Managerial discretion and optimal financing policies

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Received February 1990, final version received August 1990

I analyze financing policies in a firm owned by atomistic shareholders who observe neither cash flows nor management's investment decisions. Management derives perquisites from investment and invests as much as possible. Since it always claims that cash flow is too low to fund all positive net present value projects, its claim is not credible when cash flow is truly low. Consequently, management is forced to invest too little when cash flow is low and chooses to invest too much when it is high. Financing policies, by influencing the resources under management's control, can reduce the costs of over- and underinvestment.

1. Introduction

Recent developments in finance have emphasized the importance of managerial objectives and asymmetric information in the workings of modern corporations. In this paper, I investigate how financing policies can be used to restrict management's ability to pursue its own objectives when it has information that shareholders do not have. I show that optimal financing policies reduce the costs shareholders bear if management tends to invest too much and that these policies depend on the distribution of cash flow in each period as well as on their present value.

Consider a firm with atomistic shareholders who observe neither the firm's cash flow nor management's investment decisions. I assume it is costly for shareholders to act collectively once cash flow has accrued, so they cannot

*I am grateful for useful discussions with Harry DeAngelo, Ronen Israel, Patricia Reagan, and Robert Vishny. I also thank George Baker, Elazar Berkovitch, Douglas Diamond, Thomas George, Robert Heinkel, Robert Hendershott, Michael Jensen (the editor), Jacques Kramer, Darrell Lee, David Mayers, Merton Miller, Dorit Samuel, Ralph Walkling, Jerold Warner, Randolph Westerfield, participants at the Harvard/JFE Conference on the Structure and Governance of Enterprise and at seminars at Boston College, the Federal Reserve Board, the University of Illinois, the University of Chicago, the University of Southern California, the Ohio State University, the Virginia Polytechnic Institute, and especially the referee, Artur Raviv, for useful comments.
force management *ex post* to pay out cash. The managers value investment because their perquisites increase with investment even when the firm invests in negative net present value (NPV) projects. Consequently, when cash flow is high, they invest in negative NPV projects rather than pay out cash. Jensen (1986) calls cash flow left over after the firm has exhausted its positive NPV projects free cash flow and argues that it creates incentives to overinvest.

The informational asymmetry between managers and shareholders leads to inefficient investment when cash flow is low because management cannot credibly convince shareholders that cash flow is insufficient to take advantage of all positive NPV opportunities. Management always tells shareholders that the firm cannot take advantage of all its positive NPV opportunities. Consequently, shareholders never believe management's assertion that cash flow is too low, because management always benefits from increasing investment.

Debt payments force managers to pay out cash flow and hence reduce investment in all states of the world. Consequently, debt payments affect shareholder wealth both positively, by reducing investment when it would otherwise be too high, and negatively, by inhibiting advantageous investment in other states of the world. The tradeoff between the cost and the benefit of debt implies that there is a debt payment that maximizes firm value. The optimal debt payment is negative when shareholders benefit by giving management more resources to decrease the probability that a good project will not be undertaken. Thus, slack can be beneficial, as in Myers and Majluf (1984), even though slack can turn into free cash flow.

My analysis shows that the firm's debt-equity ratio depends critically on the probability distribution of cash flow and on the firm's investment opportunities. In particular, shareholders of a firm with negative expected free cash flow but poor investment opportunities may want the firm to issue debt so that management will control even fewer resources, whereas shareholders of a firm with positive expected free cash flow but good investment opportunities may want management to raise more funds to decrease the probability that some positive NPV investment opportunities will be left unexploited. More volatile cash flow makes significant under- and over-investment more likely and reduces firm value for all levels of debt. This suggests that diversification across projects reduces the agency costs of managerial discretion because it makes cash flow more predictable.

I ignore the agency costs of debt through most of the paper. I argue that these costs can make it profitable for the corporation to become a private firm because, in some cases, the private-firm organization form is more efficient in controlling them. For instance, if the investment opportunity set is stochastic and is better than expected *ex post*, the management of a highly levered public corporation may be unable to raise investment funds because, as in Myers (1977), an excessive fraction of the payoffs from new investments would go to creditors. This type of agency cost is reduced if shareholders also

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1See Jensen and Smith (1985) for a review of the literature on the agency costs of debt.
hold debt that can be renegotiated at low cost before the investment decision and do not or cannot unbundle their holdings of the firm's securities. I argue that in this case the debt held by shareholders will be private debt and that there will be no public market for the firm's equity.

The model is presented in section 2. In section 3, I show that managerial discretion enables managers to invest too much in some circumstances—the overinvestment cost—and prevents them from exhausting the firm's positive NPV investment opportunities in others—the underinvestment cost. Financing policy can reduce one of these costs, but not both. I show in section 4 when shareholders will want to increase resources under management's control and in section 5 when they will try to force management to pay out funds through debt payments. Section 5 also derives comparative-statics results relating the firm's characteristics to its debt-equity ratio. Section 6 shows that the effectiveness of financing policy in reducing the agency costs of managerial discretion is, in general, inversely related to the volatility of the firm's cash flow. Section 7 extends the analysis to the case where the investment opportunity set is stochastic and shows how private firms can better control some agency costs of debt because private debt can be renegotiated before default. Section 8 demonstrates that the main results hold if shareholders observe cash flow but not the firm's investment opportunities. Section 9 provides concluding remarks.

2. The model

I consider a model with three dates: 0, 1, and 2. Capital markets are open at each date. The firm's assets in place at date 0 yield a random non-negative liquidating cash flow \( R \) at date 1 that managers can either invest in new projects or pay out. The firm can raise funds at dates 0 and 1. As discussed in detail later, the firm has investment opportunities at date 1 that managers and shareholders want to exploit, but at date 0 it does not. Funds available to management at date 0 can only be invested in riskless zero NPV financial securities. I assume from now on that funds to be invested at date 1 in addition to \( R \) are raised at date 1 and that shareholders do not let managers raise funds for investment in financial securities at date 0.²

