AEROENG 2200
Midterm 2 Review

Question 1: Converging-Diverging Nozzle

- Isentropic Relations: Can use isentropic tables if the flow is calorically perfect air ($\gamma = 1.4$). Otherwise, use the equations below.

$$\frac{\rho_o}{\rho} = (1 + \frac{\gamma-1}{2}M^2)^{\frac{1}{\gamma-1}}$$

$$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2}M^2$$

$$\frac{P_o}{P} = (1 + \frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_2}{P_1} = (\frac{\rho_2}{\rho_1})^{\gamma} = (\frac{T_2}{T_1})^{\frac{\gamma}{\gamma-1}}$$

- Area-Mach Relation: A^* is required throat area to choke flow

$$(\frac{A}{A^*})^2 = \frac{1}{M^2}(\frac{2}{\gamma+1}[1 + \frac{\gamma-1}{2}M^2])^{\frac{\gamma+1}{\gamma-1}}$$

- A^* is a property of the flow (similar to stagnation conditions) and it doesn't have to be physically present!
- The Mach number at A^* is 1!

Problems:

Consider a hot, high-pressure gas (R = 378 J/kg K, $\gamma = 1.26$) entering a rocket engine from the combustion chamber. The flow is proceeding from a subsonic combustion chamber, through a throat into a nozzle. The nozzle exit velocity is 2500 m/s and the mass flow rate is 126 kg/s, with exit conditions of $T_e = 1348\ K$ and $p_e = 1\ atm$. Assume isentropic flow.

(a) What is the stagnation temperature in K?

(b) What is the stagnation density in kg/m^3?

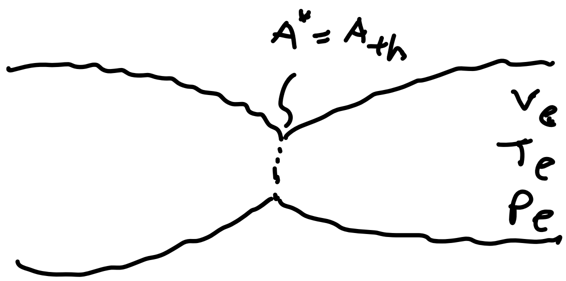
(c) What is the required throat area in m^2?

Given: $R = 378 \frac{J}{kg \cdot K}$, $\gamma = 1.26$, $V_e = 2500$ m/s, $\dot{m} = 126$ kg/s

$T_e = 1348$ K, $P_e = 101{,}325$ N/m^2

Find: $T_o = ?$ $\rho_o = ?$ $A^* = ?$

Sketch:



Equations:

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$M_e = \frac{V_e}{a_e} = \frac{V_e}{\sqrt{\gamma R T_e}} = \frac{2500}{\sqrt{1.26(378)(1348)}} = 3.12$$

$$T_o = T_{ex}\left(1 + \frac{\gamma - 1}{2} M_e^2\right) = 1348\left(1 + \frac{1.26 - 1}{2}(3.12)^2\right)$$

$$\boxed{T_0 = 3053.9\ K}$$

$$P_e = \rho_e R T_e$$

$$\rho_e = \frac{P_e}{R T_e} = \frac{101325}{378(1348)} = 0.1989\ kg/m^3$$

$$\rho_0 = \rho_e \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{1/\gamma-1} = 0.1989\left(1 + \frac{1.26-1}{2}(3.12)^2\right)^{1/0.26}$$

$$\boxed{\rho_0 = 4.62\ kg/m^3}$$

$$\dot{m} = \rho_{ex} A_{ex} V_{ex} \Rightarrow A_{ex} = \frac{\dot{m}}{\rho_{ex} V_{ex}} = \frac{126}{0.1989(2500)}$$

$$A_{ex} = 0.2534\ m^2$$

$$\left(\frac{A_{ex}}{A^*}\right)^2 = \frac{1}{M_{ex}^2}\left(\frac{2}{\gamma+1}\left[1 + \frac{\gamma-1}{2} M_{ex}^2\right]\right)^{\frac{\gamma+1}{\gamma-1}}$$

$$\left(\frac{0.2534}{A^*}\right)^2 = \frac{1}{(3.12)^2}\left(\frac{2}{2.26}\left[1 + \frac{0.26}{2}(3.12)^2\right]\right)^{\frac{2.26}{0.26}}$$

$$\boxed{A^* = 0.0385\ m^2 = A_{th}}$$

A model of an aircraft is being tested inside a supersonic wind tunnel. The reservoir of the windtunnel has a temperature of 700 K and a pressure of 8 atm. The flow is choked at the throat and a temperature of 450 K and mass flow rate off 2 kg/s are measured at the throat. The temperature of the test section was measured to be 280 K. What is the velocity at the throat, the area ~~of the throat~~ of test section, and the Mach number of the freestream flow in the test section?

Givens:

$T_0 = 700K$ $\quad$ $T^* = 450K$ $\quad$ $R_{air} = 287\ J/kgK$

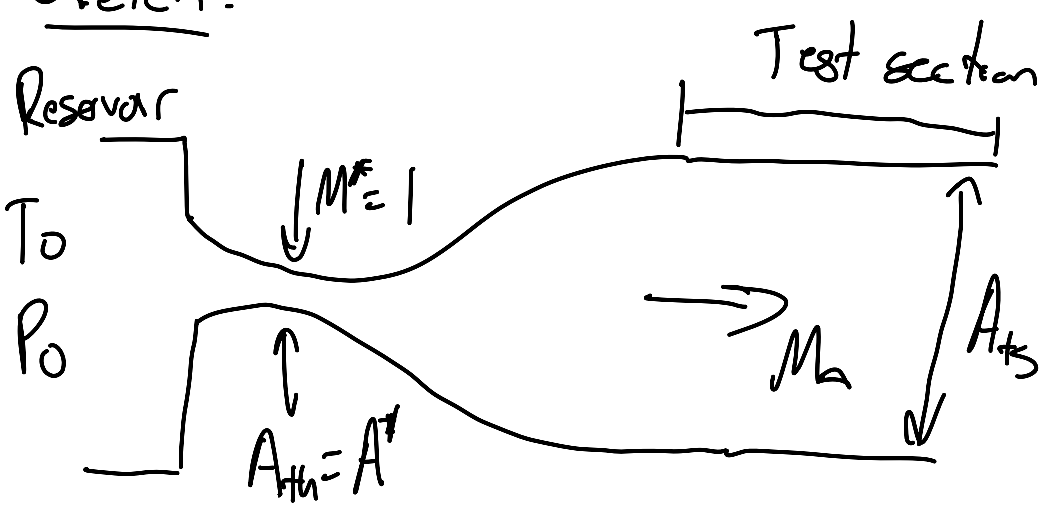
$P_0 = 8atm = 8(101325\ Pa)$ $\quad$ $\dot{m} = 2\ kg/s$ $\quad$ $\gamma = 1.4$

