2022 SGT Aero 2200 Midterm 2 Review Document – Blank Problem Set

Chapter 4: Basic Aerodynamics

4.13 Supersonic Wind Tunnels and Rocket Engines
4.14 Discussion of Compressibility 226
4.15 Introduction to Viscous Flow 227
4.16 Results for a Laminar Boundary Layer 236
4.17 Results for a Turbulent Boundary Layer 241
4.18 Compressibility Effects on Skin Friction 244
4.19 Transition 247
4.20 Flow Separation 250
4.21 Summary of Viscous Effects on Drag 255

Chapter 5: Airfoils, Wings, and Other Aerodynamic Shapes

5.1 Introduction 288
5.2 Airfoil Nomenclature 290
5.3 Lift, Drag, and Moment Coefficients 294
5.4 Airfoil Data 300
5.5 Infinite versus Finite Wings 315
5.6 Pressure Coefficient 316
5.7 Obtaining Lift Coefficient from C_P 322
5.8 Compressibility Correction for Lift Coefficient 326
5.12 Summary of Airfoil Drag 357
5.13 Finite Wings 359
5.14 Calculation of Induced Drag 363
5.15 Change in the Lift Slope 372

Chapter 6: Elements of Airplane Performance

6.2 Introduction: The Drag Polar 441
6.3 Equations of Motion 448
6.3 Thrust Required for Level, Unaccelerated Flight 450
6.4 Thrust Available and Maximum Velocity 458
6.5 Power Required for Level, Unaccelerated Flight 461
6.6 Power Available and Maximum Velocity 466

4.13 Supersonic Wind Tunnels and Rocket Engines:

Area-Velocity Relation:

1. For subsonic flow, velocity increases as area decreases
2. For sonic flow, the area is the size of the throat
3. For supersonic flow, velocity increases as area increases

$$\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}$$

Using isentropic relations for supersonic flow:

$$T_1 = T_0\left[1 + \frac{1}{2}(\gamma - 1)M_1^2\right]^{-1}$$

$$\rho_1 = \rho_0\left[1 + \frac{1}{2}(\gamma - 1)M_1^2\right]^{-1/(\gamma-1)}$$

$$p_1 = p_0\left[1 + \frac{1}{2}(\gamma - 1)M_1^2\right]^{-\gamma/(\gamma-1)}$$

Variation of Mach with area:

$$\left(\frac{A}{A_t}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2}M^2\right)\right]^{(\gamma+1)/(\gamma-1)}$$

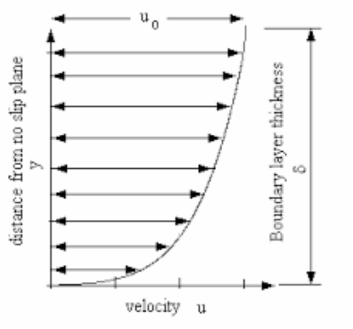
4.14 Discussion of Compressibility: Flows in which M < 0.3 are treated as incompressible because the variation of $\frac{\rho}{\rho_0}$ is less than 5%

4.15 Introduction to Viscous Flow:

Boundary Layer: region of viscous flow due to friction that gets thicker as the flow moves over the body

Shear Stress, τ_w: F/area acting tangential to surface and causing *Skin Friction Drag*

Velocity Profile: slope governs shear stress



$$\tau_w = \mu \left(\frac{dV}{dy}\right)_{y=0}$$

Absolute Viscosity Constant/Dynamic Viscosity/Viscosity

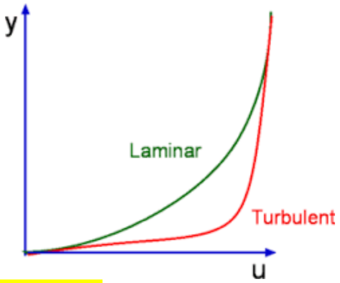
$$\mu = 1.458\left(\frac{T^{3/2}}{T+110.4}\right) * 10^{-6} \text{ kg/m*s} \qquad = 1.7894 * 10^{-5} \text{ @sea level}$$

$$\mu = 2.27\left(\frac{T^{3/2}}{T+199}\right) * 10^{-8} \text{ slug/ft*s} \qquad = 3.7373 * 10^{-7} \text{ @sea level}$$

Two Types of Viscous Flow: **Laminar** & **Turbulent**

$$\left(\left(\frac{dV}{dy}\right)_{y=0}\right)_{laminar} < \left(\left(\frac{dV}{dy}\right)_{y=0}\right)_{turbulent}$$

