# Mathematics 3345

**Meeting Time:** MWF 1:50PM - 2:45PM , Bolz Hall 124

**Website:** http://u.osu.edu/taylor.2952/teaching/math-3345/

**Instructor:** Professor Krystal Taylor

**Office:** MW 622

**Email:** taylor.2952@osu.edu (The instructor prefers that you speak to her directly before or after class or in office hours. Any emails to the instructor should be from your university email address.)

**Office Hours:** TBD

**Texts:** Official text:
**The Fundamentals of Higher Mathematics** by Neil Falkner
Recommended:
**A Concise Intro. to Pure Mathematics** by M. Liebeck
Recommended (FREE!) texts are:
**The Book of Proof** by Richard H. Hammack
**Mathematical Reasoning** by Ted Sundstrom
Relevant readings will be on the homework.

**Content:** The focus of this class will be learning how to read and write mathematical proofs. The mathematical content, while important, is secondary to the reading and writing goals. Consequently, the standards for writing in this class will be higher than in previous math classes. There will be a number of in-class activities to develop students' writing skills. Students are expected to memorize definitions and statements of theorems.

**Technology:** Typing up your homework using TEXis encouraged. Please do not surf or text during class as it is distracting to me and possibly to the people around you.

**Homework, Quizzes:** Homework will be collected weekly at the beginning of class. No late homework will be accepted. The lowest homework score will be dropped.

Homework can be handwritten (if legible), word processed, or in some variety of TEX. If wordpressed or in TEX, please use at least 12 point font. Emailed homework **will not be accepted** except under special circumstances.

Turned-in homework should have your full name and be stapled. Quizzes will be announced at least one class in advance and have the same weight as one homework.

**Writing:** There will be writing assignments due every week. The purpose of writing in mathematics is not merely to demonstrate that *you know something* but rather to *explain the material to someone who does not yet know it.* Consequently, you should provide sufficient details and aim to write with clarity. We will discuss this in class.

**Bonus:** There will be harder bonus problems on the homework and on exams. While they earn extra credit, they give students a chance to distinguish themselves if they might like to request a letter of recommendation from the instructor. Extra credit will be used to decide grades for border cases. Writing homework using TEXis another opportunity for extra credit.

**Exams:** There will be two in-class midterm exams and a comprehensive final.

| | | |
|---|---|---|
| Midterm 1 | Friday, February 9 | in class |
| Midterm 2 | Friday, March 9 | in class |
| Final Exam | Tuesday, May 1 | 4 - 5:45 pm |

Exams are closed book and closed notes. Problems on the exams will generally be similar to the homework and quiz problems. The instructor will be maintaining a list of the type of problems that can be asked on the exam.

If a midterm is missed and the instructor is given prior notice and official documentation of an emergency or illness, the other exams will be re-weighted.

**Grading:**

| | |
|---|---|
| Homework and Participation | 20% |
| Midterm exams | 40% |
| Final exam | 40% |

Individual test scores will not be re-centered. Raw scores will be used to compute overall course scores. Course grades will be determined by applying cut-offs to the course scores. The instructor will not take personal factors into account when assigning course grades.

**Grade Revisions:** You may request that homework or exams be re-graded. This request must be *in writing* and turned in at the beginning of the class immediately after the work is returned to you. No late grade revision requests will be accepted.

**Academic Misconduct:** It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct" includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-48.7). For additional information, see the Code of Student Conduct at `http://studentlife.osu.edu/csc/`.

**CCS:** As a student you may experience a range of issues that can cause barriers to learning, such as strained relationships, increased anxiety, alcohol/drug problems, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may lead to diminished academic performance or reduce a student's ability to participate in daily activities. The Ohio State University offers services to assist you with addressing these and other concerns you may be experiencing. If you or someone you know are suffering from any of the aforementioned conditions, you can learn more about the broad range of confidential mental health services available on campus via the Office of Student Life Counseling and Consultation Services (CCS) by visiting `ccs.osu.edu` or calling 614-292- 5766. CCS is located on the 4th Floor of the Younkin Success Center and 4th Floor of the PAES Building. 24 hour emergency help is also available through the National 24/7 Prevention Hotline at 1-800-273-TALK or at `suicidepreventionlifeline.org`.

**Diversity:** The Ohio State University affirms the importance and value of diversity in the student body. Our programs and curricula reflect our multicultural society and global economy and seek to provide opportunities for students to learn more about persons who are different from them. We are committed to maintaining a community that recognizes and values the inherent worth and dignity of every person; fosters sensitivity, understanding, and mutual respect among each member of our community; and encourages each individual to strive to reach his or her own potential. Discrimination against any individual based upon protected status, which is defined as age, color, disability, gender identity or expression, national origin, race, religion, sex, sexual orientation, or veteran status, is prohibited.

**SLDS Statement:** The University strives to make all learning experiences as accessible as possible. If you anticipate requiring accommodations based on a disability (including mental health, chronic or temporary medical conditions), you are encouraged to register as soon as possible with the Student Life Disability Services to establish reasonable accommodations. After registration, please make arrangements with me so that they may be implemented in a timely fashion. SLDS contact information: `slds@osu.edu`; 614-292-3307; `slds.osu.edu`; 098 Baker Hall, 113 W. 12th Avenue.