²This assumption involves no loss of generality. In the model developed in this paper, investment in securities by management at date 0 can affect firm value only to the extent that it changes investment at date 1, but any effect of investment in riskless securities on investment at date 1 can be offset if disadvantageous or replicated if advantageous through the firm's financing policy for date 1. Hence, shareholders do not benefit from letting management undertake investment in riskless securities at date 0. If managers could invest in risky securities at date 0, the analysis would be more difficult because it would have to address an additional agency problem resulting from management's discretion to use investment in risky securities at date 0 to pursue its own objectives. However, the analysis of section 6 suggests that, in general, shareholders would not want management to have funds at date 0 to invest in risky securities whose payoffs at date 1 are uncorrelated with \( R \).
\( N \) is the firm's net financing at date 1, defined as the amount of cash raised minus the amount of cash paid out at that date. If \( N > 0 \), the firm raises funds at date 1 through the sale of debt or equity so that, at that date, it can invest more than the cash flow from assets in place. If \( N < 0 \), the firm pays back debt issued at date 0 or pays a dividend. I assume that default on debt at date 1 brings about the liquidation of the firm.\(^3\) If the firm liquidates at date 1, the cash flow \( R \) is the firm's liquidation value, so that liquidation at date 1 involves the loss of the firm's investment opportunities. If the firm does not default at date 1, it liquidates at date 2. To make it possible to distinguish sharply between equity and debt financing, I assume that default at date 2 has an arbitrarily small, strictly positive cost. With this assumption, the firm never issues debt unless it has some advantage that at least offsets the default cost.

The cash flow from assets in place, \( R \), is net of payments to management, so that in contrast to Grossman and Hart (1982), management cannot expropriate \( R \). \( R \) is a nonnegative random variable with a cumulative distribution function \( G(R) \) and differentiable density \( g(R) \); \( g(R) > 0 \) for all \( R \) such that \( \approx > R > 0 \). Investment in firm projects at date 1 is \( I \). Neither \( I \) nor \( R \) is observable by outside investors at date 1. Typically, one would expect market participants to have some knowledge of \( R \) and \( I \) and managerial compensation contracts to make use of that knowledge to increase managerial incentives to maximize shareholder wealth. The model I develop ignores these complications and, consequently, applies to firms where contracting solutions have limited value and where, short of liquidating the firm, shareholders have considerable uncertainty about the firm's true value and management's investment policy.\(^4\) I assume throughout that atomistic shareholders cannot form a coalition to force liquidation, so that liquidation is not possible unless the firm defaults on debt securities.

Management is likely to value investment more than shareholders do for numerous reasons. For instance, managers of larger firms have greater visibility, have more perks to dispense to their employees, are better able to promote employees within the firm, and so forth. Rather than model the reason for which management values investment, I assume that management's utility increases with the consumption of perquisites and that this consump-

\(^3\)I therefore ignore the bankruptcy process studied in Aghion and Boulton (1988) and the possibility of renegotiation prior to default stressed by Hart and Moore (1989). Since the firm either liquidates or management invests all funds available, my model neglects the benefit of debt emphasized by Harris and Raviv (1990), namely that debt payments provide information that investors can use subsequently. The issue of renegotiation prior to default is discussed in section 7, whereas changes in the bankruptcy process are analyzed briefly in section 5.

\(^4\)Jensen and Murphy (1990) provide evidence that managerial compensation is only weakly related to managerial performance, suggesting it is unlikely that compensation contracts are used to attenuate the agency costs of managerial discretion.
tion is a function of date 1 investment only. Each unit of investment is assumed to produce a nonstochastic positive amount of perquisites that is an increasing function of the investment's NPV. This assumption insures that management invests in the positive NPV projects first. It can be motivated as follows: negative NPV projects consume corporate resources in the future whereas positive NPV projects increase these resources. Consequently, management that values investment would rather invest in projects that enable it to increase future investment. Since, in this model, the firm liquidates at date 2, making the perquisites positively related to the NPV of projects is an indirect way to account for management's preference for investment in projects that expand resources under its control. If management does not remain in place at date 1, it consumes no perquisites. With these assumptions, management benefits from being in control at date 1 and invests all cash flow unless forced to do otherwise. I assume that management receives a fixed wage and has no stake in the corporation; implicitly, I assume that no monetary rewards at date 2 can dominate management's benefits from investing more at date 1.

The specification of the return to investment is chosen to reflect the declining marginal return on investment so that overinvestment is possible. For tractability, I assume the marginal product of investment is given by a step function; consequently, to allow for overinvestment, investment in one project has to have a positive NPV and investment in the other project a negative NPV. All investment payoffs are defined as net of perquisites. The date 2 expected payoff of investment is \( Z \) per unit for the first \( I^* \) units invested and \( Y \) per unit in excess of \( I^* \). The payoff from date 1 investment is paid out to shareholders at date 2, since the firm is assumed to liquidate at that date. Without loss of generality, I let the rate of interest be zero. Consequently, I require \( Z > 1 \) and \( Y < 1 \). For simplicity, I call the investment with expected payoff \( Z \) per unit the good project and the investment with

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5 Further research should investigate the robustness of my results in models that derive management's policies for the case where management's utility function allows for an explicit tradeoff between the consumption of perquisites and the consumption of other goods. This would make it possible to assess when management finds it optimal to overinvest and when compensation policies are likely to be effective in motivating management not to overinvest. As pointed out by Myers (1990), organizations that want to grow have to find investors so there will be situations in which management will choose not to overinvest, even though it could do so, to insure that funds can be raised.

6 This assumption could be relaxed. For instance, if perquisites are random but positively correlated with the NPV of investments, my results would hold.

7 Since management has no ownership stake, a recapitalization has no impact on the distribution of the votes, in contrast to Stulz (1988).

8 I assume that the payoff of investment is uncorrelated with cash flow to simplify the analysis. This assumption is of no importance as long as contractual solutions cannot be used to affect managerial incentives.
expected payoff $Y$ per unit the bad project. I assume that investors are risk-neutral.

Investors expect management to invest all cash flow at date 1 and, if allowed to do so, to raise additional funds to invest at that date. Unless forced to do so by external circumstances, management does not sell bonds to be repaid at date 1 because doing so involves risking the loss of control at date 1. Management is willing to sell bonds at date 1 to be repaid at date 2, or to sell equity at date 1. If $N < 0$, management pays out funds at date 1; in this case, $N$ is defined to be the net amount that management pays out if the firm remains a going concern, and it is equal to the face value of debt to be paid back at date 1 if the payout takes the form of a payment to bondholders.

Since investors do not observe the cash flow at date 1, the firm's ability to raise funds at dates 0 or 1 is a function of expected cash flow, which depends on variables known at date 0. Raising funds to invest more than $I^*$ hurts existing shareholders because the new investors buy securities at their fair market price and investment in excess of $I^*$ has a negative NPV. No assumption made so far prevents management from expropriating existing shareholders in this way. I assume henceforth that management can raise funds only if existing shareholders agree, so that I focus on the financing policies that maximize shareholder wealth. Since shareholders know the distribution of cash flow and the investment payoffs, they can assess the impact of a debt or equity issue on the value of their claims and prevent management from undertaking the issue if the impact is negative. Managers always raise all the investment funds they are allowed to, irrespective of the realized cash flow at date 1.

