On the Determinants of Net Foreign Investment

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1. Introduction

ALTHOUGH THE BENEFITS of international diversification have been widely documented, little is known about the determinants of net foreign investment. Existing models of portfolio choice in open economies provide no answer to the question of why net foreign investment is positive or negative. In other words, these models do not explain why investments in productive processes in any country exceed or fall short of the wealth of the residents of that country. Most existing models of portfolio choice and asset pricing make the key assumption of a given world market portfolio, which implies that at any point in time net foreign investment itself is given. The present paper does not make this assumption.

This paper provides a simple model of net foreign investment in a world of perfect markets. The model has the useful property that production decisions are made simultaneously with portfolio choice decisions, as in Cox, Ingersoll and Ross (1978). The central feature of the model is that technologies in use differ across countries. It is assumed that pre-existing capital can be shifted costlessly between countries. Production decisions, in other words, decisions about how the existing stock of capital is invested among available technologies, determine net foreign investment in the home country. However, production decisions depend on the distribution of the returns to investments in various technologies.

Whereas this paper analyzes an issue which has always been of considerable interest in the field of international finance, it uses a different framework from that used in most of the literature on net capital flows. It recognizes explicitly the role of risk as a determinant of net foreign investment. Because political risk is not modeled explicitly, this paper constitutes only a first attempt at understanding the determinants of net foreign investment. Political risk must play a significant role as a determinant of net foreign investment, because net

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1 Net foreign investment in the domestic country is understood to mean the difference between total wealth invested in the domestic country and domestic wealth. It follows that net foreign investment is a stock and not a flow. International economists would call net foreign investment the net asset position of foreign investors in the domestic economy. Financial economists, in general, use the terminology adopted here. See, for instance, Solnik [10].

2 For references to these models, see Adler and Dumas [1] or Stulz [11]. (Notice that the model developed in Stulz [11] does not require that the world market portfolio is given.)

3 Black [2] and Stulz [12], for instance, provide discussions of barriers to international investment.

4 For references to this literature see, for instance, Buiter [4].
foreign investment is the fraction of a country’s foreign investments which is the most exposed to political risk.

Contrary to most models of international finance, the model developed in this paper does not include an exchange rate which changes over time. Exchange rate dynamics affect the demands for risky assets, and therefore, affect net foreign investment. However, it would violate the general equilibrium nature of the work presented here to introduce arbitrarily an exchange rate and exchange rate dynamics. The task of introducing national currencies in a general equilibrium framework goes beyond the ambition of the present paper. A less ambitious way of introducing an exchange rate is to look at a world with two countries and two commodities in which the exchange rate corresponds to the terms of trade. In such a world, the problem studied in this paper becomes much more complex as net foreign investment depends on the consumption preferences of investors. However, the determinants of net foreign investment stressed in the present paper remain of crucial importance if the analysis is extended to a world in which the exchange rate corresponds to the terms of trade. A complete analysis of the determinants of net foreign investment in a world with two commodities will be pursued in another paper.

The plan of the paper is as follows. In Section 2, an intertemporal model is presented in which investors maximize their expected utility of lifetime consumption. This approach allows an analysis of how net foreign investment changes over time and how changes in technologies and the possibility of future changes in technologies affect net foreign investment. In Section 3, the determinants of net foreign investment are studied using the first-order conditions of the investor’s optimization problem. In Section 4, the special case in which investors have a logarithmic utility function is examined. Section 5 offers a summary of the results and concluding remarks.

2. The Model

At any point in time one would expect different countries to produce the same commodity in different ways. Countries are endowed with different resources, some of which cannot be traded internationally. Climate, land, and some human resources in the short-run, are examples of resources that are not traded internationally. Furthermore, the structure of the property rights, including governmental regulations, differs across countries. In order to focus the analysis, it is assumed, for simplicity, that each country uses different technologies. However, there is no restriction that prevents technologies from changing through time.

In the following, a simple model which captures differences in technologies across countries is constructed. Without loss of generality, it is assumed that there are only two countries, i.e., the domestic country and the foreign country. The domestic country and the foreign country use different technologies. It is assumed that only one (possibly composite) commodity is produced in the world. The commodity is used as an input in its own production. Each technology

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5 See, for instance, Macedo [8] and Stulz [13].

