition is that the threshold-number associated with *a few* should be smaller than that associated with *many*.

This method can be used also in the case of the quantifiers compounded with *very*. Thus the threshold-number associated with *very many* will be $n+m$, with m positive, if n is the threshold-number for *many*. On the other hand, if k is the threshold for *a few*, the threshold for *a very few* will be $k-l$. Repetitions of *very* can be coped with similarly. The multiplicity of

threshold-numbers is reduced, however, by the possibility of defining some quantifiers in terms of others. Thus *nearly all* is *not many not*.

In the cases considered so far, the range of quantification is not restricted by a phrase within the sentence itself, but is determined by the context. But very frequently the range of quantification is so restricted, and in this case quantifiers such as *many* are *sortal* quantifiers. A sentence *There are many things which are both P and Q* is one in which the quantifier is not sortal, and it is logically equivalent to *There are many things which are both Q and P*. In contrast *Many Ps are Q* involves a sortal quantifier, and is not equivalent to *Many Qs are Ps*. In *Many Ps are Qs*, the quantifier's range is restricted to the set of *Ps*. Thus the set of *Ps* here becomes the domain of discourse whose size determines an appropriate threshold-number. Consequently, since the set of *Ps* may not have even nearly the same cardinal number as the set of *Qs*, the threshold-number determined by one may be different from that determined by the other. The non-equivalence of the two sortally quantified sentences is a consequence of this fact. An example will make this clear. Compare the two sentences

- (1) Many specialists in Old Norse are university officers
- (2) Many university officers are specialists in Old Norse

It seems that (1) is true, and (2) is false. Because there are far more university officers than specialists in Old Norse, the threshold-number for *many university officers* is correspondingly larger than that for *many specialists in Old Norse*. Suppose n is the threshold-number for the latter, and it is true that there are n people who are both specialists in Old Norse and university officers. Then (1) is true. But the threshold number for *many university officers* is some number $m > n$, so that for (2) to be true it must be true that there are at least m people who are both specialists in Old Norse and university officers.

From these reflections we can arrive at the extra premiss which needs to be conjoined to *Many Ps are Qs* in order validly to infer *Many Qs are Ps*. This is that there are at least as many *Ps* as there are *Qs*. Thus, to take a plausible example, if many professional men own French motorcars, and there are at least as many professional men as owners of French motorcars, then it follows that many owners of French motorcars are professional men.

These points about relative sizes of threshold-numbers were seen in Altham (1971), but the treatment there in chapter 4 is unsatisfactory because incompletely formal. What is needed, and what Altham did not then have, is a way of representing a sentence such as *There are at least as many professional men as owners of French motorcars*. This defect is

effectively remedied by the (I,k) -quantifiers explained by Tennant in the second part of this paper.

In the second part, a method of Hookway is reported which enables sortal quantifiers to be replaced everywhere by other, more complex, quantifiers that are not sortal. Here I indicate two informal possibilities of such reduction. First, it is clear that the sortal *Most Ps are Qs* is logically equivalent to *There are more things which are both P and Q than there are things which are P and not Q*, which involves a non-sortal $(I,2)$ -quantifier. Similarly, we may think that many Ps are Q if not far from half the Ps are Q. In this case, we could give, as logically equivalent to (1),

- (3) There are at least nearly as many specialists in Old Norse who are university officers as there are specialists in Old Norse who are not university officers.

(3) is not sortal. This transcription renders patent the lack of equivalence between (1) and (2), for (2) emerges as

- (4) There are at least nearly as many university officers who are specialists in Old Norse as there are university officers who are not specialists in Old Norse.

Part 2

Between them, philosophers of language and linguists have identified certain expressions in natural language as *grammatical* quantifiers; that is, as expressions which combine with substantival general terms to produce new expressions which do not behave like naming phrases, either logically or grammatically. Some linguists have made a case for their incorporation as a category of operators in deep structure.

The logician has a wide array of *logical* quantifiers which may be offered as capturing their logical forces as they are ideally used in reasoning in natural language. A logical quantifier is here loosely understood as an expression in a logical system which binds variables in formulae, and which possesses an 'interpretation' given by the formal semantics for the system. In effect, this interpretation is a mapping f from ordered pairs of cardinals to the set $\{t, f\}$ of truth values; so that $Q_{f,x}Fx$ is true if and only if $f(\overline{A}, \overline{D-A}) = t$, where A is the set of members of the domain of discourse D of which F holds. Such quantifiers I shall call (I,I) -quantifiers because they bind one variable in one formula. They are due to Mostowski (1957). It is easy to define (I,k) -quantifiers binding one variable in an ordered k -tuple of formulae, thus:

$$Qx(P_1x, \dots, P_kx).$$

Such quantifiers are induced by mappings of ordered k -tuples of ordered pairs of cardinals to the truth values in the mathematically obvious way.