# MATH 3345H SYLLABUS
# HONORS FOUNDATIONS OF HIGHER MATHEMATICS

Math 3345H is a Foundations of Higher Mathematics class aimed at strong, enthusiastic students. Its primary goal is to be a systematic introduction to problem solving and proof-writing through *interesting, non-trivial mathematics*. This goal should help the class serve as a bridge between a problem-oriented Calculus class and a more conceptual Linear Algebra class. Its secondary goal is to generate excitement about mathematics by exposing students to some of math's greatest intellectual successes, many of which do not easily fit into standardized undergraduate classes.

While many students taking this class are interested in majoring in a technical field and have seen Calculus, this class does not rely on much background material. For the most part, the class will make do with high school algebra. Instead, the focus is becoming more fluent in and comfortable with abstraction.

Homework involves routine practice, proof-writing, and some non-trivial problems. Optional bonus problems will be provided for students who want a challenge.

Students will be evaluated on the basis of two midterm exams, one final exam, weekly homework, and a writing project. The writing project is an expository report of eight to ten pages on a mathematical topic not covered in class (some possibilities are among the additional topics below). The intended audience for such a report would be another strong undergraduate student with no training in mathematics besides a typical high school course of study. Students will be graded on the correctness of their mathematics and quality of their exposition.

There are a number of textbooks that give different introductions to mathematics. Martin Liebeck's *A Concise Introduction to Pure Mathematics* will be the main textbook. Affordable supplemental texts to provide more background and interesting topics for reports are the following:

(1) Richard Hammack, *Book of Proof* explains proof techniques,
(2) Ted Sundstrom, *Mathematical Reasoning* is a very detailed text on the core topics,
(3) Mark Kac and Stanislaw Ulam, *Mathematics and Logic* touches on many beautiful mathematical topics while giving a feel for the unity of mathematics,
(4) Martin Aigner and Günter Ziegler's *Proofs from the Book* provides many additional topics.

By the end of the semester, the students should have seen the following core techniques in a mathematical context:

(1) Quantifiers;
(2) Proof techniques: direct proof, contrapositive, proof by contradiction;
(3) Proving universal and existential statements;
(4) Disproving statements by finding counterexamples;
(5) Principle of mathematical induction, complete induction, the well-ordering principle;

(6) Proof by smallest counterexample (descent);
(7) Basic set theory: sets, set building notation, containment proofs;
(8) Functions: surjections, injections, bijections, composition of functions, pointwise operations on functions;
(9) The pigeonhole principle;
(10) Infinite sets: countable and uncountable;
(11) Indexed summation, union, and intersection;
(12) Equivalence relations.

A number of mathematical topics will be covered to give the students experience writing proofs and solving problems. The follows topics will certainly be covered:

(1) Basic number theory: divisibility, congruence, definition of primes, infinitude of primes, division algorithm, unique factorization;
(2) Cardinality and countability;
(3) Existence of uncountable sets;
(4) Rational numbers and their arithmetic;
(5) Irrationality proofs.

There are a number of topics that can be covered at the instructor's discretion. Some possibilities include the following:

(1) Discrete mathematics
    (a) Inclusion/exclusion and counting
    (b) Generating series and change-making;
    (c) Binomial coefficients, Pascal's triangle, binomial theorem, binomial coefficients in probability;
    (d) Sperner's Lemma;
    (e) Pick's theorem;
    (f) Planar graphs and Euler's formula;
    (g) Ramsey's theorem;
    (h) Solving recurrences with initial conditions; the Fibonacci recurrence and formulas for $F_n$;
(2) Number theory
    (a) The relationship between rationality/irrationality and decimal expansions (repeated/non-repeating digits);
    (b) Euler's proof of the infinitude of primes by harmonic series;
    (c) The rational roots theorem and irrationality of non-integer roots of monic integral polynomials;
    (d) Irrationality of $e$;
    (e) Liovuille's theorem and examples of transcendental numbers;
    (f) Fermat's little theorem, Euler's $\phi$ function, and RSA encryption.
    (g) Pythagorean triples and parameterization of the circle;
    (h) Linear Diophantine equations
(3) Set theory
    (a) Algebraic numbers and their cardinality, Cantor's argument for existence of transcendental numbers;
    (b) The Schröder-Bernstein theorem;
(4) Geometry
    (a) Platonic solids, definition and classification;
    (b) Tilings of the plane;
    (c) Hilbert's Third Problem and Dehn's invariant;

(5) Analysis
  (a) Cantor sets;
  (b) The definition of continuity and nowhere continuous functions;
  (c) Applications of the intermediate value theorem to surjectivity of functions;
  (d) Applications of the mean value theorem to injectivity of functions;
(6) Abstract algebra
  (a) Symmetric groups, symmetry groups of geometric figures;
  (b) Solvability of the cubic by discriminants and symmetry breaking in $S_3$;
  (c) Quadratically constructible numbers and the impossibility of doubling the cube;
  (d) Signs of permutations and the 15 puzzle;
(7) Applications to economics
  (a) Fixed point theorems;
  (b) Nash equilibria and basic game theory;
(8) Computer science
  (a) Examples of NP-complete problems and their equivalences;
  (b) Turing machines and decidability.