SGT AERO 2200 MIDTERM 1 REVIEW

Question 1 content to review:

- Steady Level Flight: When an aircraft is not accelerating and there is no net forces acting upon it
  L = W , T = D
- Lift equation: $L = \frac{1}{2}\rho V^2 S C_L$
  - Ambient density (ρ)
  - Flight velocity (V)
  - Wing area (S)
  - Lift coefficient (C_L)
- Drag equation: $D = \frac{1}{2}\rho V^2 S C_D$
  - Ambient density (ρ)
  - Flight velocity (V)
  - Wing area (S)
  - Drag coefficient (C_D)
- Aspect ratio: $AR = \frac{b^2}{S}$
  - Wing span (b)
  - Wing area (S)
- Wing loading: $WL = \frac{W}{S}$
  - Weight: $W = mg$
  - Wing area (S)
- Linear Interpolation: $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

	English	SI
Time	**s**	**s**
Pressure	$\frac{lb}{ft^2} = psf$	$\frac{N}{m^2}$
Temperature	**R**	**K**
Density	$\frac{slug}{ft^3}$	$\frac{kg}{m^3}$
Velocity	$\frac{ft}{s}$	$\frac{m}{s}$
Force	$lb = lb_f$	N
Mass	$slug$	kg
Energy	$ft * lb$	$Nm = J$
Power	$\frac{ft*lb}{s}$	$\frac{Nm}{s} = \frac{J}{s} = W$
Area	ft^2	m^2
Gas Constant	$\frac{ft*lb}{slug*R}$	$\frac{J}{kg*K}$

	Metric (SI)	English (lb)
Weight	Newton (N)	Pounds (lb)
Mass	Kilogram (kg)	Slug

- English units can also express mass as pound mass (lbm)
- On earth's surface a mass of 1 lbm will weigh 1 lbf
- 1 slug = 32.2 lbm

Standard Atmosphere

Altitude Definitions

- **Absolute Altitude** (h_a)**:** Distance from center of Earth to object
- **Geometric Altitude** (h_g)**:** Distance from sea-level to object
- **Geopotential Altitude** (h)**:** Mainly used in derivation (assumes g is constant)

- **Density Altitude**: Corresponding altitude with a given ambient density or vice versa
- **Pressure Altitude**: Corresponding altitude with a given ambient pressure or vice versa
- **Temperature Altitude:** Corresponding altitude with a given ambient temperature or vice versa
  - Use Appendix A and B (the Tables) to determine the altitudes corresponding to the respective pressure, density, and temperatures at a certain altitude.

Gravity Variation with Altitude:

$$g = g_o \cdot \frac{(r_e)^2}{(r_e + h_g)^2},$$ where g_o is the gravitational acceleration at sea level