6 In the theory of international trade, models in which technologies differ across countries are often characterized as Ricardian models. A survey of the literature on Ricardian models under uncertainty is given by Pomery [9]).
exhibits constant returns to scale in the sense that output is twice as high if the quantity invested in the production process doubles.

Without loss of generality, it is assumed that all domestic firms use the same technology and all foreign firms use the same technology. All firms which use an identical technology form an industry and have perfectly correlated outputs. If \( I(F) \) is the quantity of the commodity invested in the domestic (foreign) country and if the output is re-invested in the same country, \( I \) and \( F \) evolve according to:

\[
\frac{dI}{I} = \mu_I \, dt + \sigma_I \, dZ_I \quad (1a)
\]

\[
\frac{dF}{F} = \mu_F \, dt + \sigma_F \, dZ_F \quad (1b)
\]

where \( dZ_I(dZ_F) \) is the increment of a standard Wiener process.\(^7\) To simplify the notation, it is assumed that the return of investing in the domestic technology is uncorrelated with the return of investing in the foreign technology. Technologies, therefore, can change stochastically over time. However, they depend only on state variables whose value at future dates cannot be affected by the choices of investors. These state variables are assumed to be uncorrelated with the returns of technologies and can be interpreted as indices of productivity. These assumptions can be relaxed without affecting the results of this paper.

It is assumed that markets are perfect internationally and that both countries use the produced commodity as the numeraire. In this setting, an investor can choose his consumption rate and his portfolio independently of where he lives. Let the wealth of an investor be \( W \). The investor can invest in the domestic and foreign technologies. Furthermore, he can lend or borrow at the risk-free instantaneous rate of interest \( R \) per unit of time. Let \( D(A) \) be the value of the investor's investment in the domestic (foreign) technology. \( W \) evolves according to:

\[
dW = D\left(\frac{dI}{I} - R \, dt\right) + A\left(\frac{dF}{F} - R \, dt\right) + RW \, dt - C \, dt \quad (2)
\]

where \( C \) is the consumption rate per unit of time of the investor.

The investor maximizes a von Neuman-Morgenstern expected utility function of lifetime consumption and is infinitely-lived:

\[
E_t\left\{ \int_{\tau = t}^{\infty} e^{-\rho\tau} u(C, \tau) \, d\tau \right\} \quad (3)
\]

where \( E_t \) denotes the expectation operator conditional on information available at time \( t \). Let \( \mathcal{S} \) be an \( S \times 1 \) vector which includes all relevant state variables. It is assumed that the state variables follow Ito processes. Define \( J(W, \mathcal{S}, t) \) as:

\[
J(W, \mathcal{S}, t) = \max_{\{D,A,C\}} \ E_t\left\{ \int_{\tau = t}^{\infty} e^{-\rho\tau} u(C, \tau) \, d\tau \right\} \quad (4)
\]

\(^7\) Merton [8] discusses the properties of stochastic differential equations.
The first-order conditions\(^8\) for investments in industries and consumption are:

\[
J_W(\mu_l - R) + J_{WW}\sigma_l^2 D + \lambda_{I\bar{S}} \lambda_{WS} = 0 \quad (5a)
\]

\[
J_W(\mu_F - R) + J_{WWW}\sigma_F^2 A + \lambda_{I\bar{S}} \lambda_{FS} \lambda_{WS} = 0 \quad (5b)
\]

\[
J_W = u_C \quad (5c)
\]

where \(J_W = \partial J/\partial W, J_{WW} = \partial^2 J/\partial W^2\), \(J_{WS}\) is a \(S \times 1\) vector whose \(i\)th element is \(J_{WS_i} = \partial^2 J/\partial W \partial S_i\) and \(u_C = \partial u/\partial C\). \(\lambda_{I\bar{S}}(\lambda_{FS})\) is a \(S \times 1\) vector of instantaneous covariances between the return on the domestic (foreign) technology and changes in state variables.

