Far-Field Synthesis from Complex Point Sources via Transformation Metamaterials*

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***Abstract*—It is well known that an electromagnetic field can be expanded as a weighted sum of Gaussian beams. Convergence to a given field distribution is guaranteed by adding enough beams and controlling individual beam characteristics such as beam waist and propagation direction. At the same time, a Gaussian beam can be recognized as the field produced paraxially by a complex point source with imaginary displacement along the axial beam direction. In this work, we propose a design methodology for metamaterials (flat transformation media) enabling the synthesis of electromagnetic far-field patterns based on such complex source representations. The methodology is based on the application of complex transformation optics to derive constitutive tensors that map point sources in real-space to effective complex point sources.**

I. Introduction

Gaussian beams can be used as basis elements in an electromagnetic (EM) field expansion [1], [2]. Convergence to a desired field distribution can be achieved by adding enough beams and controlling beam characteristics such as displacement, tilt, and waist. At the same time, a Gaussian beam can be recognized as the field produced paraxially by a complex point source with imaginary displacement along the axial beam direction [1], [3]. A complex point source is an hypothetical source that is located at a complex-valued coordinate point. It was recently shown [4], [5] that the effect of complex sources can be mimicked by certain metamaterials [6].

Transformation optics (TO) unveils a duality between coordinate transformations and material properties [7], [8]. By exploring such duality, TO-based media have been designed for diverse applications [9], [10]. More recently, *complex* transformation optics (CTO) has been proposed to extend the range of functionalities of TO [4], [5], [11]–[16]. In contrast to TO, which is restricted to real-valued coordinate transformations, CTO is based on complex-valued transformations. CTO can provide wave amplitude control through distributed absorption/gain effects [5], [16] in addition to phase control. CTO stipulates that any coordinate transformation that displaces the (real-valued) spatial coordinates of a point source to (equivalent) complex-valued spatial coordinates can be mimicked by a properly chosen metamaterial.

In this work, we propose a general methodology to produce arbitrary far-field patterns from point sources placed next to CTO metamaterial slabs. Each point source is equivalently

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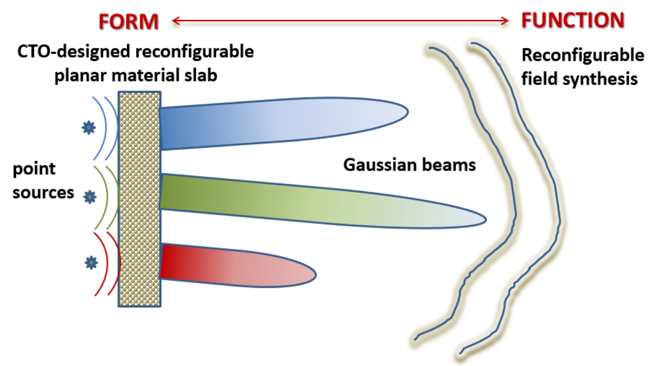


Fig. 1: CTO-based engineered metamaterial slab for field synthesis. The proposed metamaterial slab maps point sources to (effective) complex point sources that produce Gaussian beams as field solutions in the paraxial region. By properly designing the metamaterial, individual beam characteristics such as displacement, tilt, and beam waist can be controlled. A desired far-field distribution can be approximated by a superposition of such beams.

mapped to complex-valued point and, as a result, produces Gaussian beams. Judicious superposition of such point sources can be used to produce a desired field distribution. The proposed methodology yields a direct mapping of EM structure (metamaterial properties) to function (field synthesis), as illustrated in Fig. 1. This mapping is (a) predictive, (b) parametric, and (c) invertible in the sense that the EM field synthesis is based on the location of the complex point source which has a biunivocal relation to the required material constitutive parameters [5]. Morevover, the proposed metamaterial is impedance-matched to free space, which increases the overall efficiency for wireless power transfer applications for example.