3. The two costs of managerial discretion

If the firm does not have access to capital markets, its value is given by

$$ V = I^*(Z - 1) + E(R) - \int_{I^*}^{\infty} (R - I^*)(1 - Y)g(R)dR $$

$$ - \int_0^{I^*} (I^* - R)(Z - 1)g(R)dR. $$

The first two terms in (1) correspond to the value of the firm at date 0 if management maximizes shareholder wealth. To maximize shareholder wealth, management makes an investment of $I^*$ in the good project with a net present value of $I^*(Z - 1)$. If the firm's cash flow exceeds $I^*$, management pays out all free cash flow as a dividend; whereas otherwise it borrows to insure that the positive NPV investment opportunity is exploited fully. Hence, if management maximizes shareholder wealth, the existing sharehold-
ers' claim is equal to the NPV of the good project plus the present value of cash flow.

In this model, however, management does not maximize shareholder wealth but maximizes investment. Management can pursue its objective because shareholders observe neither cash flow nor investment. The costs of managerial discretion correspond to the last two terms in (1). The third term, which I call the overinvestment cost of managerial discretion, is the expected cost to shareholders that arises because management invests cash flow in excess of \( I^* \) in negative NPV projects. The fourth term, called here the underinvestment cost of managerial discretion, is the expected cost to shareholders from management's inability to exhaust the positive NPV project when available funds are below \( I^* \).

Financing policy can be used to reduce both costs of managerial discretion, but not simultaneously. If shareholders let managers raise funds at date 1, it becomes more likely that the firm will exhaust its positive NPV investment opportunities and invest in a negative NPV project, so that the underinvestment cost falls and the overinvestment cost increases. If managers could credibly communicate cash flow to shareholders, financing policies could eliminate the underinvestment cost. It is always in management's interest to try to convince shareholders that cash flow is low, however, so shareholders never believe managers when they announce that cash flow is low. If shareholders can force managers to pay out funds at date 1, underinvestment becomes more likely and overinvestment less likely. Whether positive or negative net financing at date 1 maximizes shareholder wealth depends on the firm's costs of managerial discretion.

4. The case of positive net financing

In this section, I investigate the conditions that must hold for shareholders to increase resources under management's control, and provide comparative-statics results for net financing when it is positive. In the next section, I provide conditions for net financing to be negative and argue that negative net financing takes place through a debt issue at date 0 to be repaid at date 1. Resources under managerial control at date 1 can be increased either through a debt issue that matures at date 2 or through an equity issue. Since debt that matures at date 2 has no effect on managerial decisions but has a cost if default occurs, shareholders always want managers to issue equity rather than debt that matures at date 2. With equity financing, the value of...

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9The existence of a small default cost at date 2 simplifies the exposition. In the absence of the default cost, however, all my results hold as long as the debt that does not restrict managerial discretion is viewed as equity.
equity for the old shareholders at date 0, \( V(N) \), is the present value of the payoffs from investment in the good and bad projects at date 1 minus the present value of the funds raised for investment:

\[
V(N) = \int_{I_* - N}^{I_*} [(R + N - I^*)Y + I^*Z] g(R) \, dR + \int_{0}^{I_* - N} (R + N)Zg(R) \, dR - N. \tag{2}
\]

Since all investors are risk-neutral, investors purchase equity from the firm at date 1 for \( N \) and expect to receive \( N \) back at date 2 because the riskless rate is assumed to be zero. Shareholders receive no new information at date 1 on which they could condition the amount of funds they authorize management to raise. Hence, the value of \( N \) that maximizes the value of the firm for its existing shareholders, \( N^* \), is known at date 0.

A necessary and sufficient condition for shareholders to let management issue equity is that the derivative of the value of the firm with respect to net financing is positive for \( N = 0 \):

\[
V_N(0) = Y[1 - G(I^*)] + ZG(I^*) - 1 > 0. \tag{3}
\]

The interpretation of this condition is straightforward. An additional dollar in the hands of management is invested in the bad project with probability \( 1 - G(I^*) \) and in the good project with probability \( G(I^*) \). Hence, the expected payoff from giving management an additional dollar is \( Y[1 - G(I^*)] + ZG(I^*) \). If this payoff exceeds one dollar, it pays for shareholders to increase managerial resources by at least one dollar. If the payoff is less than one dollar, shareholders would benefit from reducing resources under managerial control if the deadweight cost of doing so is small enough.

For a given distribution of cash flow and given \( I^* \), shareholders want to increase resources under managerial control for all pairs \((Y, Z)\) in the shaded region of fig. 1. The investment opportunity set improves as one moves up and to the right. For any value of \( Y \), there is always a value of \( Z \) for which

\[\text{This condition is both necessary and sufficient because the appendix shows that } V_N(N) \text{ is negative for } N > 0, \text{ so that the gain from increasing managerial resources by one dollar is never greater than for } N = 0.\]
Z and Y are the expected payoffs per dollar of investment in the positive and the negative NPV project. The solid line (a, b) that separates the debt and equity financing regions is drawn for a given probability distribution of cash flow. Along the line, the marginal cost of an increase in resources under management's control (i.e., the increase in overinvestment in some states of the world) equals the marginal benefit (i.e., the decrease in underinvestment in other states of the world). A decrease in the probability of free cash flow lowers the solid line to (c, b) as shown in the figure.

shareholders want net financing to be positive irrespective of I*. Hence, if the good project is good enough, management is allowed to raise funds. Alternatively, for a given good project, management can raise funds if the bad project is not too bad. As I* increases, management can raise funds for worse pairs (Y, Z). As the probability of free cash flow in the absence of an equity issue increases, i.e., as G(I*) falls, management becomes less likely to be able to raise funds because the probability that it can exhaust the positive NPV investment opportunity increases. Management may not be allowed to raise funds even when the probability that it can exhaust the positive NPV investment opportunity is small, however, and may be allowed to do so when that probability is large. Hence, the probability that a firm will have free cash flow is not a sufficient statistic for the extent to which a firm has a free-cash-flow problem. This probability has to be considered jointly with the expected marginal product of the firm's investment opportunities.