3. Net Foreign Investment

It is assumed, without loss of generality, that there is only one domestic investor and one foreign investor.\(^9\) The superscript \(D(F)\) is used to designate the domestic (foreign) investor. The first-order conditions can be used to obtain the domestic and the foreign investors’ investments in the domestic country and abroad:

\[
D^i = \frac{1}{\sigma_l^2} \left(\frac{-J_{W}}{J_{WWW}}\right) (\mu_l - R) + \frac{1}{\sigma_l^2} \lambda_{I\bar{S}} \left(\frac{-J_{WS}}{J_{WWW}}\right) i = D, F \quad (6a)
\]

\[
A^i = \frac{1}{\sigma_F^2} \left(\frac{-J_{W}}{J_{WWW}}\right) (\mu_F - R) + \frac{1}{\sigma_F^2} \lambda_{I\bar{S}} \left(\frac{-J_{WS}}{J_{WWW}}\right) i = D, F \quad (6b)
\]

If investors differ across countries, the distribution of wealth across countries is a state variable.\(^10\) Investors can construct a portfolio whose instantaneous return is perfectly correlated with instantaneous changes in the distribution of world wealth. It cannot, therefore, be assumed that changes in this state variable are uncorrelated with the returns to investments in risky assets. Given the assumptions made in this paper, in general, risk-averse investors hedge against unanticipated changes in one state variable, designated by \(S\) in the following.\(^11\) The

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\(^8\) For the sake of brevity, we assume that \((\mu_l - R)\) and \((\mu_F - R)\) are always non-negative. A more complete analysis would deal explicitly with the non-negativity constraints associated with the investments in industries and with the problems which arise when these constraints are binding.

\(^9\) This assumption is made to simplify the notation. As markets are assumed to be perfect, perfect competition holds on capital markets.

\(^10\) The state of the world is completely characterized by the state variables which determine the properties of the technologies, the stock of the commodity and domestic (or foreign) wealth. It follows that the state variables which characterizes the distribution of wealth across countries can be viewed as the stock of the commodity. If the foreign investor is the same as the domestic investor, the economy considered here is the same as the economy considered by Cox, Ingersoll and Ross [5]. In the economy described in Section 2, if all investors are the same, there is no state variable whose changes are correlated with the returns of technologies and, consequently, \(V_{m_U} = V_{FS} = 0\), for all \(i\)’s.

\(^11\) In this paper, an investor is said to “hedge” against unanticipated changes in the \(i\)th state variable iff some asset return is correlated with changes in this state variable and \(J_{WS} \neq 0\). A more detailed analysis would take into account the fact that some investors reverse-hedge. Let \(\epsilon_S\) be the compensating variation in wealth required following a change in the \(i\)th state variable to maintain the investor’s lifetime expected utility constant:
Foreign Investment

only way that the distribution of world wealth affects the portfolio choices of an individual investor who has a given amount of wealth is through its impact on the instantaneous rate of interest. As some investor borrows while some other investor lends, a change in the instantaneous rate of interest affects investors in opposite ways.\(^{12}\)

In this economy, world wealth, written \(W^w\), is equal to the available stock of the commodity. For the remainder of this section, it is useful to solve for the interest rate in equilibrium, i.e., where the total stock of the commodity is invested in productive processes:

\[
R(S) = (\sigma_F^2 + \sigma_I^2)^{-1}(\sigma_F^2 \mu_I + \sigma_I^2 \mu_F + \sigma_F^2 \sigma_I^2 (G - W^w)/T^w)
\]  

(7)

where

\[
T^w = \frac{-J^D_W}{J^D_W} + \frac{-J^F_W}{J^F_W}
\]  

(8)

\[
G = \frac{\sigma_{IS}}{\sigma_I^2} \left( \frac{-J^F_{WS}}{J^F_{WW}} + \frac{-J^F_{WS}}{J^F_{WW}} \right) + \frac{\sigma_{FS}}{\sigma_F^2} \left( \frac{-J^D_{WS}}{J^D_{WW}} + \frac{-J^D_{WS}}{J^D_{WW}} \right)
\]  

(9)

Notice that \(\sigma_{IS}(\sigma_{FS})\) is the instantaneous covariance per unit of time between the rate of return on the domestic (foreign) technology and the changes in the state variable.

