

Constitutive Modeling for Design and Control of Magnetostrictive Galfenol Devices

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Abstract. A dynamic, nonlinear model for magnetic induction and strain response of cubic magnetostrictive materials to 3-D dynamic magnetic fields and 3-D stresses is developed. Dynamic eddy current losses and inertial stresses are modeled by coupling Maxwell's equations to Newton's second law through a nonlinear constitutive model. The constitutive model is derived from continuum thermodynamics.

Introduction

Magnetostrictive materials (MM) deform in response to magnetic fields and change magnetization in response to mechanical stress. These effects have been effective for creating sensors, actuators, and adaptive structures with high specific power and rugged operation [1]. Galfenol ($\text{Fe}_x\text{Ga}_{1-x}$, $13 \leq x \leq 29$ at%) is a unique MM because it exhibits moderate magnetostriction and steel-like mechanical properties. It can be machined, rolled, deposited, and welded and is capable of bearing compressive, tensile, bending, torsion, and shock loads [2]. Despite these advantages, dynamic devices utilizing Galfenol are expected to be heavily influenced by dynamic thermal and eddy current effects as the operating frequency is increased [3]. Fundamental models incorporating these effects are therefore needed for design, characterization, and control of future Galfenol devices.

In this article we develop a fully-coupled model that characterizes the nonlinear and dynamic strain and magnetization of Galfenol in response to time-varying magnetic fields and mechanical stresses. Dynamic effects include eddy current losses, magnetic after-effects, and the mechanical dynamics of the transducer and load. The model provides a framework for characterization, design, and control of Galfenol devices subjected to combined dynamic magnetic field and stress loading. While the model is applied to Galfenol it can be directly applied to general magnetostrictive materials which exhibit cubic anisotropy.

Eddy current losses and mechanical transducer dynamics

The magnetic induction and strain (or displacement) of Galfenol is dependent on the magnetic field and stress. The field and stress state depends on the magnetic and mechanical boundary conditions and the geometry of the transducer. The field is calculated from the boundary value problem (BVP) described by Maxwell's equations and the stress from a BVP described by Newton's second law. In dynamic operation, eddy current losses cause a spatial distribution in the magnetic field. Eddy currents can be described by Maxwell's equations,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \mathbf{J} = \frac{1}{\rho_e} \mathbf{E}, \quad \nabla \times \mathbf{H} = \mathbf{J}. \quad (1)$$

A changing magnetic induction \mathbf{B} creates an electric field \mathbf{E} which in turn creates a current density which finally causes a magnetic field \mathbf{H} opposing the changing magnetic induction. This causes an

instantaneous power loss per unit volume $\mathbf{J} \cdot \mathbf{E}$ and a spatial variation in the magnetic field. The spatial variation may be described by combining Maxwell's equations into a single BVP

$$\nabla \times (\nabla \times \mathbf{H}) = -\frac{1}{\rho_e} \frac{\partial \mathbf{B}}{\partial t}(\mathbf{H}, \mathbf{T}), \quad \mathbf{H}(\Omega, t) = \mathbf{H}_\Omega(t), \quad \mathbf{H}(\mathbf{X}, 0) = \mathbf{H}_0(\mathbf{X}), \quad (2)$$

where \mathbf{X} is the spatial location and Ω denotes the boundary. To solve the boundary value problem, a material constitutive law needs to be formulated, e.g.,

$$\mathbf{B}(\mathbf{H}, \mathbf{T}) = \mu_0 [\mathbf{H} + \mathbf{M}(\mathbf{H}, \mathbf{T})], \quad (3)$$

where the first term is the induction due to the magnetic field and the second term is the stress (\mathbf{T}) and field dependent magnetization of Galfenol. Because of the stress dependent magnetization, a time-varying stress results in a time-varying magnetic induction and hence eddy current losses.

