Math 3345 - Spring 2018 - Taylor
Assignment 12

Assignment 12 is due in class on Friday, April 13.
The following proof journal assignment is due in class on Wednesday, April 18.

1. **Proof Journal Assignment**: A set S is called *countable* if there exists an injective function from S to the natural numbers, $\mathbb{N}$. Use an argument similar to Cantor's Diagonal Lemma (found in Chapter 3, section 15) to prove that the set of all infinite sequences consisting of $0'$s and $1'$s is uncountable. i.e. $\{0,1\}^{\mathbb{N}}$ is uncountable.

2. **Permutations** - From the book:

   (a) page 155, exercise 11

3. **Infinity** - From the book:

   (a) page 160-161, exercises 1-4 (note: if an exercise was done in class, then you should check your understanding and write up the solution neatly).

4. **Review: Images of functions** - From the book:

   (a) page 138, exercise 16
   (b) page 138, exercise 17

5. **Review: Graphs of functions** Recall, the *graph* of a function f is the set of ordered pairs (x, y) so that x is in the domain of f and $y = f(x)$. i.e.

$$Graph(f) = \{(x, f(x)) : x \in Domain(f)\}.$$

   Let f be a bijection from $\mathbb{R}$ to $\mathbb{R}$ so that f has a well-defined inverse, f^{-1}. Prove that the graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

6. **More on surjections** - From the book:

   (a) page 143, exercise 29 (b) (you can read the Axiom of choice on page 142)

7. **Review: Pascal triangle, binomial theorem, and modular arithmetic**

   (a) Let $n \in \mathbb{N}$ and $k \in \mathbb{N} \bigcup \{0\}$. Give two different proofs that $\binom{n}{k}$ is an integer.
   (b) Let $p \in \mathbb{N}$ and $x \in \mathbb{R}$. Prove that

$$(1+x)^p \equiv (1+x^p) \mod p.$$

8. Consider the following argument. Is it correct?

$$\begin{aligned}
&(x,y) \in (C \times D) \setminus (A \times B) \\
\iff &(x,y) \in (C \times D) \text{ and } (x,y) \notin (A \times B) \\
\iff &(x \in C \text{ and } y \in D) \text{ and } (x \notin A \text{ or } y \notin B) \\
\iff &[(x \in C \text{ and } y \in D) \text{ and } (x \notin A)] \text{ or } [(x \in C \text{ and } y \in D) \text{ and } (y \notin B)] \\
\iff &[(x \in C \backslash A \text{ and } y \in D)] \text{ or } [(x \in C \text{ and } y \in D \backslash B)] \\
\iff &(x,y) \in [(C \backslash A) \times D] \bigcup [C \times (D \backslash B)]
\end{aligned}$$

In the same style of the argument above, prove that

$$(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)\,.$$