$M_{th} = M^* = 1$ $\quad$ $T_{ts} = 280K$ $\quad$ $C_p = 1005\ J/kgK$

Find:

$V^* = V_{th} = ?$, $A_{ts} = ?$, $M_A = M_{ts}$

Sketch:



Calc;

$$M^* = M_{th} = 1 = \frac{V^*}{a^*} = \frac{V^*}{\sqrt{\gamma R T^*}}$$

$$V^* = \sqrt{\gamma R T^*} = \sqrt{(1.4)(287)(450)}$$

Answer:

$$\boxed{V^* = 425 \text{ m/s}}$$

$$C_p \underline{T_{ts}} + \frac{1}{2} V_{ts}^2 = C_p \underline{T^*} + \frac{1}{2} \underline{V^*}^2$$

$$V_{ts} = \sqrt{2\left(C_p (T^* - T_{ts}) + \frac{1}{2} V^{*2}\right)}$$

$$= \sqrt{2\left(1005(450-280) + \frac{1}{2}(425)^2\right)}$$

$$V_{ts} = 722.72 \text{ m/s}$$

$$M_{ts} = M_{\infty} = \frac{V_{ts}}{a_{ts}} = \frac{V_{ts}}{\sqrt{\gamma R T_{ts}}} = \frac{722.72}{\sqrt{(1.4)(287)(280)}}$$

Answer:

$$\boxed{M_{ts} = 2.15}$$

$$\dot{m} = 2\ kg/s = \rho_{ts} \underline{A_{ts}} V_{ts}$$

$$\frac{\rho_0}{\rho_{ts}} = \left(1 + \frac{\gamma-1}{2} M_A^2\right)^{\frac{1}{\gamma-1}}$$

$$P_0 = \rho_0 R T_0 \rightarrow \rho_0 = \frac{P_0}{R T_0} = \frac{(8(101325))}{(287)(700)}$$

$$\rho_0 = 4.035\ kg/m^3$$

$$\rho_{ts} = \rho_0 \left(1 + \frac{\gamma-1}{2} M_A^2\right)^{-\frac{1}{\gamma-1}}$$

$$= (4.035)\left(1 + \frac{1.4-1}{2}(2.15)^2\right)^{-\frac{1}{1.4-1}}$$

$$\rho_{ts} = 0.7853\ kg/m^3$$

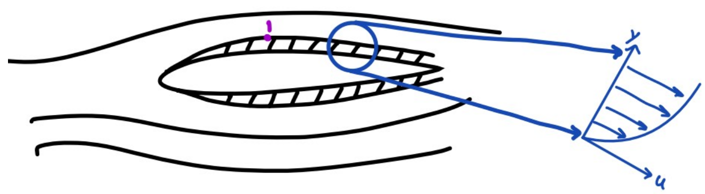
Answer:

$$A_{ts} = \frac{\dot{m}}{\rho_{ts} V_{ts}} = \frac{(2)}{(0.7853)(722.72)} \rightarrow \boxed{A_{ts} = 0.0035\ m^2}$$

Question 2: Viscous Flow

4.15.1 Introduction to Viscous Flow

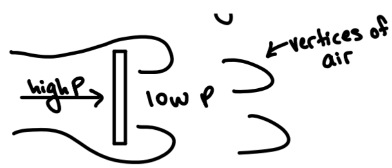
- Viscous Flows
  - Air imparts force on an aircraft through pressure (acts normal to surface) and friction (acts tangential to surface).
- Boundary Layer
  - Thin region of fluid that is impacted by viscosity.
  - Inviscid assumption *cannot* be made for flow that is in the boundary layer.
- Boundary Layer on an Airfoil
  - Boundary layer has a thickness of 0 at the leading edge that gradually increases.
  - Boundary layer on an airfoil (not to scale)



  - Pressure at the edge of the boundary layer equals the pressure acting on the surface of the airfoil (e.g., point 1 in diagram above).

4.15.2 Pressure Drag

- Pressure Drag
  - Higher for a blunt body (large surface area is exposed to flow).
  - Net pressure difference exerts a force on the coin to push it backwards.



  - It is important to minimize the effects of pressure drag.

4.15.2 Viscous Drag

- Viscous Drag
  - More flow goes past the coin on the surfaces parallel to the flow so there is more viscous (skin friction) drag and less pressure drag.

- An airfoil is designed by aerodynamicists to minimize pressure drag but we just have to live with the viscous drag.
- No slip condition → 0 relative velocity between the flow and the surface it goes over.

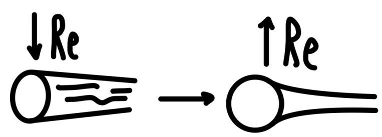
- Wall shear stress (τ):
  - $\tau = \mu(\frac{du}{dy})_{wall}$ where μ = dynamic viscosity and $(\frac{du}{dy})_{wall}$= change in velocity at the wall
  - $D = 2\tau A$

4.15.3 Sutherland's Law for Dynamic Viscosity

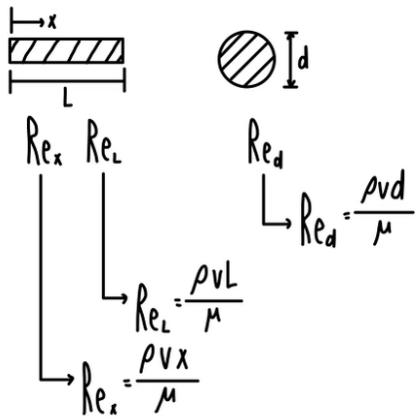
- Viscosity changes with temperature
- Standard sea level viscosity values
  - $\mu = 1.7894 \times 10^{-5} \frac{kg}{m \cdot s}$
  - $\mu = 3.7373 \times 10^{-7} \frac{slug}{ft \cdot s}$
- Temperature in K → $\mu = 1.458 \times \left[\frac{T^{1.5}}{T+110.4}\right] \times 10^{-6} \frac{kg}{m \cdot s}$
- Temperature in R → $\mu = 2.27 \times \left[\frac{T^{1.5}}{T+199}\right] \times 10^{-8} \frac{slug}{ft \cdot s}$
- Making a gas warmer increases the viscosity because the molecules are running into each other more