An improvement in the investment opportunity set or a decrease in the probability that management will be able to exhaust the positive NPV investment opportunity leads to an increase in the funds management can
raise if condition (3) holds. More precisely:

**Result 1**

If eq. (3) holds, the amount of equity management is allowed to raise is unique and, for any distribution of cash flow:

1. It increases with the marginal product of the firm's investment opportunities and with the maximum amount that can be invested in the good project.

2. It increases if the probability that the firm will have free cash flow decreases.

*Proof.* See appendix.

For some cash flow distributions, the probability of free cash flow is an increasing function of expected cash flow. The lognormal distribution is an example. With such a cash-flow distribution, there is a number $K$ such that, for otherwise identical firms, firms with expected cash flow smaller than $K$ issue equity and firms with expected cash flow larger than $K$ do not. Among the firms that issue equity, the size of the issue is inversely related to expected cash flow. Since $K$ can be larger or smaller than $I^*$, firms with positive expected free cash flow, i.e., $E(R) - I^* > 0$, may be allowed to raise funds and firms with negative expected free cash flow may be prevented from raising funds.

5. **Optimal capital structure**

If the value of the firm increases when shareholders can costlessly reduce management's resources by one dollar, i.e., if eq. (3) holds with the inequality reversed, optimal net financing at date 1 is negative and the value of the firm is higher if managers make a commitment to pay out funds at date 1. Whereas a promise by management to pay out funds is not credible because it is contrary to management's objective to maximize investment at date 1, management can credibly commit itself to a date 1 payout by issuing debt at date 0 to be repaid at date 1, with the proceeds of the issue paid out as a dividend at date 0 or used to finance the assets in place. Management's commitment is credible because if it cannot repay the debt at date 1, the firm is bankrupt and management loses its perquisites. This commitment has a deadweight cost, however, since bankruptcy brings about the loss of investment opportunities.

If management found a way to commit itself to a dividend policy under which, if it did not pay the promised dividend at date 1, it would lose its job,
the policy would be a perfect substitute for debt. I rule out dividend policies that imply the automatic removal of management unless a particular dividend is paid under the presumption that such policies can be enforced only through shareholder action. If shareholders can remove management if it does not pay an appropriate dividend, the commitment by management to pay the dividend is superfluous. Consequently, the issue is whether shareholders prefer a debt issue at date 0 or action at date 1 to insure an appropriate payout.

Since management prefers to have no outstanding debt that matures at date 1, it issues the optimal amount of debt only if forced to do so. Shareholders will force management to issue debt at date 0 if their costs of collective action are low at date 0, but not at date 1. If it is cheap for shareholders to impose a payout policy on managers at date 1, debt doesn't play a distinctive role in the model, since its role is to force a payout from management. Collective action might be expensive for shareholders at date 1 because management in place has a strategic advantage – for instance, it might be able to postpone a shareholders’ meeting or to delay the implementation of decisions made at the shareholders’ meeting – or because it is always expensive, except possibly when the firm is formed. Collective action might be cheap at date 0 because the firm has just been formed, or it might be unnecessary because the entrepreneur organizing the firm wants to commit management to pay out funds at date 1 to maximize his wealth.

If the costs of collective action for shareholders are always high, the market for corporate control can help force management to issue the appropriate amount of debt. For instance, management could voluntarily issue debt at date 0 to raise firm value so that shareholders do not tender their shares to a bidder that offers less than the firm is worth with the optimal amount of debt. Alternatively, a large shareholder could find it profitable to acquire shares to force management to issue the optimal amount of debt.\(^{11}\) Irrespective of why the optimal amount of debt is issued, firm value increases through a payout commitment at date 1 only if management earns rents when it invests suboptimally. Otherwise management would have to be compensated for the change in investment policy and the resulting effect on shareholder wealth would be ambiguous.

If the firm issues debt at date 0 to be repaid at date 1, the face value of the debt is \(F\). If \(F\) is paid back at date 1, \(F\) is the firm's negative net financing, i.e., \(F = -N\). Whereas in section 4 \(N\) is always raised, now it is possible for the firm to default and not pay \(F\). If the firm defaults, it loses its investment opportunities, so that the cases of negative and positive net financing are not symmetric. The value of the firm at date 0, including the dividend to be paid

\(^{11}\)For an analysis of the role of large shareholders, see Shleifer and Vishny (1986).
out with the proceeds of the debt issue, can be written as a function of $F$:

$$V(F) = E(R) - \int_{F-I^*}^{\infty} (R - F - I^*)(1 - Y)g(R) \, dR$$

$$+ \int_{F}^{\infty} (R - F)(Z - 1)g(R) \, dR$$

$$- \int_{F+I^*}^{\infty} (R - F - I^*)(Z - 1)g(R) \, dR. \quad (4)$$

Eq. (4) states that the value of the firm is equal to the present value of the date 1 cash flow plus the expected NPVs from investing in the bad and good projects. Debt decreases the probability that management will invest more than $I^*$; this benefit of debt is positive even if the firm's expected free cash flow is negative, since there is always some probability that the firm will overinvest. Besides the deadweight cost of bankruptcy, the cost of debt is that it increases the probability that the firm will not be able to exhaust its positive NPV investment opportunity.

The firm chooses to sell an amount of debt at date 0 so that the marginal benefit of debt equals its marginal cost:

$$\int_{F-I^*}^{\infty} (1 - Y)g(R) \, dR = \int_{F}^{F+I^*}(Z - 1)g(R) \, dR. \quad (5)$$

The marginal benefit of debt is the decrease in the loss of firm value resulting from the overinvestment cost of managerial discretion, whereas the marginal cost of debt is the increase in the loss of firm value caused by the underinvestment cost. For the firm to issue debt at all, the marginal benefit of debt has to exceed the marginal cost when $F = 0$. This condition means that the left-hand side of (5) evaluated at $F = 0$ exceeds the right-hand side. The reader can verify that this condition is the opposite from the one that must hold for the firm to issue equity - i.e., it is (3) with the inequality reversed.

The region of fig. 1 where debt is issued is therefore the region below the solid line. Consequently, firms that issue debt have a worse investment opportunity set than those that issue equity. For a given pair $(Y, Z)$, a change in the distribution of cash flow that increases the probability that the firm will have free cash flow ex post makes it more likely that the firm will issue debt, whereas a decrease in $I^*$ has the opposite effect.