Let \(N^D\) be the net foreign investment in the domestic country. By definition, \(N^D\) is equal to the investments made in the domestic country minus domestic wealth \(W^D\) (in the presence of riskless borrowing or lending by the domestic country, the wealth of the domestic country is not equal to the domestic country's investments in industries, i.e., \(W^D \neq D^D + A^D\)).

\[
N^D = D^D + D^F - W^D
\]  

(10)

The remainder of this section is devoted to an analysis of the determinants of net foreign investment, i.e., \(N^D\), using (6a), (6b), and (7). However, there is no simple explanation for the sign and size of \(N^D\). Suppose, for instance, that the domestic country is richer than the foreign country, i.e., \(W^D > W^F\). In this case, inspection of (6a), (6b), and (10), shows that it is impossible to predict the sign of \(N^D\) based upon the relative wealth of the two countries.

To understand the determinants of net foreign investment, it is useful to start

\[
\epsilon_S = \frac{1}{W} \left. \frac{\partial W}{\partial S} \right|_{\text{J}}
\]

In this case, Breeden [3] shows that:

\[
\frac{J_{WS_i}}{J_{W}} = W(1 - T^R)\epsilon_S
\]

where \(T^R\) is the coefficient of relative risk-tolerance of the investor. If the investor's coefficient of relative risk-tolerance is smaller than one, the investor reverse-hedges, i.e., makes his expected lifetime utility more dependent on the \(i\)th state variable than if \(J_{WS} = 0\).

\(^{12}\) However, if investors in one country reverse-hedge, their hedging demands do not partially offset the hedging demands of investors in the other country.
from a situation of complete symmetry between the two countries. Such a situation occurs if all investors have the same utility function and the same wealth. Furthermore, it is required that \( \mu_I = \mu_F \) and \( \sigma_I = \sigma_F \). As changes in technologies are uncorrelated with returns on investments, by assumption, it follows that if complete symmetry obtains the sum of the demands for risky assets to hedge against unanticipated changes in the distribution of world wealth, i.e., \( G \) is equal to zero. Complete symmetry also implies that the net foreign investment in the domestic country is equal to zero. In the remainder of this section, the effect of departures from the assumption of complete symmetry on net foreign investment is studied.

**Result 1.** If complete symmetry obtains and if the coefficient of absolute risk aversion of the indirect utility function of wealth of investors does not depend on the value of the state variables, an increase in the expected return on investments in the foreign industry, i.e., an increase in \( \mu_F \), decreases net foreign investment in the domestic country.

From equation (7), which defines the interest rate in equilibrium, an increase in \( \mu_F \) increases the instantaneous rate of interest. If the interest rate does not increase, for constant absolute risk tolerance, an increase in \( \mu_F \) leads investors to invest more in productive processes than the existing stock of commodity, which is not possible. An increase in the interest rate decreases the domestic investment of investors by \((1/\sigma^2_T)T^w\) times the increase in the interest rate. By assumption, \( T^w \) is unaffected by the change in \( \mu_F \) or in the interest rate, which completes the proof.

This first result is not really surprising. As markets are perfect, if a technology becomes more productive, investment in that technology, *ceteris paribus*, increases. In this case, if the technology is a foreign technology, capital flows to the foreign country. In equilibrium, it is not possible for investments to increase in one technology and not decrease in the other technology, because the available stock of commodity is fixed. The decrease in investments in the domestic industry is brought about by an increase in the interest rate.

**Result 2.** Under the same assumptions as those for Result 1, an increase in the instantaneous variance of the return of the foreign technology, i.e., \( \sigma^2_F \), increases net foreign investment at home.

An increase in \( \sigma^2_F \) decreases the investment in the foreign industry for a constant interest rate. It follows that the interest rate must fall to keep all the available stock of the commodity invested. The fall in the interest rate increases the investment in the domestic industry.

It follows from this result that if the foreign technology becomes more risky, in the sense that the instantaneous variance of the return of the existing portfolio of investors increases, investors invest less in the foreign country and more in the domestic country. Another implication of this result is that net foreign investment differs in a world in which investors are risk-averse with respect to a world in which investors are risk-neutral. In a world of risk-neutral investors, a change in the instantaneous standard deviation of the return of a technology has no effect on net foreign investment.