4.15.4 Reynold's Number Definition

- $Re = \frac{\rho_\infty v_\infty l}{\mu_\infty} = \frac{inertial\ forces}{viscous\ forces} \sim \frac{KE}{\tau}$
  - ρ_∞ = density, v_∞ = freestream velocity, l = length scale, μ_∞ = dynamic viscosity
  - Relations between Re and other aerodynamic characteristics:
    - ↓Re ↑viscous forces ↓inertial forces
    - ↑Re ↓viscous forces ↑inertial forces
    - ↑Re ↓separation point (separation point moves toward TE as ↑Re)

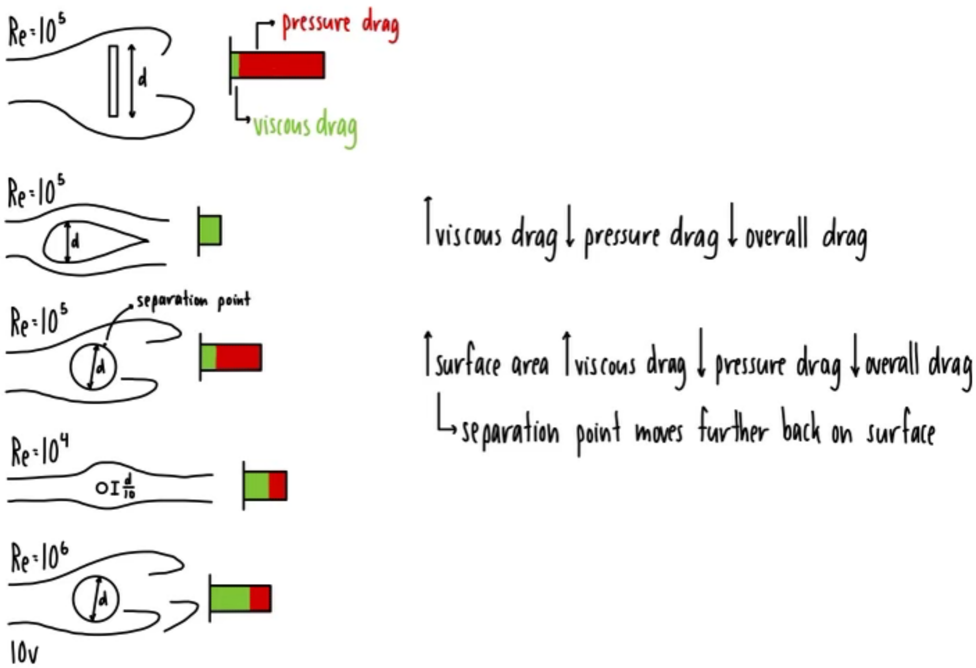


- Length scales for Reynolds number:



- ν = kinematic viscosity
  - $\nu = \frac{\mu}{\rho}$
  - $Re = \frac{v\,l}{\nu} = \frac{velocity \cdot length\ scale}{kinematic\ viscosity}$

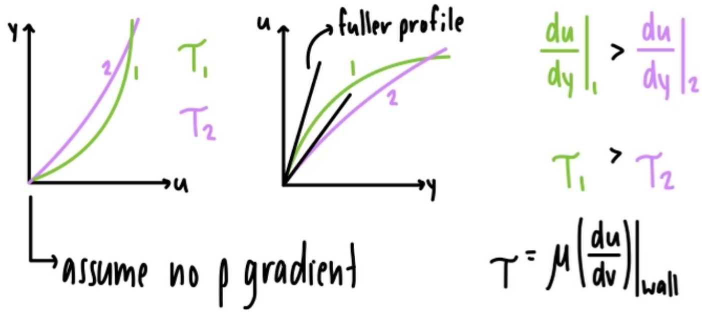
4.15.5 Drag on Various Aerodynamic Bodies at Various Re



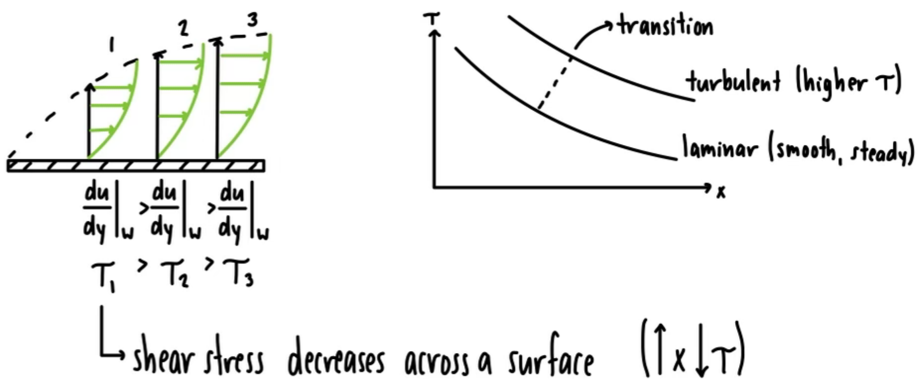
- When given the chance minimize pressure drag even if viscous drag goes up because the total drag will be reduced
- Re is a similarity parameter, which means we match it in a wind tunnel to test a scale model
  - The other main similarity parameter is mach number

4.15.6 Boundary Layers on Flat Plates

- Use a flat plate as a substitute for the surface of a wing/fuselage
  - Works on a small scale
  - Assumption that there is no pressure gradient

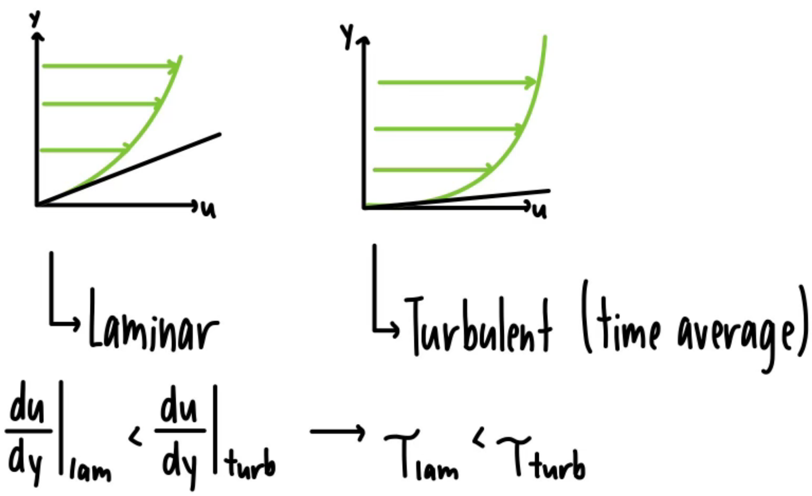


- Shear stress is a local value that is changing as you move across the surface



4.15.7 Laminar and Turbulent Boundary Layers

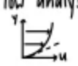
- Turbulent boundary layer has more shear stress than a laminar boundary layer
  - $\tau_{laminar} < \tau_{turbulent}$