The presence of stress in the magnetic BVP couples it to the mechanical BVP defined by Newton's second law,

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = -\nabla \cdot \mathbf{T} - c_D \frac{\partial \mathbf{u}}{\partial t} - \mathbf{F}, \quad \mathbf{u}(\Omega, t) = \mathbf{u}_\Omega(t), \quad \mathbf{u}(\mathbf{X}, 0) = \mathbf{u}_0(\mathbf{X}) \quad (4)$$

where \mathbf{u} is displacement, ρ is density, c_D is the Kelvin-Voigt damping coefficient and \mathbf{F} represents body forces. The strain \mathbf{S} is the sum of the purely elastic strain obeying Hooke's law, $\mathbf{S}_e = \mathbf{c}^{-1} \mathbf{T}$ (where \mathbf{c} is the compliance tensor), and the field and stress dependent magnetostriction $\mathbf{S}_m(\mathbf{H}, \mathbf{T})$ [4]. For small strains $\mathbf{S} = \nabla \mathbf{u}$, the stress is

$$\mathbf{T} = \mathbf{c} [\nabla \mathbf{u} - \mathbf{S}_m(\mathbf{H}, \mathbf{T})]. \quad (5)$$

The implicit relationship for stress, Eq. 5, couples Eq. 4 to Eq. 2. The boundary value problems and constitutive relationships in Eqns. 3 and 5 provide a general framework for modeling dynamic induction and strain response of magnetostrictive materials to an external field \mathbf{H}_Ω and body force \mathbf{F} . In the sections that follow, a computationally efficient and 3-D constitutive model for $\mathbf{M}(\mathbf{H}, \mathbf{T})$ and $\mathbf{S}_m(\mathbf{H}, \mathbf{T})$ incorporating rate-dependent thermal effects is developed.

Constitutive Model

Ferromagnetic materials below the Curie temperature have regions of uniform magnetization M_s where the atomic magnetic moments are aligned with each other. These regions of uniform magnetization are called magnetic domains. Magnetic domains have preferred orientations which depend on the magnetic anisotropy, magnetic field and stress. When thermodynamic equilibrium is achieved, the steady-state macroscopic magnetization \mathbf{M}_{ss} and magnetostriction $\mathbf{S}_{m,ss}$ of a material having r equilibrium domain orientations $\hat{\mathbf{m}}^k$ is the sum of the magnetization $M_s \hat{\mathbf{m}}^k$ and magnetostriction $\hat{\mathbf{S}}_m^k$ due to each equilibrium orientation, weighted by the volume fraction $\hat{\xi}^k$ of domains in each orientation,

$$\mathbf{M}_{ss} = M_s \sum_{k=1}^r \hat{\xi}^k \hat{\mathbf{m}}^k, \quad \mathbf{S}_{m,ss} = \sum_{k=1}^r \hat{\xi}^k \hat{\mathbf{S}}_m^k. \quad (6)$$

Equilibrium Domain Orientations. The internal energy of a magnetic domain with orientation $\mathbf{m} = [m_1 \ m_2 \ m_3]$ is due to the magnetocrystalline anisotropy energy. After considering the cubic crystal symmetry and neglecting higher order terms, the internal energy for cubic materials is [4]