Which of these quantifiers may prove useful in the analysis of natural language? The following suggest themselves:

If $f_I(\langle a_1, b_1 \rangle, \langle a_2, b_2 \rangle) = t$ iff $a_1 = a_2 = I$, then $\iota x(Px, Px \ \& \ Qx)$ can be given the reading *The P is Q*.

If $f_{\geq}(\langle a_1, b_1 \rangle, \langle a_2, b_2 \rangle) = t$ iff $a_1 \geq a_2$, then $\geq x(Px, Qx)$ can be given the reading *There are at least as many Ps as Qs*; similarly for *as many as* and *more than*.

If $f_0(\langle a_1, b_1 \rangle, \langle a_2, b_2 \rangle, \langle a_3, b_3 \rangle) = t$ iff $a_1 = a_2 = a_3 = I$, then $Ox(Px, Qx, Px \ \& \ Qx)$ can be given the reading *Only the P is Q*.

If $f_o(\langle a_1, b_1 \rangle, \langle a_2, b_2 \rangle, \langle a_3, b_3 \rangle) = t$ iff $a_1 = I, a_2 = a_3 = o$, then $\alpha x(Px, Px \ \& \ Qx, -Px \ \& \ -Qx)$ can be given the reading *All but the P are Q*.

If $f_{Mo}(\langle a, b \rangle) = t$ iff $a \geq b$, then $Mo \ x \ Px$ can be given the reading *Most things are P*; if $f_M(\langle a, b \rangle) = t$ iff $a \geq n$ (where n depends on \overline{D}) then $MxPx$ can be given the reading *Many things are P*; and if $f_N(\langle a, b \rangle) = t$ iff $b < n$ then $NxPx$ can be given the reading *Nearly all things are P*; these are Altham's *plurality* quantifiers (1971).

When Quine's policy of eliminating singular terms is followed, *John is bald* becomes $\iota x(jx, jx \ \& \ Bx)$, just as *The King of France is bald* becomes $\iota x(KFx, KFx \ \& \ Bx)$. The burden of the difference between proper names and descriptive phrases will then be shifted to the definition of *interpretation of non-logical constants in a model* by incorporating a *conventional constraint* on models to the effect that names, as predicates, are lexically identified as *naming* predicates and must, under any interpretation of the non-logical constants in any model, have no more than one member each in their extensions. Another conventional constraint on models is to the effect that E , the existence predicate, always has as extension the whole domain.

The range of quantification in natural language is usually restricted by a phrase after the quantifier, as noted by Harman (1970). Moreover, certain quantifiers in natural language are *irreducibly sortal* in the sense that a sentence containing a restricted quantification is not logically equivalent to a sentence containing an unrestricted quantification using the same quantifier. For example, *all* is not irreducibly sortal, because *All Ps are Qs* is equivalent to $\forall x(Px \supset Qx)$; but *many* is irreducibly sortal, because no

such paraphrase is available for *Many Ps are Qs*. Since natural language contains such irreducibly sortal quantifiers, and since the hope prevails that logic may come to be a theory not only of truth conditions of sentences, but also of their deep structures, logical syntax and semantics should be developed accordingly.