- Laminar flow is smooth and steady with all streamlines in parallel

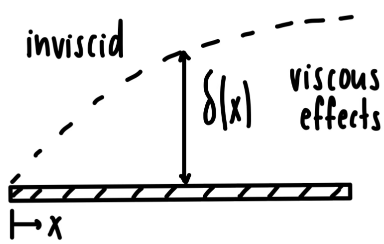
- A laminar flow at some point will transition to a turbulent flow
- Main difference is how much high viscosity fluid is getting near the surface
  - Turbulent flow has a lot more
- Reduce shear stress and viscous drag by delaying transition from laminar flow to turbulent flow as far back as possible on the airfoil

4.15.8 Review of Viscous Flow Terminology

- Pressure Drag results from a pressure imbalance in front and behind the body. → result from viscosity of air
- Viscous Drag results from friction along the surface of a body. → result from viscosity of air
- Boundary Layer is the very thin region of air near the surface of a body impacted by viscosity with a velocity deficit.
  → viscous flow analysis
- BL Profile 

- Velocity Gradient at the Wall $\left.\frac{du}{dy}\right|_{wall}$ for $y=0$ helps to determine shear stress.
- Dynamic Viscosity (μ) and Kinematic Viscosity ($\nu=\frac{\mu}{\rho}$)
- Wall Shear Stress (τ) where $\tau=\mu\left.\frac{du}{dy}\right|_{wall}$
- Reynolds Number (Re) is the relation of inertial forces to viscous forces. $\uparrow v \uparrow$ inertial forces $\downarrow$ viscous forces $\downarrow$ boundary layer thickness $Re=\frac{\rho_\infty V_\infty l}{\mu_\infty}$
- Laminar Boundary Layer has smooth flow and parallel streamlines. Flow naturally begins as laminar. It can delay separation of the flow from the body. → ordered motion
- Turbulent Boundary Layer results from inherent instabilities in the flow that exceed the ability of other viscous effects to dampen them and is characterized by chaotic flow. It mixes high speed flow further from the BL closer to the surface. There is more momentum near the surface. → chaotic motion
- Transition is the natural process from laminar to turbulent flow. It will always occur for flow over a plane. $\tau_{turb} > \tau_{lam}$ → delay transition to reduce skin friction drag
- Separation occurs when the flow streamlines separate/detach from the surface of the body. The flow is less likely to separate under turbulent conditions.

4.16.1 Laminar Boundary Layer - Thickness and Skin Friction Coefficient



- δ is boundary layer thickness
- Flat plate correlations
  - $\delta(x) = \frac{5.2x}{\sqrt{Re_x}}$
  - C_{fx} is local skin friction coefficient
    - Non-dimensional form of the wall shear stress

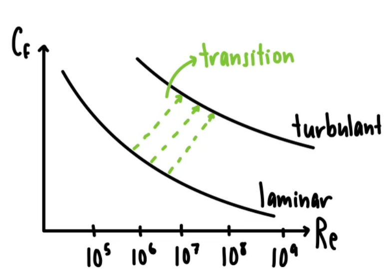
- $C_{fx} = \frac{\tau(x)}{\frac{1}{2}\rho_\infty v_\infty^2} = \frac{0.664}{\sqrt{Re_x}}$

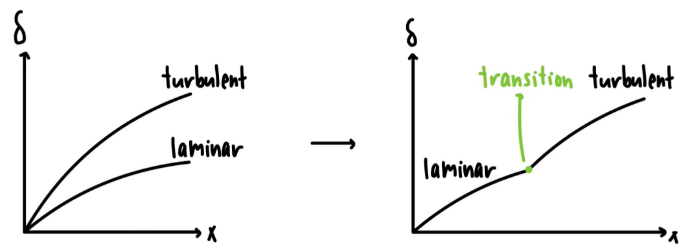
4.16.2 Laminar Boundary Layer - Total Skin Friction Drag

- Skin friction drag $D_F = \frac{1.328}{\sqrt{Re_L}} q_\infty S$
- Skin friction drag coefficient $C_F = \frac{1.328}{\sqrt{Re_L}}$

4.17 Correlations for Laminar and Turbulent Boundary Layers

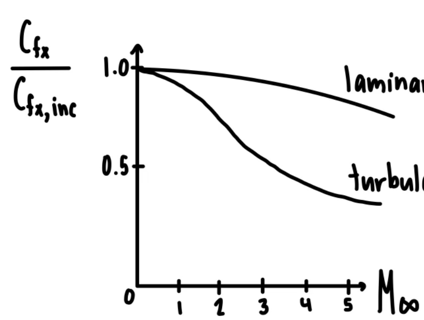
	Laminar	Turbulent	
Boundary Layer Thickness	$\delta = \frac{5.2x}{\sqrt{Re_x}}$	$\delta = \frac{0.37x}{Re_x^{0.2}}$	Recall:
Local Skin Friction Coefficient	$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$	$C_{fx} = \frac{0.0592}{Re_x^{0.2}}$	$C_f = \frac{\tau}{q_\infty}$
Total Skin Friction Coefficient	$C_F = \frac{1.328}{\sqrt{Re_L}}$	$C_F = \frac{0.074}{Re_L^{0.2}}$	$C_F = \frac{D_F}{q_\infty S}$





4.18 Compressibility Effects on Skin Friction

- The local coefficient decreases as Mach number increases. This has a larger impact on turbulent boundary layers.