$$U(\mathbf{m}) = K_4 (m_1 m_2 + m_2 m_3 + m_3 m_1), \quad (7)$$

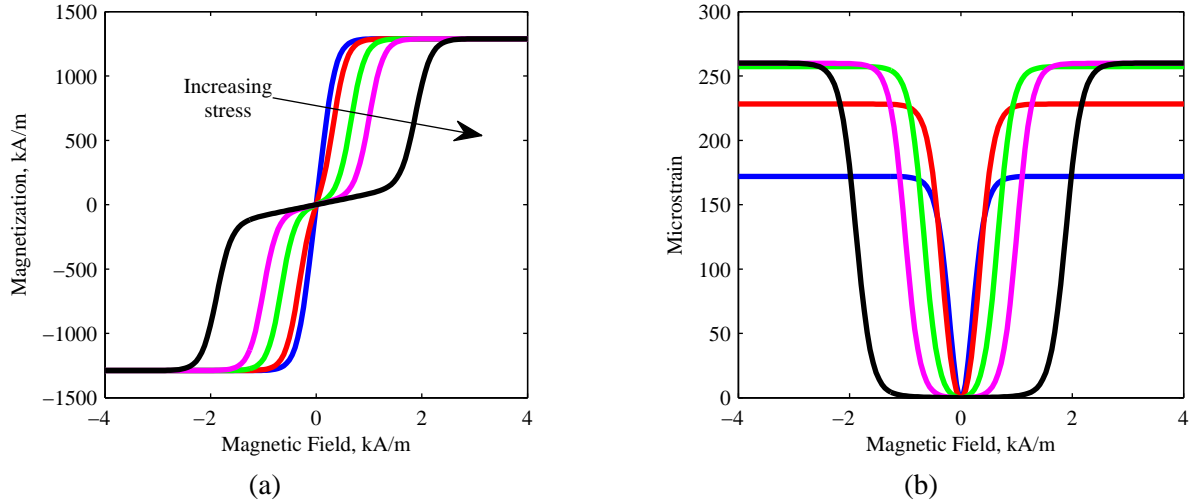


Figure 1: Steady-state simulations of (a) magnetization and (b) magnetostriction with varying field and constant stress levels of 0, -5, -15, -25, and -40 MPa.

where K_4 is the fourth-order, cubic anisotropy constant. The Gibbs free energy is

$$G(\mathbf{H}, \mathbf{T}) = U(\mathbf{m}) - \mathbf{S}_m \cdot \mathbf{T} - \mu_0 M_s \mathbf{m} \cdot \mathbf{H}, \quad (8)$$

where \mathbf{T} is the six-element stress vector with the first three components the longitudinal stresses and the last three the shear stresses, \mathbf{H} is magnetic field and $\mathbf{S}_m = \mathbf{S}_m(\mathbf{m})$ is the magnetostriction with longitudinal components [4]

$$S_{m,i} = -\frac{3}{2} \lambda_{100} m_i^2, \quad i = 1, 2, 3 \quad (9)$$

and shear components

$$S_{m,4} = -3\lambda_{111} m_1 m_2, \quad S_{m,5} = -3\lambda_{111} m_2 m_3, \quad S_{m,6} = -3\lambda_{111} m_3 m_1. \quad (10)$$

These expressions are derived by balancing the magnetic anisotropy, elastic, and magnetoelastic coupling energies [4]. The equilibrium domain orientations ($\hat{\mathbf{m}}^k; k = 1, \dots, r$), needed for calculation of the bulk magnetization in Eq. 6, are obtained through the conditions $\partial G / \partial m_i = 0$ with the constraint $m_1^2 + m_2^2 + m_3^2 = 1$. The equilibrium magnetostrictions ($\hat{\mathbf{S}}_m^k; k = 1, \dots, r$), needed for calculation of the bulk magnetostriction in Eq. 6, are obtained by evaluating Eqns. 9 and 10 using the equilibrium domain orientations, $\hat{\mathbf{S}}_m^k = \mathbf{S}_m(\hat{\mathbf{m}}^k)$.

Equilibrium Domain Volume Fractions. The domain volume fractions in each equilibrium can be determined by minimization of a total Gibbs energy potential which includes the entropy of a collection of domains. The entropy of a system with r energy states is

$$\eta = -\frac{k_B}{V} \sum_{k=1}^r \xi^k \ln \xi^k. \quad (11)$$

The total Gibbs energy and equilibria are given by

$$\bar{G}(\mathbf{H}, \mathbf{T}) = -\eta\theta + \sum_{k=1}^r \xi^k G^k(\mathbf{H}, \mathbf{T}), \quad \hat{\xi}^k = \frac{e^{-G^k(\mathbf{H}, \mathbf{T})V/k_B\theta}}{\sum_{j=1}^r e^{-G^j(\mathbf{H}, \mathbf{T})V/k_B\theta}}. \quad (12)$$