Syntax

1. Definition of well-formed formula:

- (i) If P is an n -place predicate letter then Px_{i_1}, \dots, x_{i_n} is a wff with exactly x_{i_1}, \dots, x_{i_n} as free variables;
 - (ii) If X is a wff, then so is $\neg X$, with the same variables free;
 - (iii) If X and Y are wffs then so are $X \& Y$, $X \vee Y$, $X \supset Y$, and $X \equiv Y$, with exactly the distinct free variables of X and Y free;
 - (iv) If X and Y are wffs with x free, and if Q is a (I, I) -quantifier symbol, then $Q[X]^x Y$ is a wff with exactly the distinct free variables of X and Y , save x , free;
 - (v) If X , Y and Z are wffs with x free, then $\iota[X]^x(Y, Y \& Z)$ is a wff with exactly the distinct free variables of X , Y and Z , save x , free;
 - (vi) Similarly for whatever other (I, k) -quantifiers are adopted.
2. X is a *constituent* of X , $\neg X$, $X \& Y$, $X \vee Y$, $X \supset Y$, $X \equiv Y$, $Q[X]^x Y$, $Q[Y]^x X$, $\iota[X]^x(Y, Y \& Z)$, $\iota[Y]^x(X, X \& Z)$, etc.; $X \& Z$ is a constituent of $\iota[Y]^x(X, X \& Z)$, etc.; and the constituent relation is transitive.
 3. X is a *sortalizer* of $Q[X]^x Y$, $\iota[X]^x(Y, Y \& Z)$, etc.
 4. x is *bound* in X if and only if x is not free in X but is free in some constituent of X .
 5. x *dangles* in X if and only if x is free in X but is bound in some constituent of X .
 6. X is a *proper* wff if and only if no variable dangles in X .
 7. *Abbreviations*
 - (i) $\iota[X]^x(Y, Y \& Z) =_{df} \iota[X]^x \langle Y, Z \rangle$;
 - (ii) $Q[Ex]^x X =_{df} QxX$; (where E is the existence predicate)
 - (iii) $\iota[Ex]^x \langle Y, Z \rangle =_{df} \iota x \langle Y, Z \rangle$; etc.

Strictly speaking, improper wffs are interpretable as open wffs; attention is drawn to them as deviants because they exemplify the results of misguided attempts to write down the logical forms of sentences containing 'pronouns of laziness', to borrow a phrase from Geach (1972a, 1972b).

Semantics

Tarski-style rules of satisfaction are easy to give for this logic. Suppose we have, for each quantifier symbol Q , the inducing function f_Q which is its interpretation. The satisfaction rules for the connectives and for atomic wffs are as usual; while for sortal quantification there is the following difference: the infinite sequence s of objects from domain D satisfies $Q[X]^x Y$ if and only if $f_Q(\langle \text{number of sequences } (X, i)\text{-differing from } s \text{ which satisfy } Y, \text{ number of sequences } (X, i)\text{-differing from } s \text{ which do not satisfy } Y \rangle) = t$; and similarly for (I, k) -quantifiers for $k > I$; where s' (X, i) -differs from s if and only if for every $j \neq i$ $s'_j = s_j$ and s' satisfies X . It follows that if X has no free variables, then it is satisfied by all sequences or by none, in which cases it is *true* or *false* respectively, with respect to the given interpretation of non-logical constants in the model at hand.

With this sortal logic we can provide many English sentences with more congruous logical forms than they would receive in the classical predicate logic. There are also English sentences which now can be coped with but which could not be accommodated at all in the old system. Geach provides a fund of examples, some of which we shall now consider.

- (1) Almost every man who borrows a book from a friend eventually returns it to him

If one tries to write out the wff representing the logical form of (1) by turning the relative clause *who borrows a book from a friend* into a conjunct of a sortalizer, and then treating *it* and *him* as occurrences of the variables ranging over *book* and *friend* respectively, the result is

- (2) $AE [Mx \& \exists [Bk y]^y \exists [Fzx]^z Bxyz]^x ERxyz$.

y and z dangle in (2). On the most likely reading

- (3) Almost every man who borrows a book from a friend eventually returns each book thus borrowed to the friend from whom he borrowed it

the correct logical form is

- (4) $AE [Mx \& \exists [Bk y]^y \exists [Fzx]^z Bxyz]^x \forall [Bk w \& \exists [Fux]^u Bxwu]^w w \langle Fxw \& Bxwv, ERxwv \rangle$.

Another example of Geach is

- (5) The only man who ever stole a book from Snead made a lot of money by selling it

Analysing this along the lines of *The only P who is Q is R*, which has logical form $\forall [Qx] \wedge x \langle Px, Rx \rangle$, we get

- (6) $\forall [\exists [By] \wedge y \wedge x \langle Sz, \text{Stole } xyz \rangle] \wedge x \langle Mx, \wedge w \langle Bw \wedge w \langle Sv, \text{Stole } xwv \rangle, \text{MLMBS}xw \rangle \rangle$.

Inspection will reveal that, according to our semantics, (6) has the same truth conditions as Geach's paraphrase

- (7) As regards some book, the man of whom it holds good that this was the book he stole from Snead and that nobody else ever stole a book from Snead made a lot of money by selling it.