4.19.1 Boundary Layer Transition

- Relate Re at transition (Re_{tr}) to location of transition (x_{tr})
    - $Re_{tr} = \frac{\rho\ v\ x_{tr}}{\mu}$
        - x_{tr} is assumed to be an exact point but transition occurs over a finite distance in reality
        - Re_{tr} occurs over a region and range of Re
- First check Re at the end of the plate. If $Re_L < Re_{tr}$ then the flow never transitions
- Take ratio of Re to find what at what percent of the length of the plate the flow transitions at
    - $\frac{Re_{tr}}{Re_L} = \frac{\frac{\rho x x_{tr}}{\mu}}{\frac{\rho v L}{\mu}} = \frac{x_{tr}}{L}$

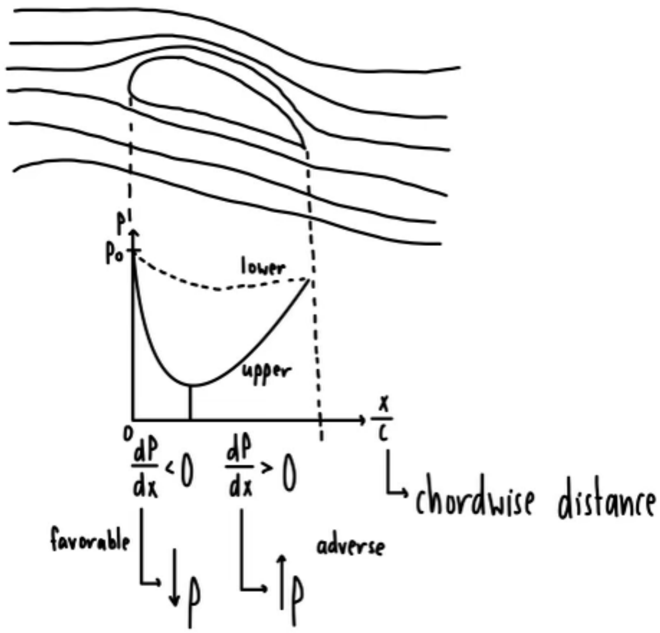
4.19.2 Reynolds Number Revisited - Transition Re

- Uses of Re:
    - Express relative importance of viscosity
        - ↑Re means that inertial forces have more of an impact relative to viscosity
    - Locate transition on plate (Re_{tr})
    - Define the type of boundary layer (laminar or turbulent)
        - $lam\ \alpha\ \frac{1}{\sqrt{Re}}$ and $turb\ \alpha\ \frac{1}{Re^{0.2}}$
    - Wind tunnel scaling so model data matches flight data
- Re subscripts
    - Re_L = full length of flat plate
    - Re_x = local Re somewhere along the length of flat plate
    - Re_d = round object like a cylinder or sphere
    - Re_c = chord, distance from leading edge to trailing edge
    - Re_{tr} = Re_x at x_{tr}
        - Typical range for a flat plate: 200,000 to 1,000,000
            - Depends on surface roughness, smoothness of flow, any disturbances imparted on the

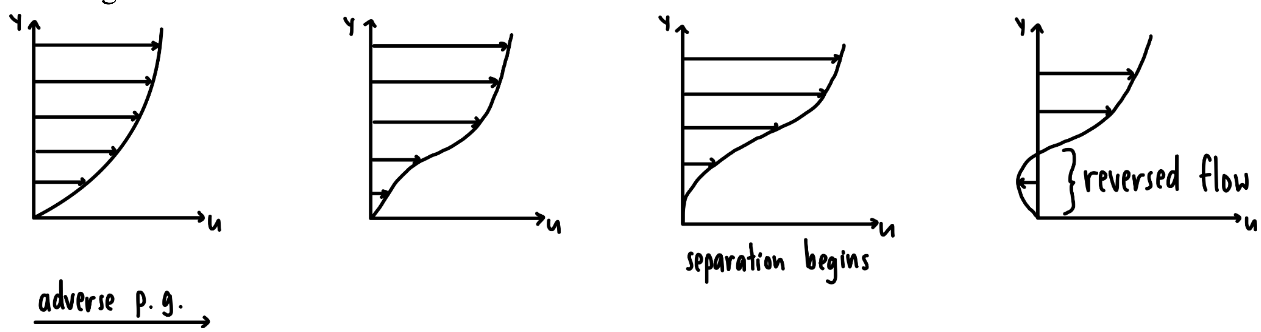
4.20 Flow Separation

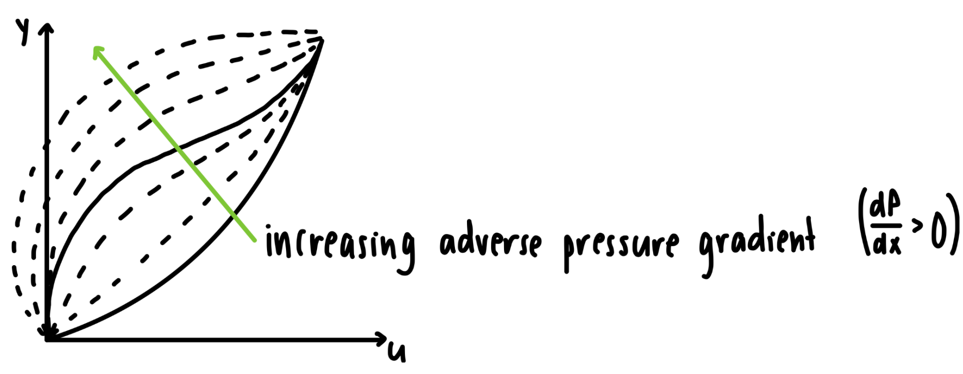
- A pressure gradient can be used to control the behavior of the flow over the airfoil.

4.20.1 Pressure Gradient Effects on BL



- The height and speed of the reversed flow increases with an increasing adverse pressure gradient.



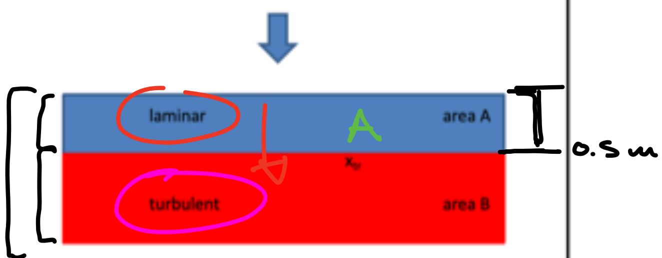


Question 2 Practice Problem:

Givens

- Flat plate (wing simulant)
  - Span = 3 m
  - Chord = 1 m
- V_∞ = 10 m/s
- ρ_∞ = 1 kg/m³
- μ_∞ = 1 x 10⁻⁵ kg/m·s
- Re_{tr} = 500,000
- Find total drag over the wing