**Result 3.** An increase in the absolute risk tolerance coefficient of the indirect
utility function of wealth of the domestic investor leaves net foreign investment in the domestic country unchanged if (a) the absolute risk tolerance coefficient of foreign investors is maintained constant, (b) the effect of the change on the net aggregate demands for assets as hedges against unanticipated changes in state variables, i.e., $G$, is negligible, and (c) complete symmetry holds before the change.

Notice from equation (7) that an increase in the coefficient of absolute risk tolerance of an investor increases the interest rate. With complete symmetry, the increase in the interest rate must be such that the total investment in each country is left the same after the increase in the coefficient of absolute risk tolerance of an investor. Complete symmetry implies that initially the stock of the commodity is invested equally among countries. The change in the absolute risk tolerance coefficient and in the interest rate leaves the instantaneous expected returns and variances of the returns of investments in the domestic and foreign technology unchanged, so that world wealth is still invested in the same way after it occurs, provided that $G$ does not change in a significant way. It is important to understand that an increase in the absolute risk tolerance coefficient of the domestic investor implies an increase in the investor's investments in risky assets. The domestic investor finances his new holdings of risky assets by borrowing from foreign investors at the interest rate $R$. The foreign investor can lend to the domestic investor at the interest rate $R$ because he decreases his investments in risky assets as a result of the increase in the interest rate.

This result is important, because it shows that differences in risk tolerance coefficients are not sufficient to make net foreign investment in the home country different from zero. However, if the risk tolerance coefficient of investors differs, investors want to hedge against unanticipated changes in interest rates and this affects their asset demands. When $G$ differs from zero, investors hedge against unanticipated changes in the interest rate; whereas $G$ is equal to zero if the symmetry assumption holds. If investors hedge in opposite ways, however, one would expect $G$ to be small.

Result 4. If the complete symmetry assumption holds initially, an increase in the wealth of the domestic investor accompanied by an equivalent decrease in the wealth of the foreign investor, decreases net foreign investment, provided that the effect of the change in the distribution of world wealth on $G$ is negligible.

The change in wealth affects risk tolerance coefficients and the interest rate. Since the distribution of returns on investments in industries is unaffected by the change in the distribution of wealth, it follows that the total investment in each industry stays the same, if the assumption of complete symmetry holds. However, net foreign investment is equal to the total investment in a country's industry minus the wealth of the investors of that country. As the domestic wealth increases and total investment in the domestic industry stays constant, net foreign investment in the domestic country decreases.

4. A Special Case

The generality of the previous results is obtained at the cost of assumptions about the partial derivatives of the indirect utility function of wealth. In this
section, results are obtained which do not require such assumptions. However, assumptions about the functional form of the utility function of investors are made. It is assumed that investors maximize the following expected utility function of lifetime consumption;

\[ E_i \int_{\gamma=t}^{\infty} e^{-\rho \tau} \ln C(\gamma) \ d\tau \quad i = D, F \]  

(11)

If investors have a logarithmic utility function, they can differ only with respect to their coefficient of time preference and with respect to wealth. It is well-known that if investors maximize (11), they do not hold risky assets to hedge against unanticipated changes in state variables, so that \( G = 0 \). If investors maximize (11), their asset demands are given by:

\[ D^i = \frac{1}{\sigma_i^2} (\mu_i - R)W^i \quad i = D, F \]  

(12)

\[ A^i = \frac{1}{\sigma_i^2} (\mu_F - R)W^i \quad i = D, F \]  

(13)

The interest rate \( R \) is given by:

\[ R = (\sigma_i^2 \mu_i + \sigma_i^2 \mu_F - \sigma_i^2 \sigma_F^2) \left( \frac{1}{\sigma_i^2 + \sigma_F^2} \right) \]  

(14)

Using equation (12), it follows that net foreign investment in the domestic country is equal to:

\[ N^D = \frac{1}{\sigma_i^2} (\mu_i - R)W^w - W^D \]  

(15)

Examining equation (15) it is possible to ascertain that:

(a) Net foreign investment in the domestic country increases if the instantaneous expected return of investing at home, i.e., \( \mu_i \), increases.

(b) Net foreign investment in the domestic country falls if the instantaneous standard deviation of the return of investing at home, i.e., \( \sigma_i \), increases.

(c) A change in the distribution of world wealth which decreases domestic wealth and increases foreign wealth increases net foreign investment in the domestic country.