Temperature Distribution in the Standard Atmosphere

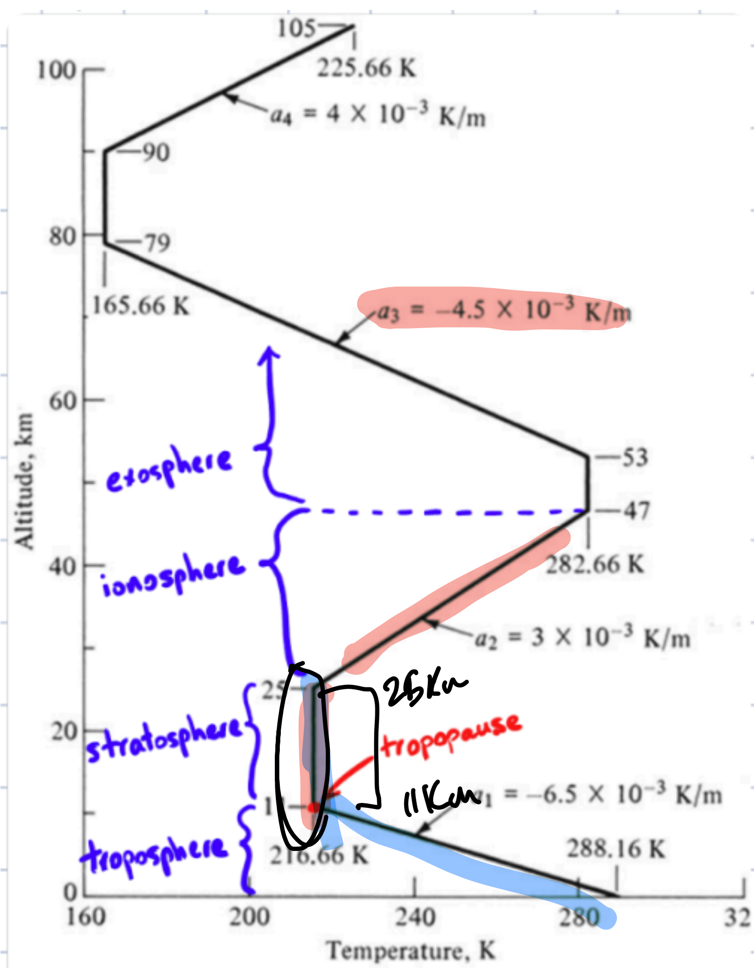

Figure 3.4 Temperature distribution in the standard atmosphere.

- Note: Finding “temperature altitude” is potentially troublesome because unlike pressure and density, which have an exponential relationship with altitude. Temperature in certain ranges of altitude is linear OR constant (isothermal). It’s these isothermal sections that makes it difficult to find “temperature altitude”, since an altitude of 11,000 m and 25,000 , have the same temperature.

Isothermal Regions

- **Temperature** → $T = Constant$
- **Pressure** → $\frac{P}{P_1} = e^{\left(-\frac{g_0}{RT}(h-h_1)\right)}$
- **Density** → $\frac{\rho}{\rho_1} = e^{\left(-\frac{g_0}{RT}(h-h_1)\right)}$

Gradient Regions

- **Temperature** → $T = T_1 + a(h - h_1)$
- **Pressure** → $\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{\left(-\frac{g_0}{aT}\right)}$
- **Density** → $\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(1 + \frac{g_0}{aT}\right)}$

The planform area of the Vought F4U Corsair (*Fig. 2*) is 29.17 m^2 and a takeoff weight of 6,592 kg. What is the wing loading in SI and English units?

SI

Given

- Take off weight = 6,592 [kg]
- wing area = 29.17 [m^2]

Find

wing loading a) in SI

b) Eng

Equation

$$\text{wing loading} = \frac{W}{S}$$

Solution

$$\text{a)}\quad WL = \frac{W}{S} = \frac{\overset{m}{6592\,[kg]} \cdot \overset{g}{9.81\,[m/s^2]}}{29.17\,[m^2]} = 2216.9\,[Pa]$$

$$6592\,[kg] \cdot \frac{2.205\,[lb_f]}{1\,[kg]} = 14532.63\,[lb_f]$$

$$29.17\,[m^2] \cdot \left(\frac{1\,[ft]}{0.3048\,[m]}\right)^2 = 313.98\,[ft^2]$$

$$WL = \frac{14532.63\,[lb_f]}{313.98\,[ft]} = 46.19\,\left[\frac{lb}{ft^2}\right]$$

Eng. units.

Consider an airplane flying at some real, geometric altitude. The outside (ambient) pressure and temperature are 5.3×10^4 N/m^2 and 253 K, respectively. Calculate the pressure and density altitudes at which this airplane is flying.

$$\text{Pressure} = 5.3 \times 10^4 \ [N/m^2]$$

$$\text{Ambient Temp} = 253 \ [K].$$

Find :

pressure altitude :

density altitude :

linear

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

pressure altitude

given P

$$P = 5.3 \times 10^4$$

$$h_p = h_{G_1} + \left(\frac{h_{G_1} - h_{G_2}}{P_2 - P_1} \right)(P - P_1)$$

$$= 5100 + \left(\frac{5200 - 5100}{(5.2621 - 5.3331) \times 10^4} \right)(5.3 - 5.3331) \times 10^4$$

$$\underline{h_p = 5146.6 \ [m]}$$

Density Altitude

$$P = \rho R T$$

$$\rho = \frac{P}{RT} = \frac{(5.3\times10^{4})}{(287)(253)} = \underline{0.72992} \ [kg/m^3]$$

$$h_\rho = h_{G1} + \left(\frac{h_{G2} - h_{G1}}{\rho_2 - \rho_1}\right)(\rho - \rho_1)$$

$$h_\rho = 5000 + \left(\frac{5100 - 5000}{(7.2851 - 7.3643)\times10^{-1}}\right)(7.2992 - 7.3643)\times10^{-1}$$

$$\underline{h_\rho = 5082.2 \ [m]}$$

A Boeing 747-8 is flying 600 mph at steady level flight at an altitude where the ambient temperature and pressure are 391° R and 4.80×10^2 psf, respectively. The lift generated is 900,000 lb, and the wing area is 554 m^2.