TOP View



$$Re_{tr} = \frac{\rho_\infty V_\infty x_{tr}}{\mu_\infty} \Rightarrow x_{tr} = \frac{(Re_{tr})(\mu_\infty)}{\rho_\infty V_\infty}$$

$$x_{tr} = \frac{(500{,}000)(10^{-5})}{(1)(10)}$$

$$\boxed{x_{tr} = 0.500 \ [m]}$$

1) Drag for the whole wing (assuming turbulent)

$$D = q_\infty S C_f$$

$$C_f = \frac{0.074}{Re_L^{0.2}} = \frac{0.074}{(10^6)^{0.2}} = 0.00467$$

$$D_{f,turb} = \underbrace{\overbrace{\left(\tfrac{1}{2}\right)(1)(10^2)}^{\frac{1}{2}\rho_\infty V_\infty^2}}_{q_\infty}\underbrace{(1\cdot 3)}_{\text{wing Area}}\underbrace{(0.00467)}_{C_f} = 0.7\ \text{N}$$

2) turbulent flow until x_{tr}.

$$D = q_\infty S C_f$$

$$C_{f_{x_{tr}}} = \frac{0.074}{(500000)^2} \Rightarrow \underbrace{C_f = 0.00536}$$

$$D_{f,\underbrace{A},turb} = q_\infty S C_f = \underbrace{\left(\tfrac{1}{2}(1)(10^2)\right)}_{q_\infty}\left(3 \cdot \overbrace{0.500}^{x_{tr}}\right)(0.00536)$$

$$D_{f,A,turb} = 0.402N$$

$$D_{f,B,turb} = \underline{0.7N} - 0.402N = \underline{D_{f,B,turb} = 0.298\,[N]}$$

3) laminar up to the x_{tr}.

$$D = q_\infty S C_f.$$

$$C_f = \frac{1.328}{\sqrt{Re_{tr}}} = \frac{1.328}{\sqrt{500000}} = 0.00188$$

$$D_{f,A,lam} = \left(\tfrac{1}{2}(1)(10^2)\right)\left(3 \cdot \underbrace{0.5}_{x_{tr}}\right)(0.00188)$$

$$D_{f,A,lam.} = 0.141\,[N]$$

$$D_{tot,f} = D_{f,A} + D_{f,B} = 0.141\,[N] + 0.298\,[N]$$

$$\Rightarrow D_{f,L} = 0.439\,[N]$$

$$D_w = 2 \cdot D_{f,L} = D_{wing} = 0.878\,[N]$$

Question 3: Finite and Infinite Wing Performance

Helpful Equations:

Airfoils, Finite Wings, & 3D Aerodynamics

$a_0 = 2\pi/\text{rad} = 0.11/\text{deg}$ OR $\boxed{\dfrac{dC_L}{d\alpha} = \dfrac{a_0}{1+\frac{57.3 a_0}{\pi AR e_1}}}$ $\qquad \alpha_{eff} = \alpha - \alpha_i$

$AR = \dfrac{b^2}{S} = \dfrac{b}{c}$ (for rectangle)

$C_{p\,point} = \dfrac{P_{local} - P_\infty}{q_\infty}$

$P_0 = P_\infty + \frac{1}{2}\rho_\infty V_\infty^2$ (incompressible)

$C_p = \dfrac{C_{p\,m\infty}}{\sqrt{1-M_\infty^2}}$ ← Incompressible (Compressible)

$C_L = C_N \cos\alpha - C_N \sin\alpha \qquad C_D = C_N \sin\alpha + C_N \cos\alpha \qquad C_L \approx C_N$ (for small α)

$C_L = \dfrac{C_{L\,m\infty}}{\sqrt{1-M_\infty^2}}$ (compressible ⟷ incompressible)

$D_i = L \sin\alpha_i$ (induced drag)

$C_{Di} = \dfrac{C_L^2}{\pi AR e} \qquad C_D = C_d + \dfrac{C_L^2}{\pi AR e}$ (profile + induced drag)

$D_{total} = D_f + D_p + D_w$

$D_{profile} = D_f + D_w$

Helpful Terms:

a_0: Lift curve slope (2d)

a: Lift curve slope (3d)

C_L: Coefficient of lift (3d wing)

C_l: Coefficient of lift (2d wing)

