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THERMODYNAMIC MODEL FOR THE SENSING BEHAVIOR IN FERROMAGNETIC SHAPE MEMORY NI-MN-GA

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ABSTRACT
A magnetomechanical model for the sensing effect in the ferromagnetic shape memory alloy (FSMA) Ni-Mn-Ga is presented. The model is based on continuum thermodynamics with the constitutive response being derived from the total (magnetic and mechanical) free energy. The model predicts the stress-strain and induction-strain response at various bias fields. Only two experiments are required to obtain the model parameters.

MODEL DEVELOPMENT
FSMAs have been considered for actuation applications because of their large strains (> 6%) and high bandwidth compared to heat activated SMAs. A recent experimental study by the authors [1] has shown the feasibility of utilizing commercial single-crystal Ni-Mn-Ga for sensing applications, an area which has received little attention. This paper presents a thermodynamic model which describes the measurements in Ref. [1]. This paper differs from prior thermodynamic models for Ni-Mn-Ga [2–4] in the construction of new energy functions with strain and field as independent variables, as required for sensor modeling.

Fig. 1 shows a simplified two-variant FSMA microstructure consisting of field preferred (ξ) and stress preferred variants (1-ξ). In each variant, the magnetic domains (α, 1-α) are generated to minimize the net magnetostatic energy. The applied magnetic field is oriented in the x direction, and the magnetization vectors in the field-preferred variant are constrained to be in the direction of the field or opposing it. However, the magnetization vectors in stress-preferred variants can rotate by an angle θ, which is equal and opposite in the two domains. These postulates are reached after assuming four different rotation angles and equating the four driving forces to zero.

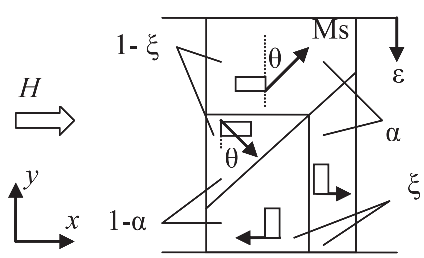

Figure 1. SIMPLIFIED TWO-VARIANT FSMA MICROSTRUCTURE.

The magnetic field (H) and externally applied strain (ε) constitute the independent variables, with ε the sum of elastic (ε_e) and detwinning (ε_{tw}) components. The latter is given by $\varepsilon_0(1-\xi)$, with ε_0 being the lattice distortion (5.8%). The stress (σ) and magnetization (M) are the dependent variables. The volume fraction (ξ), domain fraction (α) and magnetization rotation angle (θ) relative to the crystallographic c-axis are the internal variables. The modified form of the reduced Clausius-Duhem inequality for the isothermal process is given by

$$-\rho\dot{\phi}+\sigma\dot{\varepsilon}-\mu_0 M\dot{H}\geq 0, \quad (1)$$

in which the second term represents the rate of work due to applied strains and the third term represents the rate of work due to applied fields. The total thermodynamic potential or magnetic Gibbs free energy is the sum of mechanical and magnetic (Zee-

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man, magnetostatic and anisotropy) contributions

$$\rho\phi = \frac{E}{2}\varepsilon_e^2 + \frac{a}{2}\varepsilon_{tw}^2 - \mu_0 MH + \frac{\mu_0}{2}NM^2 + K_u(1-\xi)\sin\theta^2. \quad (2)$$

The parameters needed for quantification of the mechanical energy are obtained from the experimental stress-strain curve at zero bias field. Other parameters include the *average* elastic modulus of the sample (E) and the stiffness associated with detwinning strain (a). Other studies have considered linearly-varying compliance ($S(\xi)$), e.g., Ref. [3]. This implies the use of $\int[1/S(\xi)]\varepsilon_e d\varepsilon_e$ in lieu of the elastic term $E\varepsilon_e^2/2$ in eq. (2), which significantly decreases model tractability without necessarily improving model accuracy. Parameter N represents the demagnetization factor. The anisotropy constant K_u is evaluated experimentally as the difference in the area of the easy and hard axis magnetization curves. Thus, K_u represents the energy associated with pure rotation of the magnetization vectors. Using the Coleman-Noll procedure [5], eq. (2) can be written as,

$$(\sigma - \frac{\partial(\rho\phi)}{\partial\varepsilon})\dot{\varepsilon} + (-\mu_0 M - \frac{\partial(\rho\phi)}{\partial H})\dot{H} + \pi^{\alpha}\dot{\alpha} + \pi^{\theta}\dot{\theta} + \pi^{\xi}\dot{\xi} \geq 0 \quad (3)$$