(a) What is the coefficient of lift?

(b) If the lift to drag ratio is 18, how much thrust is produced by each engine?

Given: $T = 391°R$, $L = 900{,}000$ lb

$P = 4.8 \times 10^2$ psf, $V = 600$ mph, $S = 554\ m^2$

$\frac{L}{D} = 18$, * steady level flight *

Find: $C_L = ?$, $T_{each} = ?$

Equations: $P = \rho R T$, $L = W$, $T = D$

$$L = \frac{1}{2}\rho V^2 S C_L$$

$$V = 600 \frac{\cancel{mi}}{\cancel{hr}} \cdot \frac{5280\ ft}{1\ \cancel{mi}} \cdot \frac{1\ \cancel{hr}}{3600\ s} = 880\ ft/s$$

$$S = 554\ m^2 \left(\frac{1\ ft}{0.3048\ m}\right)^2 = 5963.21\ ft^2$$

$$L = \frac{1}{2}\rho V^2 S C_L \qquad \rho = \frac{P}{RT} = \frac{4.8 \times 10^2\ psf}{(1716)(391)}$$

$$\rho = 7.154 \times 10^{-4}\ slug/ft^3$$

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{900000}{\frac{1}{2}(4.754\times10^{-4})(880)^2(5963.27)}$$

$$\boxed{C_L = 0.549}$$

$$\frac{L}{D} = 18 \Rightarrow D = \frac{L}{18} = \frac{900000}{18} = 50000\ \text{lb}$$

$$T = D = 50000\ \text{lb} = T_{total}$$

$$T_{each} = \frac{T_{total}}{4} = \frac{50000}{4} = 12500\ \text{lb}$$

$$\boxed{T_{each} = 12500\ \text{lb}}$$

AEROENG 2200 General Advice

- → Remember the assumptions for each equation and use them only if these criteria are met.
- → Note the flight regime for each problem.
  - → $M < 0.3$: Incompressible
  - → $0.3 < M < 1$: Compressible
  - → $M > 1$: Supersonic
- → Clearly mark your givens at the start of each problem.

Basic Theory of Fluid Flow

4.1 Continuity Equation

- Conservation of Mass: Mass cannot be created or destroyed.
  - → Continuity Equation: $\dot{m} = \rho V_n A$
    - → Assumptions: (1) Steady State

4.2 Incompressible Flow

→ Note: No flow is truly incompressible - it's just a simplifying assumption for this regime.

- Incompressible Flow: The density is (assumed) to be constant throughout the flow field.
- For an incompressible flow only, the continuity equation simplifies to: $A_1 V_1 = A_2 V_2$
  - → The simplifying assumption can be used when $M < 0.3 \approx 100\frac{m}{s} \approx 300\frac{ft}{s}$ for V_{max}; useful for low-speed wind tunnels, etc.
  - → $\rho_1 = \rho_2$ for $\underbrace{\rho_1 V_1 A_1}_{\dot{m}_1} = \underbrace{\rho_2 V_2 A_2}_{\dot{m}_2} \rightarrow A_1 V_1 = A_2 V_2$

4.3 Momentum Equation

- Conservation of Momentum: The force acting on a body is equal to the time rate of change of momentum.
  - → Newton's 2nd Law: $\sum F = \frac{d}{dt}(mV) = ma$
  - → Bernoulli's Equation: $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$
    - → Assumptions: (1) Steady State (2) Inviscid (3) $\vec{F}_{BODY} = 0$ (4) Incompressible Flow (5) Flow is along a streamline

- From Bernoulli's Equation, $P_0 = p + q = p + \frac{1}{2}\rho V^2$
  - ↳ Total (Stagnation) Pressure = Static Pressure + Dynamic Pressure

4.4 Summary I

- Ideal Gas Law: $p = \rho R T$
  - ↳ Needs one point in the flow.
- Continuity Equation: $\dot{m} = \rho V_n A$
  - ↳ Needs two points in the flow.
- Bernoulli's Equation: $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$
  - ↳ Needs two points in the flow.

4.6 Isentropic Flow

- Isentropic Flow: No change in entropy (disorder) of the flow.
- There are two necessary conditions:
  - ↳ (1) Adiabatic: No heat transfer through system boundaries ($\delta q = 0$).
  - ↳ (2) Reversible: Inviscid (no friction), no flow across shocks.
    - ↳ "Properties going out are the same coming in."
- Isentropic Flow Relations: $\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$
  - ↳ Assumptions: (1) Steady State (2) Isentropic Flow (Adiabatic AND Reversible) (3) Ideal Gas Behavior
  - ↳ The relations are derived with the ideal gas law.