α: Geometric angle of attack

α_{eff}: Effective angle of attack

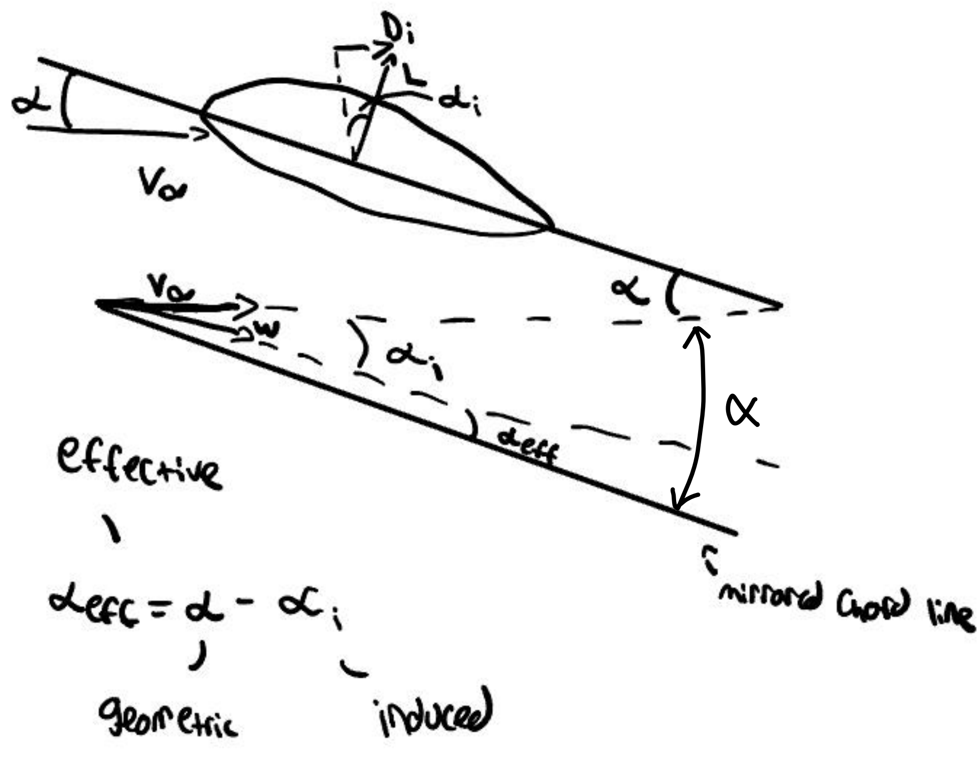
α_i: Induced angle of attack

e: Spanwise efficiency factor

AR: Aspect ratio

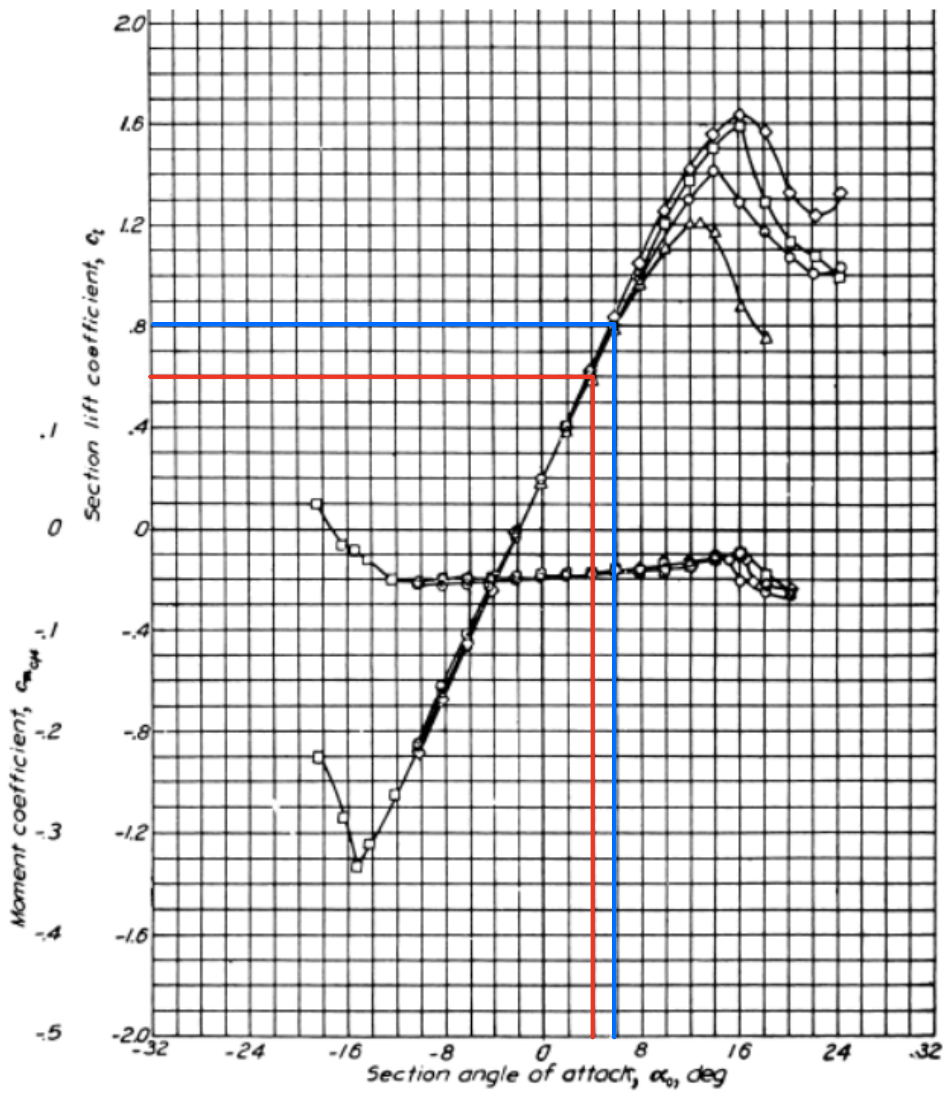
b: span
S: Wing area
c: Chord length
q_∞: Freestream dynamic pressure
C_{p,M_∞} or C_{p,M_0}: Incompressible pressure coefficient
C_{p,l_∞} or C_{p,l_0}: Incompressible lift coefficient

General Concepts:
Infinite wing - Flow is 2 dimensional
Finite wing - Flow field is 3 dimensional, must account for induced drag
Lift is generated as a result of the pressure distribution.
The lower surface of the wing is the higher pressure surface
e for an elliptical wing = 1
Geometric angle of attack is ALWAYS greater than the effective angle of attack
As a result the lift coefficient is lower and there is increased drag on finite wings
You CANNOT use l and L lift terms interchangeably, pay attention to the notation



Problems:

1. An airplane with the wing cross section of a NACA 2415 airfoil is flying at 205 ft/s in steady level flight at standard sea level conditions. It also has an aspect ratio of 8 in a rectangular shape with a wing span of 36 ft. and weighs 4,855 lbs. We also know from a 2D analysis that the NACA 2415 airfoil provides a lift of 180 lbs. under the same conditions.
    - a). Find C_L
    - b). Find α_{eff}
    - c). Find α
    - d). Find α_i



NACA 2415 Wing Section

A).

$$L = W \;, \quad L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \;, \quad AR = \frac{b^2}{S}$$

$$8 = \frac{36^2}{S}$$

$$S = 162 \text{ ft}^2$$

$$4855 = \frac{1}{2}(2.3764 \times 10^{-3})(205)^2(162)\, C_L$$

$$\boxed{C_L = 0.6}$$

B).

$$\boxed{\alpha_{eff} = 4^\circ}$$

C).