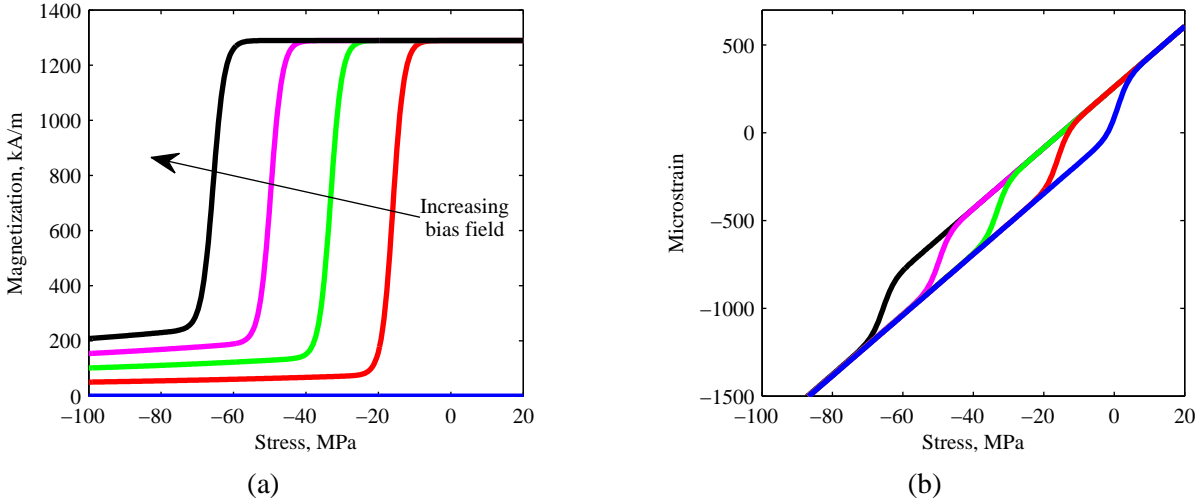


Figure 2: Steady-state simulations of (a) magnetization and (b) magnetostriction with varying stress and constant field values of 0, 10, 20, 30, and 40 kA/m.

where θ is temperature and $G^k(\mathbf{H}, \mathbf{T})$ is the Gibbs energy of the k^{th} equilibrium domain orientation given by Eq. 8.

Model predictions for the steady-state magnetization and magnetostriction in the [100] direction in response to a magnetic field at constant compressive stress are shown in Figs. 1(a) and 1(b). The responses have rounded elbows and exhibit a kinked shape due to 90 degree domain orientations in agreement with the magnetization and magnetostriction measurements [3]. Prediction of the magnetization and strain in the [100] direction in response to stress at constant magnetic field is shown in Figs. 2(a) and 2(b).

Basso et al. [5] have shown that rate-dependent thermal effects result in hysteresis losses in the magnetic constitutive behavior. For time-varying magnetic fields and stresses there is a delay associated with the evolution of the domain volume fractions as moments overcome anisotropy barriers. The dynamic magnetization \mathbf{M} and magnetostriction \mathbf{S}_m are characterized by two first-order differential equations with time constant τ , driven by the steady-state constitutive models in Eq. 6,

$$\tau \frac{d\mathbf{M}}{dt} + \mathbf{M} = \mathbf{M}_{ss}, \quad \mathbf{M}(\mathbf{H}, \mathbf{T}, 0) = \mathbf{M}_{ss}(\mathbf{H}, \mathbf{T}), \quad (13)$$

$$\tau \frac{d\mathbf{S}_m}{dt} + \mathbf{S}_m = \mathbf{S}_{m,ss}, \quad \mathbf{S}_m(\mathbf{H}, \mathbf{T}, 0) = \mathbf{S}_{m,ss}(\mathbf{H}, \mathbf{T}). \quad (14)$$

Dynamic Tonpilz transducer Model

The general boundary value problems in Eqs. 2 and 4 may be simplified to provide a low-order dynamic model for a theoretical Tonpilz transducer composed of a cylindrical Galfenol rod in a closed magnetic circuit, loaded by a spring with stiffness k_L , damping coefficient c_L , and mass m_L (see Fig. 3). The rod is approximated as infinitely long and is energized by a magnetic field H_{ext} at the surface, applied along the rod length. This is an accurate assumption when the magnetic circuit is closed. In this case, Maxwell's equations are reduced to