This discussion leads to the following result:

**Result 2**

If managers are not allowed to issue equity for investment at date 1, there is at most one finite face value of debt $F^*$ to be paid at date 1 that
This figure shows firm value as a function of the face value of debt to be paid out of the cash flow from assets in place for five combinations of the maximum initial investment for the positive NPV project, $I$, and the NPVs of $a$ and $b$ for two alternative projects. The present value of that cash flow from assets in place is 100, with a monthly variance of 0.01%. The cash flow is paid in three years and the interest rate is 10% per annum.

maximizes the date 0 value of the firm and is chosen by managers. The proceeds of the date 0 debt issue with face value $F^*$ are paid out to shareholders as a dividend or used to finance the assets in place.

Proof. See appendix.

Fig. 2 plots the value of the firm as a function of the face value of the debt maturing at date 1 for five sets of parameters for the firm's investment opportunities, assuming that cash flow is distributed lognormally. In all cases, the expected cash flow at date 1 exceeds what can be invested in the good project, so that expected free cash flow is always positive. Yet, the firm may choose not to issue debt because the good project is too valuable (this is the case where $I^* = 60$ and the firm's projects have NPVs of, respectively, 0.3

The computations for the figure also assume a positive interest rate to show that there is no loss of generality in assuming that the interest rate is zero.
The figure also shows the polar case where the optimal amount of debt is infinite, so that the firm is liquidated. Whenever the optimal amount of debt is finite, continuing the firm's operations is better than liquidation. In three cases, the value of the firm is maximized with a finite positive amount of debt. In the examples, the present value of the date 1 cash flow is 100. Hence, in one case, the value of the firm is less than 100 without debt and more than 100 with the optimal amount of debt. This case illustrates best that issuing debt to restrict investment dominates liquidation for a firm that expects to have free cash flow but also has a sufficiently valuable positive NPV project.

In this model, debt decreases management's ability to invest in the bad project at the expense of its ability to invest in the good project. Since debt and equity issues have opposite effects, one would expect the comparative statics for debt to be the opposite of those for equity presented in Result 1. \(^{13}\) Debt falls as the firm's good projects become more profitable and increases as its bad projects become less profitable. An increase in the probability that the firm will have free cash flow increases debt because the probability of overinvestment increases.

When debt is issued that matures at date 1, there is a positive probability that the firm will be liquidated at date 1 and hence that the positive NPV investment opportunity will be lost. If none of the positive NPV investment opportunity is lost when the firm defaults, the optimal amount of debt becomes infinite, because default enables the debtholders to start a new firm at date 1 with resources equal to \( J^* \). Hence, the smaller the amount of the positive NPV investment opportunity lost through default and liquidation, the larger the amount of debt maturing at date 1.

6. Cash-flow volatility and the costs of managerial discretion

The previous two sections demonstrate that an optimal financing policy can reduce the costs of managerial discretion. For some firms, the optimal financing policy involves increasing the resources under managerial control, whereas for others, it amounts to reducing these resources. Throughout the analysis, I take the volatility of the firm's cash flow as given. Since the cash-flow volatility can be affected by firm policies, I investigate here how a change in cash-flow volatility affects the effectiveness of financing policies in reducing the costs of managerial discretion.

As discussed in section 3, the value of the firm if managers maximize shareholder wealth, which we define here to be \( V^* \), is simply the value of the firm's positive NPV investment opportunity plus the present value of the

\(^{13}\) See the appendix for a proof of the following results.
firm's cash flow:

\[ V^* = I^*(Z - 1) + E(R). \] \hspace{1cm} (6)

Consequently, a change in cash-flow volatility has no impact on firm value if managers maximize shareholder wealth. The costs of managerial discretion equal \( V^* - V(N^*) \), where \( N^* \) is the firm's optimal net financing at date 1. In general, since \( V(N^*) \) is a function of cash-flow volatility, the costs of managerial discretion depend on cash-flow volatility.

To understand the relation between the costs of managerial discretion and cash-flow volatility, it is useful to look at the polar case in which cash flow is nonstochastic. In this case, shareholders know \( R - I^* \) and can choose a financing policy such that \( N^* = I^* - R \). With such a policy, there are no agency costs of managerial discretion because management's available funds are always equal to \( I^* \). Hence, if shareholders could implement policies making cash flow riskless, they could eliminate the costs of managerial discretion.

In general, a decrease in the dispersion of cash flow reduces the costs of managerial discretion even if that decrease in dispersion does not make cash flow riskless. More precisely:

**Result 3**

For any distribution of cash flow \( G(R) \):

1. If the firm issues equity, the costs of managerial discretion fall if \( G(R) \) is replaced by a distribution that dominates \( G(R) \) in the sense of second-order stochastic dominance.

2. If the firm issues debt, the costs of managerial discretion fall if \( G(R) \) is replaced by a distribution that dominates \( G(R) \) in the sense of second-order stochastic dominance, provided that the present value of the investment in the positive NPV project either increases or does not fall too much with the decrease in the dispersion of cash flow.

*Proof*. See appendix.

The present value of the investment in the positive NPV project is the present value of a random amount equal to cash flow minus the face value of debt up to \( I^* \). Hence, the present value of the investment in the good project is equivalent to the present value of debt with face value \( I^* \) subordinated to the debt with face value \( F \). It follows that an increase in volatility will reduce
the value of the firm if the value of subordinated debt with face value $I^*$ falls with an increase in the dispersion of cash flow. As shown by Black and Cox (1976), this will be the case if cash flow is lognormally distributed as long as the firm is not too highly levered.

The intuition for Result 3 is straightforward. Through the financing policy, shareholders select a target for funds available to management for investment. Deviations from the target are costly to shareholders, since management overinvests when funds are plentiful and underinvests when they are scarce. As the dispersion of cash flow falls, the funds available to management are more likely to be close to the target.

The negative relation between the usefulness of financing policy in reducing the costs of managerial discretion and cash flow volatility has several important implications for firm policies:

1) The value of diversification. Consider two firms that are identical except that their cash flows are imperfectly correlated. If a merger of the two firms entails no costs, it will benefit the shareholders of both firms because the resulting cash flow will be less volatile. Hence, diversification across projects and mergers for diversification purposes can increase shareholder wealth. As a firm's investment-opportunity set worsens, however, diversification becomes less valuable because the cost of underinvestment and the cost of default fall.

2) The value of hedging. Shareholders can increase their wealth by finding ways to force management to commit to a hedging policy that decreases the volatility of cash flow. The value of hedging is positive even if the firm has no debt. This is because the benefit to shareholders from giving more resources to management is inversely related to the volatility of cash flow.