$$AR = \frac{b}{c}$$

$$8 = \frac{36}{c}$$

$$c = 4.5 \text{ ft.}$$

$$l = \frac{1}{2} \rho_\infty V_\infty^2 c C_l$$

$$180 = \frac{1}{2}(2.3769\times10^{-3})(205)^2(4.5)C_l$$

$$C_l = 0.8$$

$$\boxed{\alpha = 6^\circ}$$

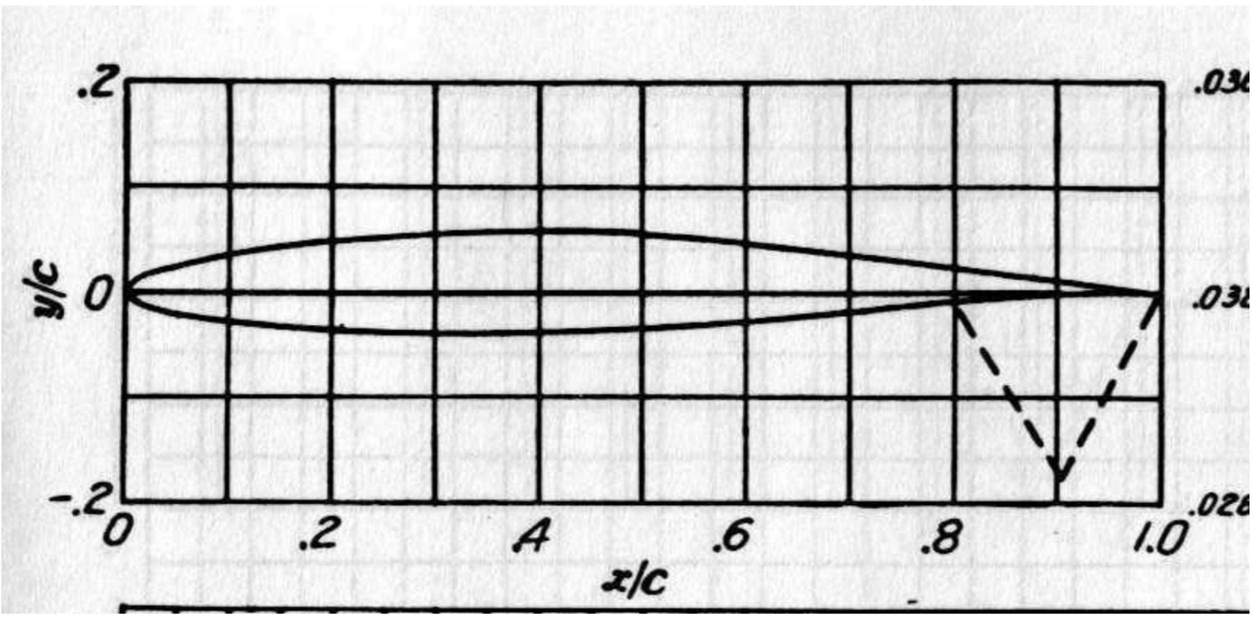
D).

$$\alpha_{eff} = \alpha - \alpha_i$$

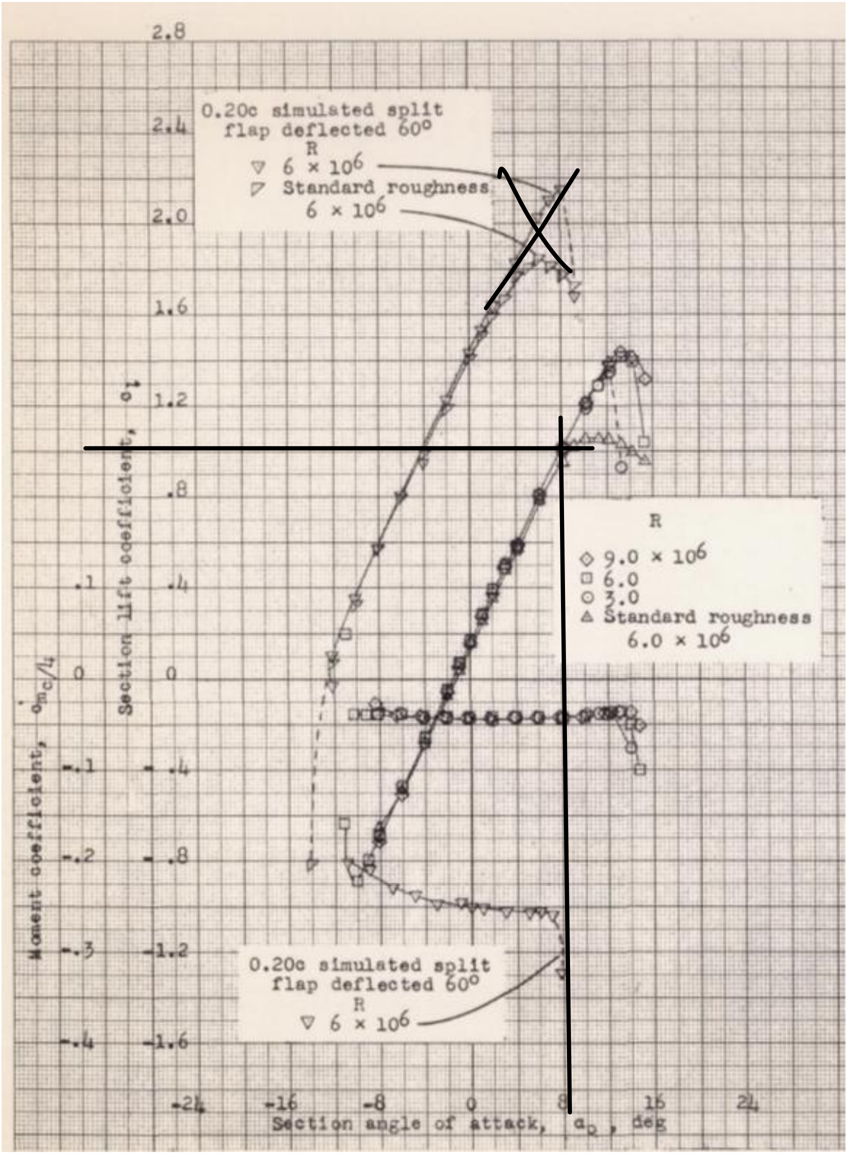
$$4 = 6 - \alpha_i$$

$$\boxed{\alpha_i = 2^\circ}$$

2. Consider an elliptic NACA 64-210 wing with a span of 10 meters and a root chord of 2.5 meters. If the wing is flying at an airspeed of 180 km/h and a ~~geometric~~ effective angle of attack of 8 degrees, compute:
    a. C_L and C_{Di}
    b. L and D_i



NACA 64-210



a. $C_L \approx 1.0$

$$C_{D_i} = \frac{C_L^2}{\pi AR e} \qquad \text{elliptical} \therefore e = 1$$

$$AR = \frac{b^2}{S} = \frac{(10\ m)^2}{(10m)(25m)} = 16$$

$$C_{D_i} = \frac{(1.0)^2}{\pi(16)(1)} = 0.3183 < C_L \checkmark$$

b. $L = \frac{1}{2}\rho v^2 S C_L \qquad D_i = \frac{1}{2}\rho v^2 S C_{D_i}$

$L = \frac{1}{2}(1.225 \frac{kg}{m^3})(50\ m/s)^2 (10m)(2.5m)(1.0)$

$= 38.281\ kN$

$\frac{L}{D} = \frac{C_L}{C_{D_i}}$

$D = L \frac{C_{D_i}}{C_L}$

$= (38.281\ kN)\left(\frac{0.3183}{1}\right)$

$= 12.185\ kN < L$ ✓