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial H}{\partial r} \right) = \frac{1}{\rho_e} \frac{\partial B}{\partial t} = \frac{\mu_0}{\rho_e} \left(\frac{\partial H}{\partial t} + \frac{\partial M}{\partial t} \right), \quad H(r_0, t) = H_{ext}, \quad H(r, 0) = H_0, \quad (15)$$

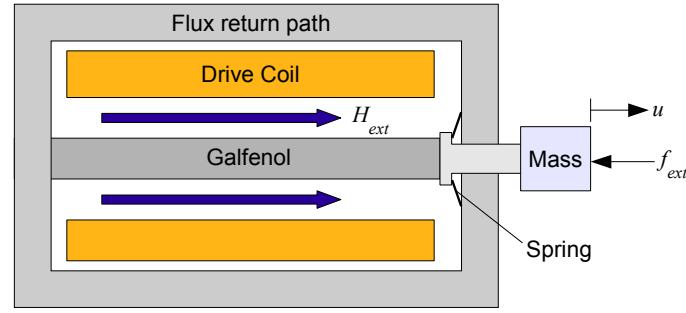


Figure 3: General Tonpilz transducer used for model development.

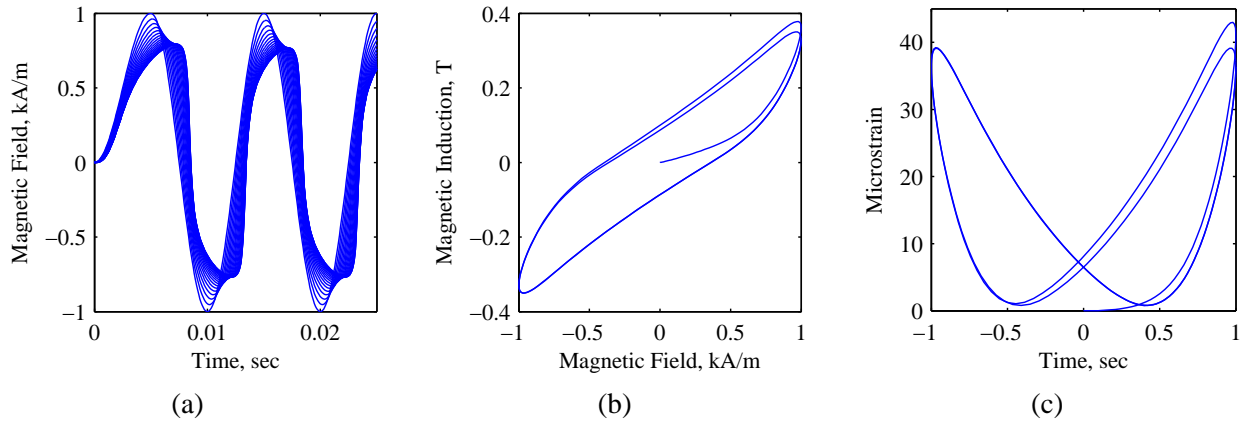


Figure 4: Tonpilz model dynamic actuation simulation with $k_l = (EA/l)/8$, a bias stress of -5 MPa and a 100 Hz external field input where (a) shows the spatial variation of the magnetic field with the smallest amplitude field at the center, (b) shows the average magnetic induction vs. applied magnetic field, and (c) shows the rod elongation u/l vs. applied magnetic field.

where r is the radial location and r_0 is the radius of the rod. If the $[001]$ crystal orientation of the Galfenol rod is oriented along the rod axis, then $\partial M/\partial t$ is given by the $[001]$ component of the dynamic constitutive model, Eq. 13. With the uniformity assumption for stresses and strains, the mechanical boundary value problem is reduced to a second-order differential equation for the rod tip displacement from equilibrium, driven by an external force f_{ext} at the rod tip and the average magnetostriction \bar{S}_m given by averaging the $[001]$ component of $\mathbf{S}_m(H(r, t), T)$ from Eq. 14 over the cross section of the rod,

$$(m_R + m_L) \frac{d^2 u}{dt^2} + (c_R + c_L) \frac{du}{dt} + (k_R + k_L) u = EA \bar{S}_m(H, T) - f_{ext}, \quad u(0) = 0, \quad (16)$$

where $m_R = (1/3)\rho A l$ is the dynamic mass of the Galfenol rod with cross-sectional area $A = \pi r_0^2$ and length l , $c_R = c_D A/l$ is the damping coefficient of the rod, and $k_R = EA/l$ is its equivalent spring stiffness. The constitutive relationship for stress in Eq. 5 reduces to $T = E(u/l - \bar{S}_m)$. The outputs are the displacement u and the magnetic induction \bar{B} averaged over the cross-section and the inputs are the externally applied magnetic field H_{ext} and the external force.