4.7 Energy Equation

- Energy Equation: $c_p T_1 + \frac{1}{2}V_1^2 = c_p T_2 + \frac{1}{2}V_2^2$
  - ↳ Assumptions: (1) Steady State (2) Isentropic Flow (3) $\vec{F}_{BODY} = 0$ (4) Flow is along a streamline (5) Constant specific heats (c_p, c_v)
  - ↳ Useful for finding changes in temperature and/or velocity of the flow.

4.8 Summary II

Incompressible Flow (constant ρ)	Compressible Flow (Variable ρ)
Continuity Equation: $\dot{m} = \rho V_n A \rightarrow A_1 V_1 = A_2 V_2$	Continuity Equation: $\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$
Bernoulli's Equation: $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$	Isentropic Flow Relations: $\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$
Ideal Gas: $p = \rho R T$	Energy Equation: $c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2$
$M < 0.3$ for V_{max} of the flow	Ideal Gas: $p_1 = \rho_1 R T_1$, $p_2 = \rho_2 R T_2$
$\gamma = \frac{c_p}{c_v}$ $R = c_p - c_v$	Compressible flow equations can be used for any airflow.

4.9 Speed of Sound, Mach Number

- The speed of sound (a) is the speed at which a pressure wave can propogate through a fluid.
  - $\rightarrow a = \sqrt{\gamma R T}$
- Mach Number: $M = \frac{V}{a} = \frac{V}{\sqrt{\gamma R T}}$
  - $\rightarrow$ Assumptions: (1) Isentropic Flow (2) Ideal Gas Behavior
  - $\rightarrow$ $M\uparrow$ at higher altitudes as $T\downarrow$ and $\rho\downarrow$ for a given true velocity (V_{true}).

Question 2 Problems:

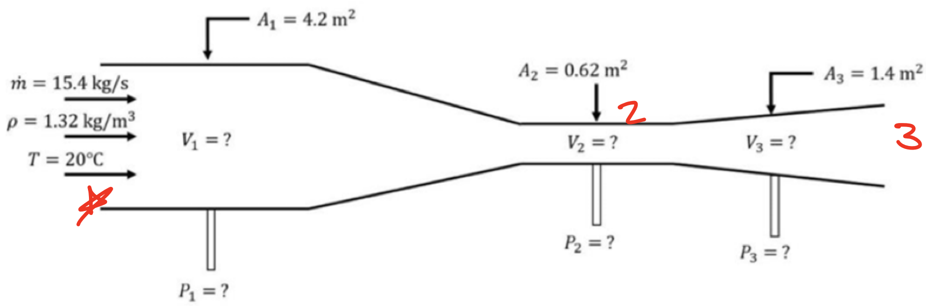

Figure 1: Wind tunnel sketch

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- A low-speed subsonic wind tunnel is operating with a mass flow rate of 15.4 kg/s, a density of 1.32 kg/m^3, and a temperature of 20 degrees C at its settling chamber. Determine the velocities (m/s) and (static) pressures (Pa) in the wind tunnel at (a) location 1 and (b) location 2 and location 3.

$$\dot{m} = \rho A V$$

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{15.4}{1.32(4.2)} = 2.778\ \frac{m}{s}$$

$$P_1 = \rho R T = (1.32)(287)(20+273) = \underline{111000.12\ Pa}$$

$$A_1 V_1 = A_2 V_2$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \rho A V$$

$$(4.2)(2.778) = (0.62)\, V_2$$

$$V_2 = 18.819\ m/s$$

$$A_1 V_1 = A_3 V_3$$

$$V_3 = 8.334$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$111000.12 + \frac{1}{2}(1.32)(2.778)^2 = P_2 + \frac{1}{2}(1.32)(18.819)^2$$

$$P_2 = 110771.47\ P_2$$

- A Pitot tube is mounting in the test section of a low-speed subsonic wind tunnel. The flow in the test section has a velocity, static pressure, and temperature of 150 mph, 1 atm, and 70 degrees F, respectively. Calculate the pressure measured by the Pitot tube

$150 \text{ mph} \rightarrow 220 \frac{ft}{s}$

$1 \text{ atm} \rightarrow 2116.22 \text{ psf}$

$70°F \rightarrow 529.67 \text{ R}$

$$\rho = \frac{P}{RT} = \frac{2116.22}{(1716)(529.67)} = 0.00233 \frac{slugs}{ft^3}$$

$$P_{total} = P + q = P + \frac{1}{2}\rho V^2 = 2116.22 + \frac{1}{2}(0.00233)(220)^2 = 2172.6 \text{ psf}$$