II. Basic Methodology and Exploratory Results

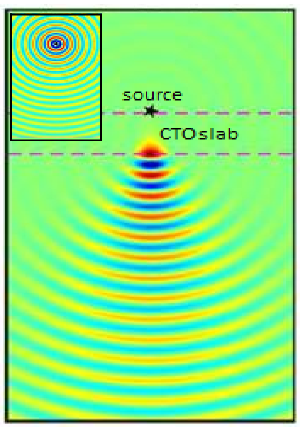
We assume the $e^{-i\omega t}$ convention. From CTO theory, the permittivity and permeability constitutive tensors of a planar metamaterial slab designed to mimic a complex point source [5] can be written as $\overline{\boldsymbol{\epsilon}} = \epsilon\overline{\boldsymbol{\Lambda}}$ and $\overline{\boldsymbol{\mu}} = \mu\overline{\boldsymbol{\Lambda}}$, with

$$\overline{\boldsymbol{\Lambda}} = diag\{s_y, s_y^{-1}, s_y\}$$

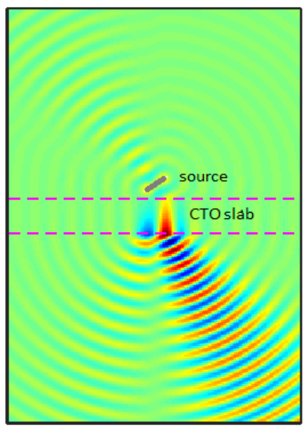
where $s_y = \partial\tilde{y}/\partial y$, and the relevant coordinate transformation is

$$y \to \tilde{y} = r(y) + i\frac{a(y)}{\omega}$$

for $0 < y < h$ assuming a planar metamaterial parallel to the xz plane with thickness h. The parameter $a(y)$ is a continuous function with $a(y) = 0$ for $y \leq 0$ and $a(y) = a(h)$ for $y > h$. The choice $a(y) < 0$ for $0 < y < h$ yields a gain-medium that amplifies the fields from point sources (or electrically small antennas) placed near to it. In particular, we obtain a Gaussian beam in the paraxial region along the negative y axis when a complex point source is present at $y > h$. Note that the coordinate transformation $y \rightarrow \tilde{y}$ is continuous and the resulting metamaterial is impedance matched to free space for all incidence angles [7]. The parameter $r(y)$ can be chosen so as to increase (if $\partial r/\partial y > 1$) or decrease (if $\partial r/\partial y < 1$) the electric thickness of the metamaterial. In particular, the electric thickness reduces to zero for $\partial r/\partial y \rightarrow 0$, which means that no phase accumulation would occur along the longitudinal direction as the wave propagates through the slab region. Interestingly, the choice $\partial r/\partial y < 0$ can be used to produce negative refraction [16].



(a)



(b)

Fig. 2: Field from a CTO metamaterial slab of thickness $d = 2\lambda$. (a) Single point source on top of the slab. (b) Array of point sources on top of the slab. See main text for further details.

Fig. 2 depicts computational results of the EM field produced by sources placed on top of a CTO metamaterial slab with $\overline{\mathbf{\Lambda}} = diag\{-0.5i, 2i, -0.5i\}$ in 2-D (TM problem). The slab has thickness $d = 2\lambda$, where λ is the wavelength. Fig. 2(a) shows the spatial distribution of E_z in the xy plane for a single point source present at the location indicated by a star (the inset shows the consequent field distribution in free-space). The emergence of a Gaussian beam for $y < 0$ along the paraxial direction can be clearly observed. The waist and other properties of the beam can be adjusted by changing the metamaterial parameters. Fig. 2(b) shows the field distribution produced by an array of five point sources in the position indicated by the gray trace. A Gaussian beam is again observed, but now tilted because of the array factor. The array source produced a sidelobe which is also amplified by the CTO slab. Although not shown here, the beam can also be directly tilted by varying the metamaterial parameters along the lateral (transversal) direction.

III. Conclusion and Look Ahead

We explored the application of complex transformation optics to derive metamaterial slabs that map point sources in real space to effective complex point sources. Each complex point source generates a Gaussian beam paraxially. Consequently, a desired far-field EM pattern can be obtained based on a standard Gaussian beam expansion where individual Gaussian beam properties (displacement, tilt, and waist) could be further adjusted by locally varying the metamaterial parameters. The proposed metamaterials correspond to doubly-anisotropic gain media and hence pose fabrication challenges. In practice, approximate metamaterial blueprints may be considered to achieve a satisfactory compromise between performance and ease of fabrication. The present CTO methodology can in principle be generalized [17] to obtain metamaterials over curved surfaces with analogous functionality.

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