Pronouns of laziness compound the difficulties with Geach's examples in (1972c). Here he is concerned to discover why (A) is valid while (B), so similar to (A) in surface form, is not:

- A. 1. Anything that counts as the personal property of a tribesman is suitable to offer to a guest by way of hospitality
 2. One thing that counts as the personal property of a tribesman is that tribesman's wife
 3. So, she is suitable to offer to a guest by way of hospitality
 B. 4. Any woman whom every tribesman admires is beautiful by European standards
 5. One woman whom every tribesman admires is that tribesman's wife
 6. So, she is beautiful by European standards

If *woman whom every tribesman admires* were a genuine complex term (in Geach's sense) then (B5) would have the overall form *One P is the Q*. In our notation this is $\wedge x \langle Qx, Px \rangle$. This naive reading of the logical form of (B5) would yield

- (8) $\wedge x \langle Wf xy, Wm x \wedge \forall [Ty] \wedge y Ad yx \rangle$

in which y dangles. The complex term *woman whom every tribesman admires* does not function as a genuine logical unit, to use another phrase of Geach. In our terminology we would say that it should not be rendered as a constituent in the wff which correctly represents the logical form of (B5). Re-written as wffs of our sortal logic, the clearly valid (A) and invalid (B) are as follows:

- A. 1. $\forall [\exists [Ty] \wedge y PPxy] \wedge x Sx$
 2. $\forall [Ty] \wedge y \wedge x \langle Wf xy, PPxy \rangle$
 3. $\forall [Ty] \wedge y \wedge x \langle Wf xy, Sx \rangle$

- B. 4. $\forall [Wm x \wedge \forall [Ty] \wedge y Ad yx] \wedge x BESx$
 5. $\forall [Ty] \wedge y \wedge x \langle Wf xy, Ad yx \rangle$
 6. $\forall [Ty] \wedge y \wedge x \langle Wf xy, BESx \rangle$

Note that if *a tribesman* were replaced in (A1) by *every tribesman* then (A) would be invalid in the same way as (B). This seems to be the real source of the difference between (A) and (B), and it is reflected in the different quantificational structures of (A1) and (B4) immediately above.

One example of quantificational structure which can be exhibited by a wff of our sortal logic, but not by one of classical logic, is that underlying the following (where italicization indicates emphasis):

- (9) Nearly all the delegates had read mostly *soft-covered* books.

This involves quantifying into a sortalizer:

- (10) $N[Dx] \wedge x M[By \wedge Rxy] \wedge y Sy$.

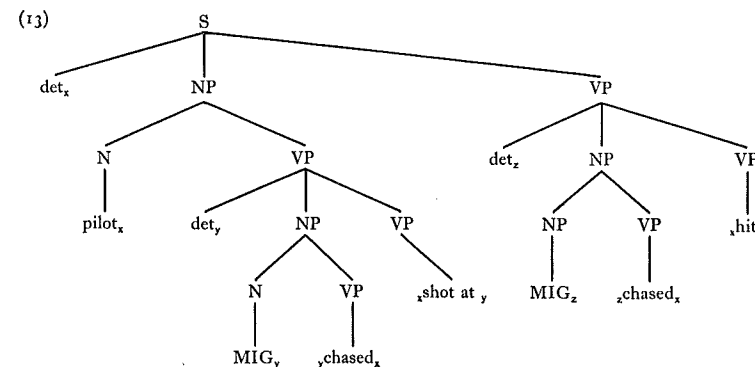
A final bothersome example which can be taken care of is the Bach-Peters sentence

- (11) The pilot who shot at it hit the MIG that chased him.

The wff in our sortal logic which represents its logical form is

- (12) $\wedge x \langle Px \wedge \wedge y \langle My \wedge Cxy, Sxy \rangle, \wedge z \langle Mz \wedge Cxz, Hxz \rangle \rangle$.

From this can be extracted the phrase-marker (13) for deep structure



which I have not yet come across in the literature on the problem. Someone may be interested in finding the transformations taking (13) to (11). Note that a direct reading of (11) is

- (14) The pilot who shot at the MIG that chased him, hit the MIG that chased him

in which the only pronominalization – *him* – is straightforwardly explained in terms of occurrences of the variable x in (11). It can then be introduced anaphorically, to get

(15) The pilot who shot at the MIG that chased him, hit it

which should be easily transformable into (11).

The same process of pronominalization seems to underlie

(16) Most boys who were fooling them kissed many girls who loved them.

This has logical form

(17) $Mo [Bx \& M[Gy \& Lyx]^y Fxy]^x M[Gz \& Lzx]^z Kxz \& Fxz$

The direct reading of (17) is

(18) Most boys who were fooling many girls who loved them kissed many girls who loved them

in which *them* comes from occurrences of the variable x in (17); anaphoric pronominalization then accounts for

(19) Most boys who were fooling many girls who loved them, kissed them

which should then be easily transformable to (16).