3) Present value of cash flows versus period-by-period cash flows. In this model, the cash-flow volatility that is important to shareholders is the volatility of date 1 cash flow. For a given volatility of date 1 cash flow, the volatility of date 2 cash flow is irrelevant to shareholder wealth — i.e., if the payoffs from investments at date 1 are stochastic, shareholder wealth does not depend on their volatility. Hence, the analysis helps to explain why firms are concerned with the distribution of each period's cash flow.

4) The value of financial engineering. The value of the firm increases if management issues securities with payments highly correlated with $R - I^*$. For instance, if the firm can sell debt indexed to some observable quantity so that the debt payment is positively correlated with $R$, the firm can reduce the costs of managerial discretion more than if it only issues debt with a fixed face value. More generally, the firm benefits from selling securities that give resources to management when $R$ is smaller than $I^*$ and withdraw resources from management otherwise.
7. Agency costs of debt, debt renegotiation, and going private

In this section, I give an example in which the firm can increase its value more by issuing private rather than public debt because of the agency costs of debt. Private debt can include provisions that enable its holders to convert the debt into equity before default occurs, whereas public debt cannot, under the terms of the Trust Indenture Act.\footnote{See Roe (1987) and Gertner and Scharfstein (1989).} Even if such provisions were allowed, they would be difficult to implement for debt with diffuse ownership, because the costs of collective action for bondholders would be high. Private debt could also be sold in the absence of agency costs. I show, however, that without these costs, the option to renegotiate private debt before liquidation is useful only if renegotiation is costly to management.

To introduce agency costs of debt, I consider the case in which \( Z \) is a random variable with a discrete distribution, so it can take several different values. At date 1 the true value of \( Z \) is revealed, and it can be better or worse than expected. I assume that the distribution of \( Z \) is common knowledge and that contracts cannot be made to depend on the realization of \( Z \), because the courts would not uphold such contracts. As before, default leads immediately to liquidation, in the sense that if the firm defaults on one security all investment opportunities are lost, and shareholders do not know \( R \) before learning whether managers default or not. Consequently, if shareholders want to reduce the probability of default, they must raise equity or renegotiate debt agreements before management has to pay back bondholders.

At date 0, the amount of debt that maximizes the value of the firm depends on the expected value of \( Z \). If \( Z \) turns out better than expected, the firm has too much debt \emph{ex post}. In that case, shareholders would like to issue equity to insure that the firm can take advantage of the positive NPV investment opportunity. If the firm is highly leveraged, however, it may experience the underinvestment problem discussed in Myers (1977), namely, that it cannot raise equity to finance a profitable investment opportunity because too much of the new funds would go to the bondholders.

A stochastic investment opportunity set introduces agency costs of debt that lower the value of the firm. If the shareholders also buy debt, the conflict between the bondholders and the shareholders may be sufficiently lessened that the underinvestment problem does not occur. It is still optimal for the firm to have debt because shareholders will want the managers to pay out funds unless \( Z \) is high. By making the debt private, shareholders can insure that its provisions can be renegotiated if the firm has too much debt \emph{ex post} because \( Z \) is high. If the debt is private, one would expect equity to be
private also. There is no advantage to having public equity, because equity trading separately from the debt is worth less than equity trading with the debt, and equity trading with the debt cannot be traded publicly. If investors are indifferent between holding public and private claims to cash flows, the underinvestment problem arising from having too much debt \textit{ex post} can be solved by having multiple debt issues, so that for each possible value of $Z$ shareholders can convert debt issues into equity, leaving the face value of the remaining debt equal to the optimal amount of debt for the realized $Z$.

To understand this, suppose that $Z$ can take values $Z_1$, $Z_2$, and $Z_3$, such that $Z_1 > Z_2 > Z_3$. In this case, the firm has three debt issues with different seniority. The most senior debt has face value $F_1$; the most junior debt has face value $F_3$; and there is debt of intermediate seniority with face value $F_2$. The equityholders also hold the debt, except for the most senior. The face value of each debt issue is chosen so that $F_1$ is the amount of debt that maximizes firm value if the investment opportunity is $Z_1$, $F_1 + F_2$ maximizes firm value if the investment opportunity is $Z_2$, and $F_1 + F_2 + F_3$ maximizes firm value if the investment opportunity is $Z_3$. If, at date 1, the investment opportunity has marginal product $Z_1$, the shareholders want the firm to have little debt because the cost of not investing in the good project is high. The shareholders therefore transform all the debt they hold into equity. If the investment opportunity has marginal product $Z_2$, the debt with face value $F_2$ is transformed into equity. Finally, if the investment opportunity has marginal product $Z_3$, all debt remains as debt. The strip-financed recapitalization enables the firm to issue state-contingent debt contracts. Hence, in this setting, the firm does not recapitalize so that it can go private, but rather goes private so that it can issue more debt.

Suppose now that the required rate of return is higher on non-tradable claims than on tradable claims. This can be the case even if shareholders are risk-neutral; for instance, trading restrictions can make it harder for an investor to compute his wealth and hence to devise appropriate investment and consumption policies. In this case, shareholders want to minimize the fraction of firm value financed with nontradable claims. A possible solution is one in which the firm has only two classes of debt, debt with face value $F_1$ and debt with face value $F_2$. Shareholders hold only the junior debt and the senior debt is publicly traded. If the investment opportunity is $Z_3$, all the debt contracts are enforced. If the investment opportunity is $Z_2$, the debt held by shareholders is transformed into equity. Finally, if the investment opportunity is $Z_1$, the firm goes public and issues equity.

If the required expected rate of return on private claims is higher, the firm will go public after paying off its creditors at date 1. At that date, the agency costs of debt become irrelevant but the equity is valued less by investors if it has trading restrictions. Hence in this setting, a firm is private only temporar-
ily, when being private reduces the agency costs of debt. The extension of the model discussed in this section can therefore provide a rationale for the well-documented phenomenon of firms going private through a leveraged-buyout only to go public again at some later date.