These results are consistent with the results obtained in Section 3.

It is possible to use equation (15) to derive the dynamics of net foreign investment in the domestic country. Let \( a_i = (\mu_i - R) / \sigma_i^2 \) and \( a_F = (\mu_F - R) / \sigma_F^2 \). Differentiating (15) using Ito’s Lemma yields:

\[ dN^D = a_i dW^w + W^w da_i + \text{Cov}(da_i, dW^w) - dW^D \]  

(16)

It is known from equation (14) that changes in the interest rate are uncorrelated with changes in world wealth. It follows from this that, given the assumptions made in Section 2, \( \text{Cov}(da_i, dW^w) \) is equal to zero. Equation (16) implies that an increase in the expected excess return per unit of variance for the domestic
technology increases net foreign investment in the domestic country. Furthermore, if the increase in foreign wealth is larger than \((1 - a_t)/a_t\) times the increase in the domestic wealth, net foreign investment in the domestic country increases. This result follows from the fact that if the increase in foreign wealth is large enough, new investments in the domestic country from the foreign investor exceed new investments abroad from the investor.

Notice that \(dW^i, i = F, D\), is a decreasing function of the rate of time preference of the \(i\)th investor, i.e., \(\rho^i\).\(^{13}\) It follows from this observation that net foreign investment in the domestic country is a decreasing (increasing) function of the foreign (domestic) rate of time preference. This result is not surprising as it is shown in Section 3 that a decrease in domestic wealth relative to foreign wealth increases net foreign investment. If the domestic rate of time preference is higher than the foreign rate of time preference, \textit{ceteris paribus}, the expected rate of change in foreign wealth is higher relative to the expected rate of change of domestic wealth.

5. Conclusion

This paper presents a simple model of net foreign investment. It is shown that an increase in the expected return of investments in the foreign technology decreases net foreign investment in the domestic country, whereas an increase in the variance of the return of investments in the foreign technology increases net foreign investment in the domestic country. An increase in foreign wealth accompanied by an equivalent decrease in domestic wealth is shown to increase net foreign investment in the domestic country. Finally, it is shown that an increase in the risk tolerance coefficient of the domestic investor has, to a first approximation, no effect on net foreign investment in the domestic country. A closed-form solution is given for the case in which all investors have a logarithmic utility function and is used to derive the dynamics of net foreign investment.

One direction in which the present model could be extended consists in taking into account the fact that it is harder for domestic investors to enforce property rights abroad than at home. If property rights cannot be enforced abroad by domestic investors and at home by foreign investors, net foreign investment is always equal to zero, even though gross foreign investment differs from zero. Whereas the differential cost of enforcing property rights at home and abroad for domestic investors creates incentives for foreign investors to take actions which reduce the value of the net investment abroad by domestic investors, domestic investors can impose substantial penalties on foreign investors if they engage in confiscatory actions. The suggested extension of the analysis involves the development of a concept of equilibrium that accounts for the dependence of the actions of domestic investors on what they believe foreign investors will do.\(^{14}\)

\(^{13}\) If investors maximize (11), \(C^i = \rho^i W^i\).
\(^{14}\) Eaton and Gersovitz [6] provide an analysis of international lending and borrowing along these lines.
REFERENCES


DISCUSSION

BERNARD DUMAS*: General equilibrium financial theory in its present form is incapable of explaining institutions. The Value Additivity Principle which guarantees the absence of externalities in valuation, implies that nothing is gained by pooling several risks into one corporate entity. A pure financial institution in this context is inconceivable since no value can possibly be gained (except via a fiscal factor perhaps) by trading financial assets.

The existence of banks and similar institutions may be rationalized by using one or several of the following stories none of which can be easily handled in a general equilibrium framework:

(a) Banks provide a payment mechanism. Of course we cannot explain why in a world where trading is frictionless, payments could not be made by means of securities. Even if some "money" is designated as legal tender, one could still go in and out of money costlessly and never hold it. So, transactions costs must be at the heart of the issue. Banks are in the transactions business.

(b) Banks provide a richer menu of securities than could possibly be achieved in a financial market. Each loan has a special individual risk attached to it. Without the banks these risks would not be traded. In this view, incomplete markets would be at the heart of the issue.

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