I hope these few examples have indicated that a logic of sortal quantification can contribute to an understanding of the grammar of complex terms and pronominalization.

Before we conclude, however, that sortality is a primitive phenomenon in the quantificational structure of sentences of a certain complexity, and that the syntactical device of sortalizers along with the corresponding modifications of the rules of satisfaction is the only way to cope with it, we should consider the implications of the following result obtained by investigating a suggestion of Chris Hookway (private communication): For every wff $Q_f[X]^x(Y_1, \dots, Y_n)$ with Q_f a (I, n) -quantifier induced by f , there is an equivalent wff $Q_{f'}[X, X \& Y_1, X \& Y_2, \dots, X \& Y_n, X \& -Y_n]$ with $Q_{f'}$ a $(I, 2n+1)$ -quantifier induced by f' , with f and f' interdefinable thus:

$$f(\langle a_1, b_1 \rangle, \dots, \langle a_n, b_n \rangle) = t \text{ (} a_i + b_i \text{ constant)}$$

iff

$$\text{for all } c_0, c_1, \dots, c_{2n} \text{ such that } a_1 + b_1 + c_0 = a_1 + c_1 = \\ = b_1 + c_2 = \dots = a_n + c_{2n-1} = b_n + c_{2n},$$

$$f'(\langle a_1 + b_1, c_0 \rangle, \langle a_1, c_1 \rangle, \langle b_1, c_2 \rangle, \dots, \langle a_n, c_{2n-1} \rangle, \langle b_n, c_{2n} \rangle) = t;$$

and for all $a_0, \dots, a_{2n}, b_0, \dots, b_{2n}$ such that $a_i + b_i$ is constant and

$$a_1, \dots, a_{2n} \leq a_0, \\ f'(\langle a_1, b_2 \rangle, \langle a_1, b_1 \rangle, \dots, \langle a_{2n}, b_{2n} \rangle) = t$$

iff

$$f(\langle a_1, a_2 \rangle, \langle a_3, a_4 \rangle, \dots, \langle a_{2n-1}, a_{2n} \rangle) = t.$$

Thus every restricted (I, n) -quantification can be treated as an unrestricted $(I, 2n+1)$ -quantification. If we opt always for the latter, we need not have sortalizers bracketed off in the syntax, but need only ensure that the sortalizing wff appears alone in the first position, and then as a conjunct in the appropriate fashion in each of the following $2n$ positions.

This underlines the indeterminacy of the syntactical representation of logical form; a syntactical structure represents *the* logical form of a sentence only modulo the formal semantics. We may feel that it is more 'natural' to bracket off the sortalizer as in the system developed above, and to introduce the notion of sequences (X, i) -differing from one another, but in Hookway's view this is to indulge a mere prejudice – we should not be concerned at all with mirroring even the most striking features of surface form in the wff representing logical form. All that matters is that the wff be so interpreted by the formal semantics that it capture the truth conditions of the sentence.

Moreover, which wff, or which abbreviation of it is chosen as a deep structure is immaterial since such notational variants can easily be transformed into one another. What *is* important is that there is a variable-binding configuration which is invariant with respect to the method of its representation by syntactical complexes; and it is this configuration which both logic and grammar must lay bare.

APPENDIX

Lewis' (this volume) adverbial quantifiers would appear to be easily representable as (n, I) -quantifiers in my sense:

$$Q[\Phi]^{x_{i_1}, \dots, x_{i_n}} \Psi$$

Extra clauses in the recursive definition of wff are easily formulable, as are the corresponding clauses in the recursive definition of satisfaction:

$$s \text{ satisfies } Q[\Phi]^{x_{i_1}, \dots, x_{i_n}} \Psi \text{ iff } f_Q(\langle a, b \rangle) = t$$

where $a = \text{card } \{s' \mid s'(\Phi, i_1, \dots, i_n)\text{-differs from } s \text{ and } s' \text{ satisfies } \Psi\}$ and $b = \text{card } \{s' \mid s'(\Phi, i_1, \dots, i_n)\text{-differs from } s \text{ and } s' \text{ does not satisfy } \Psi\}$, where $s'(\Phi, i_1, \dots, i_n)$ -differs from s iff for all $j \neq i_1, \dots, i_n$ $s'_j = s_j$ and s' satisfies Φ .

$Q[\Phi] \wedge x_{i_1}, \dots, x_{i_n}$ has the reading ' Q^* -ly, if Φ then Ψ ' where e.g. if Q is *most* then Q^* -ly is *usually*.

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