This section illustrated that private debt is useful in the presence of some types of agency costs. Could it also be useful in the absence of agency costs? Suppose the true value of $R$ is $R'$, $R' < F$, and managers can convince shareholders of this fact. In this case, shareholders would be better off if they could agree to reduce $F$ to $F'$, so that $R' - F' = I^*$. If shareholders hold the debt and the debt is private, shareholders can achieve such an outcome if their costs of collective action are low at date 1. This reasoning seems to suggest that there are benefits to private debt and renegotiation even if there are no agency costs of debt. The reasoning rests on a fallacious assumption, however. If managers can convince shareholders that $R = R' < F$ when shareholders know nothing about $R$, they will always do so even if $R > F$. Consequently, with our assumptions, managers cannot credibly communicate earnings to shareholders and shareholders never want to renegotiate $F$ downward. Renegotiation would increase shareholder wealth if shareholders' costs of collective action at date 1 were low enough and if they could impose costs on managers for renegotiating that were high enough that managers would always prefer to pay $F$ than renegotiate. In this setting, going private is advantageous if it reduces shareholders' costs of collective action and increases their ability to impose costs on management if management cannot pay off the debt.

8. An alternative approach

In the earlier sections, I assumed that investors know neither the firm's cash flow nor its investment. In this section, I show that, in a simple case, my main results hold if shareholders observe both cash flow and investment, but do not know the firm's investment opportunity set. To simplify the analysis, all the earlier assumptions hold except that $R$ is deterministic and observable, whereas $I^*$ is stochastic and not observable by shareholders. $I^*$ is the most that can be invested in the project that has expected payoff $Z$ per unit invested. I assume that $H(I^*)$ is the cumulative distribution function for $I^*$ and that $h(I^*)$ is the associated density function, with $h(I^*) > 0$ for strictly positive but finite values of $I^*$.

In the setting of this paper, management always wants to invest too much. Consequently, if $I^*$ is small, management invests in excess of $I^*$. Depending on whether $R$ is large in relation to the mean of $I^*$, shareholders want management to pay out funds at date 1. More formally, the payout $F$ that
maximizes firm value is the one that maximizes

\[ V = F + \int_{0}^{R-F} (R - I^* - F) Yh(I^*) dI^* + \int_{0}^{R-F} I^* Zh(I^*) dI^* + \int_{R-F}^{\infty} (R - F) Z h(I^*) dI^*. \] (8)

Here, \( F \) is riskless because \( R \) is nonstochastic. The second term in (8) is the present value from the payoff of investment in the bad project, whereas the third and fourth terms correspond to the present value from the payoff of investment in the good project. There is a unique value of \( F \) that maximizes the value of the firm in this case. The comparative statics for \( F \) are straightforward:

Result 4

If \( R \) is nonstochastic and observable by shareholders, whereas \( I^* \) is random and observable by management only, the optimal face value of debt:

1. Falls if the marginal product of the investments available to the firm increases.
2. Increases if cash flow increases.
3. Increases if the probability that the firm will have free cash flow increases.

Proof. See appendix.

As in section 6, an improvement in the firm's investment opportunities and a decrease in the probability of free cash flow lead to less debt.

The analysis of this section could be expanded to make \( R \) stochastic. In this case, when \( R \) turns out to be low, the firm raises funds at date 1. Debt would still be useful as long as the costs of collective action to shareholders are high at date 1. This case differs in a fundamental way, however, from the cases analyzed earlier. If \( R \) is nonstochastic or if it is not observable, shareholders gain no benefit from acting at date 1 rather than at date 0. Hence, shareholders choose to act when their costs of collective action are lowest. Thus, a cost of collective action at date 1 exceeding the cost of collective action at date 0 by an arbitrarily small amount is sufficient to lead shareholders to force management to issue debt at date 0. In the case where \( R \) is unknown at date 0 but observable at date 1, shareholders benefit from acting at date 1, because they have more information. Consequently, it may
pay for shareholders to act at date 1 rather than at date 0 even if the costs of collective action are substantially higher at date 1 than at date 0. Presumably, the advantage of acting at date 1 increases with the volatility of cash flow, so that firms with highly volatile cash flow may have no debt at date 0. Alternatively, instead of acting at date 1, shareholders could force management to issue debt contingent on the realization of cash flow at date 0, since the cash flow will be observable. It follows that if the investment opportunity set is not observable but cash flow is, the case for debt with a fixed face value becomes somewhat weaker than when cash flow is not observable. When cash flow is observable, the extent to which debt is used to reduce the costs of managerial discretion depends on the costs of collective action and/or on the difficulties in writing debt contracts in which debt payments depend on cash flow.

The analysis of section 5 could also be expanded by making $I^*$ stochastic. If the realization of $I^*$ is observable, shareholders can infer from $I^*$ some information about $R$ if $R$ and $I^*$ are correlated. This additional information may reduce the cost of underinvestment because more funds could be raised at date 1 when shareholders suspect $R$ is low. If the realization of $I^*$ is not observable, the optimal amount of debt would be lower if $I^*$ and $R$ are positively correlated, because cash flow is likely to be high when the firm has good investment opportunities.

9. Concluding remarks

In this paper, financing policy matters because it reduces the agency costs of managerial discretion. These costs exist when management values investment more than shareholders do and has information that shareholders do not have. Managerial discretion has two costs: an overinvestment cost that arises because management invests too much in some circumstances and an underinvestment cost caused by management’s lack of credibility when it claims it cannot fund positive NPV projects with internal resources. A debt issue that requires management to pay out funds when cash flows accrue reduces the overinvestment cost but exacerbates the underinvestment cost. An equity issue that increases resources under management’s control reduces the underinvestment cost but worsens the overinvestment cost. Since debt and equity issues decrease one cost of managerial discretion and increase the other, there is a unique solution for the firm’s capital structure.

Whereas much of the finance literature focuses on the present value of cash flows, this paper shows that the distribution of cash flows matters period by period, because shareholders want to optimize resources under managerial control each period to maximize their wealth. Consequently, this approach could be used to develop a theory of the optimal maturity of a firm’s debt. Further, the analysis provides a rationale for policies that reduce
As the volatility of a given period's cash flow falls, it becomes less likely that resources available to management will differ significantly from the resources shareholders expect management to have.

Appendix

Proof of Result 1

The first-order condition for \( N^* \) is

\[
V_{N^*} = \int_{I^* - N^*}^{\infty} Yg(R) \, dR + \int_{0}^{\infty} Zg(R) \, dR - 1 + \lambda = 0, \tag{A.1}
\]

where \( \lambda \) is the Lagrangian multiplier for the constraint that net financing must be nonnegative. Evaluated at \( N^* = 0 \), (A.1) yields eq. (3). To investigate the properties of an interior solution, we use the implicit function theorem to get

\[
\frac{dN^*}{dY} = -\frac{V_{N^*}}{V_{N^*}}. \tag{A.2}
\]

For \( N^* < I^* \), the second-order condition for a maximum holds:

\[
V_{N^*, N^*} = Yg(I^* - N^*) - Zg(I^* - N^*) < 0, \tag{A.3}
\]

since \( g(I^* - N^*) > 0 \) and \( Y < Z \). Shareholders never allow \( N^* \geq I^* \). Since \( V_{N^*, N^*} \) is negative, the sign of \( \frac{dN^*}{dZ} \) is the same as the sign of \( V_{N^*, Z} \).