where the terms $\pi^{\alpha} = -\partial(\rho\phi)/\partial\alpha$, $\pi^{\theta} = -\partial(\rho\phi)/\partial\theta$, $\pi^{\xi} = -\partial(\rho\phi)/\partial\xi$ represent thermodynamic driving forces associated with the internal state variables α, θ, and ξ, respectively. This leads to the constitutive equations

$$\sigma = \frac{\partial(\rho\phi)}{\partial\varepsilon} = E(\varepsilon - \varepsilon_{tw}) \quad (4)$$

$$M = -\frac{1}{\mu_0}\frac{\partial(\rho\phi)}{\partial H} = M_s(2\xi\alpha - \xi + \sin\theta - \xi\sin\theta), \quad (5)$$

where M_s is the saturation magnetization. Because the magnetization curves of commercial Ni-Mn-Ga show negligible hysteresis [1], the processes associated with evolution of the corresponding variables α and θ are reversible. Hence, the driving forces are zero, $\pi^{\alpha} = 0$ and $\pi^{\theta} = 0$. This gives the following closed form solutions for the variables α and θ

$$\alpha(H,\xi) = \frac{H}{2NM_s\xi} + \frac{1}{2} \quad (6)$$

$$\theta(H,\xi,\alpha) = \arcsin\left(\frac{2\mu_0 NM_s^2\alpha\xi - \mu_0 NM_s^2\xi - \mu 0 HM_s}{\mu_0 NM_s^2\xi - 2K_u - \mu_0 NM_s^2}\right). \quad (7)$$

Inequality (3) thus reduces to $\pi^{\xi}\dot{\xi} \geq 0$. Since during loading the volume fraction rate is negative ($\dot{\xi} \leq 0$), the force π^{ξ} must be negative. Twin boundary motion is initiated when π^{ξ} reaches the negative value of a yield function π^{cr}, which represents the energy barrier to initiate detwinning [2–4]. Parameter π^{cr} is estimated as 46400 J/m^3 from the detwinning stress at zero field. Values of the volume fraction (ξ) can thus be obtained by solving the equation $\pi^{\xi} = -\pi^{cr}$. During unloading, the rate is positive ($\dot{\xi} \geq 0$), and hence π^{ξ} should reach a positive critical value π^{cr} for detwinning to commence. In addition, the mechanical energy in eq. (2) is modified as the detwinning that takes place during loading is considered irreversible.

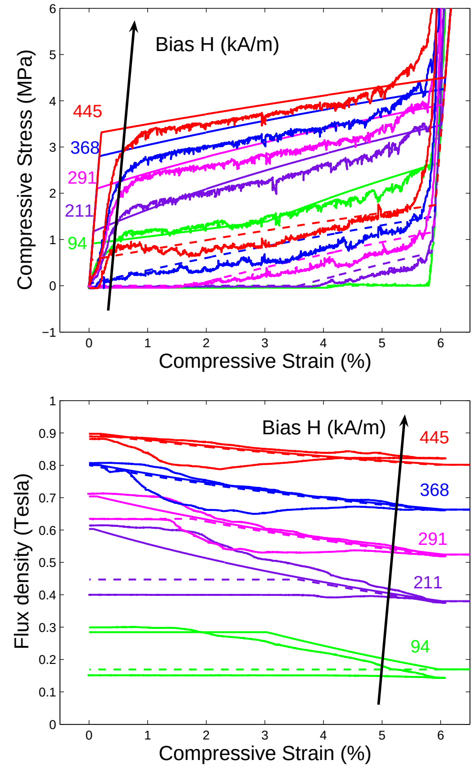

Figure 2. MODEL RESULTS: (a) STRESS-STRAIN, (b) FLUX DENSITY-STRAIN.

The model parameters obtained from experiments are M_s=620 kA/m, K_u=1.67E5 J/m^3, N=0.308, E=1600 MPa, k=16 MPa, and $a = Ek/(E-k)$. Figs. 2(a)-(b) show a comparison of model results with experimental data. The transition from irreversible (quasiplastic) to reversible (pseudoelastic) behavior occurs at a bias field of 368 kA/m. The same transition is observed in the elastic and magnetic behaviors [1]. The model accurately describes the sensing effect below and above the transition field.

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