$$\therefore (\tau_w)_{laminar} < (\tau_w)_{turbulent}$$



4.16 Results for a Laminar Boundary Layer:

$$\delta = \frac{5.2x}{\sqrt{Re_x}}$$

$$c_{f\,x} = \frac{\tau_w}{q_\infty} = \frac{0.664}{\sqrt{Re_x}}$$

$$C_f = \frac{1.328}{\sqrt{Re_L}} = \frac{D_f}{q_\infty S}$$

$Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty}$ local Reynolds number

$Re_L = \frac{\rho_\infty V_\infty L}{\mu_\infty}$ plate Reynolds number

4.17 Results for a Turbulent Boundary Layer:

$$\delta = \frac{0.37x}{Re_x^{0.2}}$$

$$c_{f\,x} = \frac{0.0592}{Re_x^{0.2}}$$

$$C_f = \frac{0.074}{Re_L^{0.2}}$$

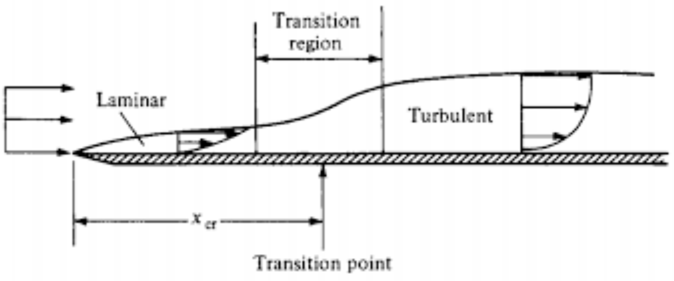
4.18 Compressibility Effects on Skin Friction:

$$C_{f_x} = \frac{f_1(M_\infty)}{\sqrt{Re_x}} \quad \text{laminar, compressible}$$

$$C_{f_x} = \frac{f_2(M_\infty)}{Re_x^{0.2}} \quad \text{turbulent, compressible}$$

1. For a constant Re, increasing M decreases C
2. The decrease in C is much greater for turbulent flow

4.19 Transition:



Critical Reynolds Number: Re at transition point

$$Re_{x_{cr}} = \frac{\rho_\infty V_\infty x_{cr}}{\mu_\infty}$$

4.20 Flow Separation:

Friction also causes flow Separation which means the flow can no longer follow the curvature of the body which leads to second form of drag that is perpendicular to the surface, Pressure Drag

Effects of Pressure Gradient:

Favorable: $\frac{dp}{dx} < 0$	Adverse: $\frac{dp}{dx} > 0$
Fuller velocity profile	Less full velocity profile
Reduced $\delta(x)$ growth rate	Increased $\delta(x)$ growth rate
Delays Transition	Promotes Transition
Delays Separation	Less resistant to Separation

Effects of Separation:

1. Drastic loss of lift
2. Major increase in drag, specifically pressure drag

Note: Laminar boundary layers separate more easily that turbulent boundary layers

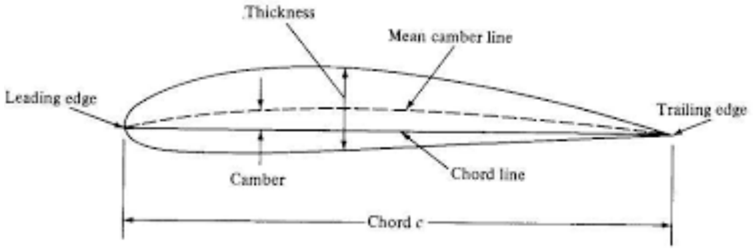
4.22 Summary of Viscous Effects on Drag

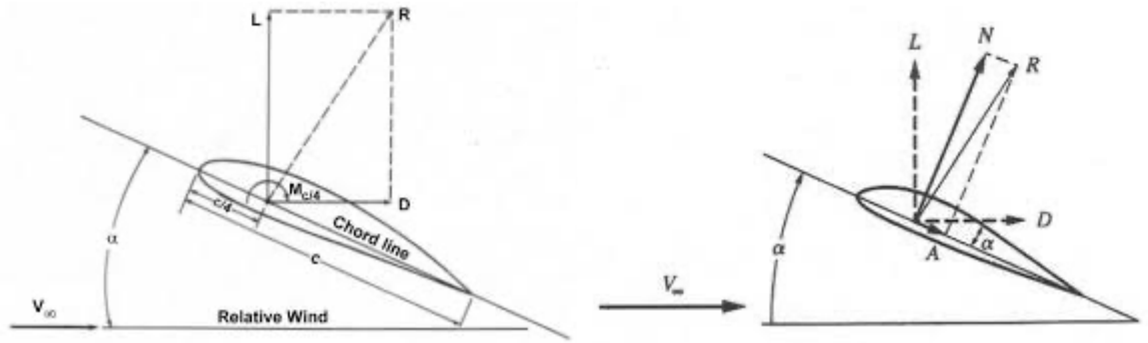
$$D = D_f + D_p$$

Total viscous drag = Skin friction drag + Pressure Drag due to separation

5.2 Airfoil Nomenclature:

Airfoil: intersection of the wing with a perpendicular plane


Thickness
Mean camber line
Leading edge
Trailing edge
Camber
Chord line
Chord c


L
R
M_c/4
D
c/4
Chord line
c
α
V∞
Relative Wind
V∞
L
N
R
D
A
α
α

- V_∞: relative wind
- α: angle of attack- angle between chord line and relative wind
- R: resultant force due to pressure and shear stress distributions
- L: Lift- component of R that is perpendicular to relative wind
- D: Drag- component of R that is parallel to relative wind
- $M_{c/4}$: resultant moment due to pressure and shear stress distributions taken about the quarter-chord point, $c/4$, *function of* α
- M_{LE}: resultant moment about the leading edge, *function of* α
- M_{ac}: resultant moment about *aerodynamic center* where it is *NOT a function of* α
- N: Normal Force- component of R that is perpendicular to the chord line
- A: Axial Force- component of R that is parallel to the chord line

$$L = N\cos\alpha - A\sin\alpha$$
$$D = N\sin\alpha + A\cos\alpha$$

5.3 Lift, Drag, and Moment Coefficients:

L, D, M depend at least on

1. Free-stream velocity, V_∞
2. Free-stream density, ρ_∞
3. Size of surface; wing area, S
4. Angle of attack, α
5. Shape of airfoil
6. Viscosity, μ_∞
7. Compressibility of airflow