- A high-speed aircraft is flying at Mach 0.95 in a standard atmosphere at 30,000 ft. Determine true airspeed

$$M = \frac{V_\infty}{a_\infty}$$

$$a_\infty = \sqrt{\gamma R T} = \sqrt{(1.4)(1716)(411.77)} = 994.75 \text{ ft/s}$$

$$T = 411.77 \text{ R}$$

$$V_\infty = M a_\infty = 0.95(994.75) = 945.01 \text{ ft/s}$$

Question 3 content to review:

- Dynamic pressure equation: $q = \frac{1}{2}\rho V^2$
- Anything over a mach number of 0.3 is considered compressible
- Compressible flow equations work for ALL cases
- Isentropic means that the flow is adiabatic and reversible
- Energy equation: $C_p T_1 + \frac{1}{2}V_1^2 = constant = C_p T_2 + \frac{1}{2}V_2^2$
  - Assumes: steady - isentropic flow, along a streamline, no body forces, constant specific heat C_p.
- Isentropic flow relations: $\frac{P_2}{P_1} = (\frac{\rho_2}{\rho_1})^{\gamma} = (\frac{T_2}{T_1})^{\frac{\gamma}{\gamma-1}}$
  - Assumes: adiabatic, reversible, steady, ideal gas
- Isentropic Mach relations:
  - $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2$
  - $\frac{P_0}{P} = (1 + \frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}}$
  - $\frac{\rho_0}{\rho} = (1 + \frac{\gamma-1}{2}M^2)^{\frac{1}{\gamma-1}}$
    - Sub 0 denotes stagnation values
    - Pitot tubes measure stagnation pressure
- Helpful compressible flow equations:
  - Continuity: $\rho_1 V_1 A_1 = mass\ flow = \rho_2 V_2 A_2$
  - Isentropic flow relations
  - Isentropic Mach relations
  - Energy equation
  - Relations using ideal gas at different points in the flow ($P = \rho RT$)
  - Mach number: $M = \frac{V}{a}$
    - a = speed of sound = $\sqrt{\gamma RT}$
- Gamma is 1.4 for air, and R is 287 (SI) or 1716 (English)
  - R = C_p - C_v

Problems:

1): Imagine a flow over an airfoil with freestream values of: V_∞ = 500 mph, P_∞ = 2116 psf, T_∞ = 519 R. (a) Determine if the flow is compressible or incompressible. There is a point A on the airfoil where P_A = 1500 psf, (b) determine the local mach number at point A.

$$M_\infty = \frac{V_\infty}{a_\infty} \quad , \quad a_\infty = \sqrt{\gamma R T}$$

$$V_\infty = 733.33 \text{ ft/s}$$

$$a_\infty = \sqrt{1.4(1716)(519)} = 1116.622 \text{ ft/s}$$

$$M_\infty = \frac{733.33}{1116.622}$$

$$\underline{M_\infty = 0.6567}$$

$$\frac{P_0}{2116} = \left(1 + \frac{1.4-1}{2}(0.6567)^2\right)^{1.4/1.4-1}$$

$$P_0 = 2826.645 \text{ psf}$$

$$\frac{2826.645}{1500} = \left(1 + \frac{0.4}{2} M_A^2\right)^{1.4/0.4}$$

$$\boxed{M_A = 0.996}$$

2): A venturi tube is attached to a plane that is in steady, level flight. The plane is flying at an altitude of 5000 ft and at a speed of 122 knots. If the venturi tube has an inlet area of 0.2 ft^2, a throat area of 0.1 ft^2, and a throat temperature of 15 degrees Fahrenheit, what is the Mach number at the throat? Can the air at the throat be considered incompressible? What is the total temperature at the throat?