Hence, for Result 1.1:

\[
V_{N^*, Z} = G(I^* - N^*) > 0, \tag{A.4}
\]

\[
V_{N^*, Y} = 1 - G(I^* - N^*) > 0, \tag{A.5}
\]

\[
V_{N^*, \cdot} = (Z - Y)g(I^* - N^*) > 0. \tag{A.6}
\]

For Result 1.2, note that \( V_{N^*, \cdot} = (Z - Y)G(I^* - N^*) + Y - 1 \). \( G(I^* - N) \) is the probability that cash flow \( R \) will be smaller than \( I^* - N \), i.e., that free cash flow will be negative. An increase in that probability increases \( V_{N^*, \cdot} \) for a given value of \( N^* \). To keep the first-order condition holding, \( N^* \) must therefore increase, since \( V_{N^*, N^*} < 0 \).

Proof of Result 2

From (3), there is an interior solution for \( F \) if there is a value of \( F^* \), such that

\[
V_{F^*} = \int_{F^* - I^*}^{\infty} (1 - Y)g(R) \, dR - \int_{F^*}^{I^*} (Z - 1)g(R) \, dR = 0. \tag{A.7}
\]
If \( V_{F^*} = 0 \) for \( F^* > 0 \), then \( V_F > 0 \) evaluated at \( F = 0 \) because for \( F = 0 \), \( V_F = -V_N \). Hence, if there is an interior solution for \( F^* \), \( V(F^*) > V(N) \) for all \( N \geq 0 \). Without distributional assumptions, it cannot be shown that there is a unique value of \( F \) that maximizes firm value. However, if there are multiple values of \( F \) that maximize firm value, management chooses the value that implies the lowest probability of default to maximize its consumption of perquisites. Hence, since we assume that \( g(R) > 0 \) for finite strictly positive values of \( R \), the value of \( F \) chosen by management is always unique. (It turns out that if \( R \) is lognormally distributed, there is at most one finite value of \( F \) that maximizes firm value.)

**Proof of comparative statics for the level of debt**

For an interior solution for \( F^* \), \( V_{F^*} \cdot < 0 \). Hence, using the implicit function theorem, \( dF^*/dX \) has the same sign as \( V_{F^*} \cdot  \), where \( X \) is the parameter for which the comparative statics are evaluated:

\[
V_{F^*Y} = -\int_{F^*+I^*}^{\infty} g(R) dR < 0 \quad \text{and} \quad V_{F^*Z} = -\int_{F^*}^{F^*+I^*} g(R) dR < 0.
\]

(A.8)

\[
V_{F^*I} = (Y - Z)g(F^* + I^*) < 0.
\]

(A.9)

Note that the first-order condition (5) can be written as

\[
V_{F^*} = (1 - Y)(1 - G(F^* + I^*))
\]

\[-(G(F^* + I^*) - G(F^*))(Z - 1) = 0.
\]

Hence, \( V_{F^*} \) is decreasing in \( G(F^* + I^*) \), which is the probability that the firm will not have free cash flow given debt \( F^* \). An increase in that probability of \( \Delta \) that does not increase \( G(F)^* \) by more than \( \Delta \) necessarily decreases \( V_{F^*} \) so that \( F^* \) has to fall.

**Proof of Result 3**

For Result 3.1, note that the cost of managerial discretion when the firm issues equity can be rewritten as

\[
W = R + N^* - I^* - C(R, I^* - N^*, 1)Y + P(R, I^* - N^*, 1)Z
\]

\[= [R + N^* - I^*](1 - Y) + P(R, I^* - N^*, 1)(Z - Y), \quad \text{(A.10)}
\]
where $C(R, I^* - N^*, 1)$ and $P(R, I^* - N^*, 1)$ denote, respectively, the price of a call option and of a put option on $R$ with maturity at date 1 and exercise price of $I^* - N^*$. $R$ is the date 1 value of a claim on $R$; with risk-neutral preferences and an interest rate equal to zero, $R = E(R)$. From Merton (1973), we know that the put price increases with the addition of a mean-preserving spread to $R$.

For Result 3.2, note that the first integral in (4) increases if a mean-preserving spread is added to the distribution of cash flow. Hence, $V(F)$ falls with the addition of a mean-preserving spread to cash flow, provided that the sum of the last two terms does not increase too much with the addition of such a spread. The sum of the last two terms can be written as $C(R, F, 1) - C(R, F + I^*, 1)$ and hence is equivalent to the value of subordinated debt with face value $I^*$. Since $W = R + I^*(Z - 1) - V(F^*)$, a decrease in $V(F^*)$ increases the agency costs of managerial discretion.

**Proof of Result 4**

The first-order condition for an interior solution for $F^*$ is

$$
V_{F^*} = 1 - \int_{0}^{R-F^*} Yh(I^*) \, dI^* - \int_{R-F^*}^{\infty} Zh(I^*) \, dI^* = 0. \tag{A.11}
$$

For $R > F^*$,

$$
V_{F^*} = (Y - Z) g(R - F^*) < 0. \tag{A.12}
$$

To obtain Result 4.1, we use the implicit function theorem:

$$
dF^*/dY = -V_{F^*}/V_{F^*} = \int_{0}^{R-F^*} h(I^*) \, dI^*/V_{F^*} < 0, \tag{A.13}
$$

$$
dF^*/dZ = -V_{F^*}/V_{F^*} = \int_{R-F^*}^{\infty} h(I^*) \, dI^*/V_{F^*} < 0. \tag{A.14}
$$

Similarly, for Result 4.2, we obtain

$$
dF^*/dR = -V_{F^*}/V_{F^*} = (Z - Y) h(R - F^*)/V_{F^*} > 0. \tag{A.15}
$$

For Result 4.3, the first-order condition can be rewritten as

$$
V_{F^*} = 1 - (Z + Y)(1 - H(R - F^*)) = 0,
$$

where $H(R - F^*)$ is the probability that $I^*$ is smaller than $R - F^*$ and hence that the firm will have free cash flow. $V_{F^*}$ is increasing in that probability.
References


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