Model Simulations. For all simulations the rod dimensions are $1/4 \times 1$ inches, $E = 60$ GPa, $\rho = 77.1$ Kg/m³, $\lambda_{100} = 173 \times 10^{-6}$, $\lambda_{111} = 20 \times 10^{-6}$, $\mu_0 M_s = 1.62$ T, $K_4 = 10$ kJ/m³, $k_B \theta/V = 200$ kJ/m³, $\tau = 1 \times 10^{-9}$ sec, and $\rho_e = 7 \times 10^{-7}$ Ω m.

As the Tonpilz transducer is actuated by a dynamic external magnetic field, eddy currents give rise to magnetic fields which oppose the applied field; this causes a spatial distribution of the magnetic

field which results in hysteresis between the external field and average induction of the rod (see Fig. 4). A dynamic force input results in a dynamic change in the magnetic induction due to stress-induced domain rotation. In response to this dynamic magnetic induction, eddy currents arise which also results in dynamic hysteresis loss as seen in Fig. 5(b), now between the applied force and the rod elongation.

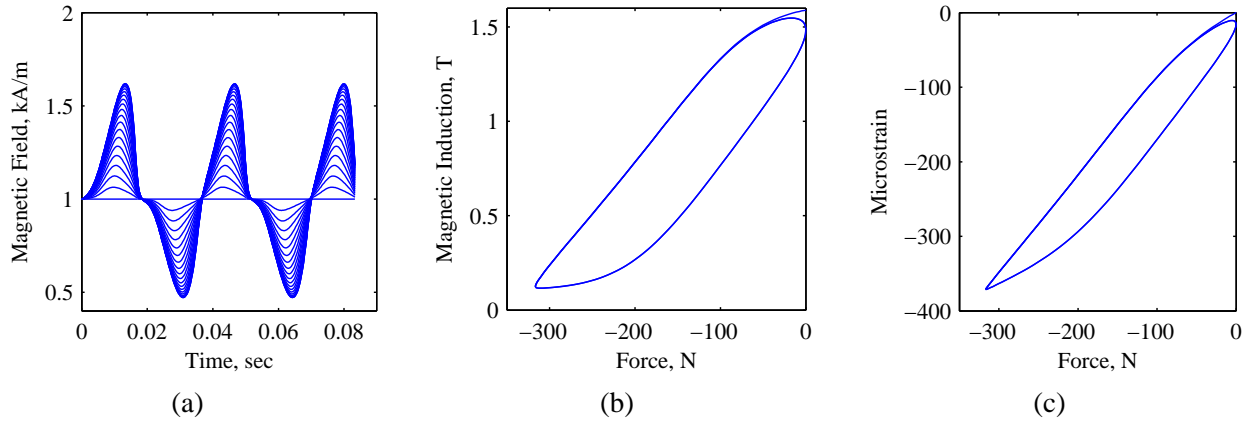


Figure 5: Dynamic sensing simulation at 100 Hz where (a) shows the spatial variation of the magnetic field with the largest amplitude field at the center (b) average magnetic induction vs. applied force and (c) rod elongation u/l vs. applied force

Conclusion

A fully-coupled, dynamic model has been developed to characterize the nonlinear and dynamic strain and magnetic induction of Galfenol in response to time-varying magnetic fields and mechanical stresses. The model provides a framework for characterization, design, and control of Galfenol devices subjected to combined 3-D dynamic magnetic field and 3-D stress loading. It may generally be applied to cubic magnetostrictive materials. We wish to acknowledge the financial support by the Office of Naval Research, MURI grant #N000140610530.

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