Givens:

$h = 5000\,ft$

$T_\infty = 500.86\,°R = 41.19°F$

$P_\infty = 1.7607 \times 10^3\ lb/ft^2$

$\rho_\infty = 2.0482 \times 10^{-3}\ slugs/ft^3$

$V_\infty = 122\ \frac{nm}{hr} \cdot \frac{6076.1\ ft}{1\ nm} \cdot \frac{3600\ s}{hr} = 205.71\ ft/s$

$A_{in} = 0.2\ ft^2$

$A_{th} = 0.1\ ft^2$

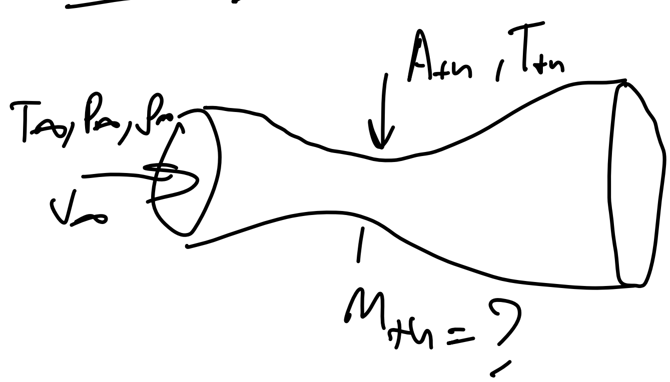
$T_{th} = 15°F + 459.67 = 474.67\ °R$

$\gamma = 1.4$

$R_{air} = 1716\ \frac{ft\,lb}{slug\,°R}$

$C_{P_{air}} = 6000\ \frac{ft\,lb}{slug\,°R}$

Sketch:



$$C_p T_{in} + \frac{1}{2} V_{in}^2 = C_p T_{th} + \frac{1}{2} V_{th}^2$$

$$V_{th} = \sqrt{2C_p(T_{in} - T_{th}) + V_{in}^2}$$

$$= \sqrt{2(6000)(500.86 - 474.67) + (205.91)^2}$$

$$V_{th} = 597.23 \ ft/s$$

$$M_{th} = \frac{V_{th}}{a_{th}} = \frac{V_{th}}{\sqrt{\gamma R T_{th}}} = \frac{597.23}{\sqrt{1.4(1716)(474.67)}} = 0.56$$

$$\boxed{M_{th} = 0.56} \quad \boxed{M_{th} > 0.3}$$

Not incompressible

$$\frac{T_{o\,th}}{T_{th}} = 1 + \frac{\gamma - 1}{2} M_{th}^2 \Rightarrow T_{o\,th} = T_{th}\left(1 + \frac{\gamma-1}{2} M_{th}^2\right)$$

$$= (474.67)\left(1 + \frac{1.4-1}{2}(0.56)^2\right)$$

$$\boxed{T_{o\,th} = 504.3 \, ^\circ R}$$

3): You are helping BSLI launch a rocket! They've asked you to do some calculations with a theoretical rocket nozzle. Assume isentropic, compressible flow. The mass flow rate of air is found to be 2.5 kg/s. The reservoir experiences a temperature of 225°C & a pressure of ~~3~~ 1.5 atm. The exit temperature is 150°C . Assume $\gamma = 1.4$ ~~and Cp = 1 (idk a good value)~~.

(a) Find the exit velocity

(b) Find the exit area

$C_p = 1008 \frac{J}{kg\,K}$ $\quad R = 287 \frac{J}{kg\,K}$

Given : $\dot{m}, T_R, T_E, P_R, R, \gamma, C_p$

a. $C_p T_R + \cancel{\frac{1}{2} V_R^2} = C_p T_E + \frac{1}{2} V_E^2$

$$V_E = \sqrt{2 C_p (T_R - T_E)}$$

$$T_R = 498.15 \text{ K}$$
$$T_E = 423.15 \text{ K}$$

$$V_E = \sqrt{2(1008)(498.15 - 423.15)} = \boxed{388.844 \text{ m/s}}$$

b. A_E

$$\frac{\dot{m}}{\rho_E V_E} = A_E$$

$$\rho_R = \frac{P_R}{R T_R}$$

$$P_R = 151987.5 \text{ Pa}$$

$$\rho_R = \frac{(151987.5)}{(287)(498.15)} = 1.0631 \frac{kg}{m^3}$$

$$\left(\frac{\rho_E}{\rho_R}\right)^{\gamma} = \left(\frac{T_E}{T_R}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\rho_E = \rho_R \sqrt[\gamma]{\left(\frac{T_E}{T_R}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$= (1.0631) \sqrt[1.4]{\left(\frac{423.15}{498.15}\right)^{\frac{1.4}{0.4}}}$$

$$= 0.707 \ \frac{kg}{m^3}$$

$$A_E = \frac{\dot{m}}{\rho_E V_E}$$

$$= \frac{(2.5)}{(0.707)(388.844)}$$

$$= 0.00909 \ m^2$$

$$= 9.094 \times 10^{-3} \ m^2$$