Using dimensional analysis of those variables:

$$c_l = \frac{L}{q_\infty S} = f(\alpha, M_\infty, Re)$$

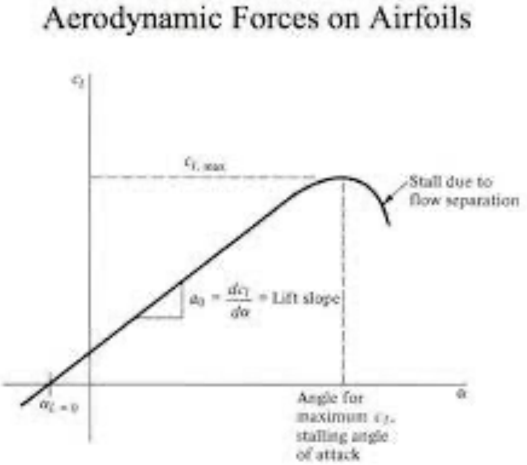
$$D = q_\infty S c_d \quad \rightarrow \quad c_d = \frac{D}{q_\infty S} = f(\alpha, M_\infty, Re)$$

$$M = q_\infty S c c_m \quad \rightarrow \quad c_m = \frac{M}{q_\infty S c} = f(\alpha, M_\infty, Re)$$

This means that to repeat a dynamically similar test, you need the **same $\boldsymbol{\alpha}, \boldsymbol{M_\infty}$, and $\boldsymbol{Re}$**

5.4 Airfoil Data:

Typical lift curve:


Aerodynamic Forces on Airfoils
c_l
$c_{l,\max}$
Stall due to flow separation
$a_0 = \frac{dc_l}{d\alpha}$ = Lift slope
$\alpha_{L=0}$
α
Angle for maximum c_l, stalling angle of attack

** For a symmetric airfoil $\alpha_{L=0}$ is located at the origin

L, D, M for an airfoil are treated as L, D, M **per unit span**, meaning $S = c(1) = c$

$$L(\text{per unit span}) = q_\infty c(1) c_l, \qquad c_l = \frac{L(\text{per unit span})}{q_\infty c}$$

5.5 Infinite versus Finite Wings:

For an infinite wing, it only varies in x and y so flow is called *two-dimensional*
For a finite wing, all real wings, the flow field is *three-dimensional*

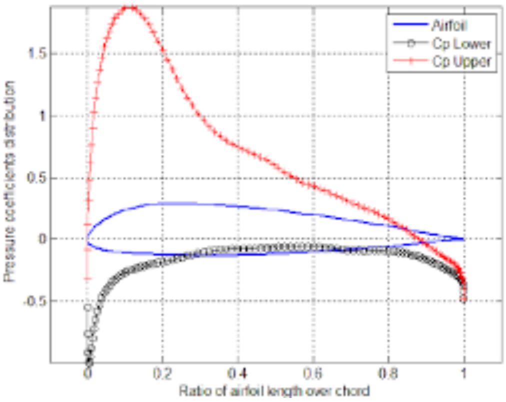
Aspect Ratio:

$$\text{AR} = \frac{b^2}{S}$$

5.6 Pressure Coefficient:

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$

Pressure coefficient distribution:



Pressure coefficient is important because its distribution leads directly to c_l and to the calculation of the effect of Mach number on c_l

Considering an airfoil at a fixed α, *Prandtl-Glauert*:

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}$$

This means that C_p does not change much for $M < 0.3$ but will increase in magnitude for $0.3 < M < 0.7$. For $M > 0.7$ its accuracy diminishes.

5.7 Obtaining Lift Coefficient from C_p:

Calculated by integrating the pressure distribution over the chord length, i.e. area between red and black lines on the above diagram.

$$c_n = \frac{1}{c}\int_0^c \left(C_{p,l} - C_{p,u}\right)dx$$

$$\& \; c_l = c_n \cos\alpha - c_a \sin\alpha$$

$$\therefore \text{ for } \alpha \leq 5°: \quad c_l \approx \frac{1}{c}\int_0^c \left(C_{p,l} - C_{p,u}\right)dx$$

5.8 Compressibility Correction for Lift Coefficient:

Similar to Prandtl-Glauert:

$$c_l = \frac{c_{l,0}}{\sqrt{1 - M_\infty^2}}$$

5.12 Summary of Airfoil Drag:

Total Drag of an airfoil:

$$D = D_f + D_p + D_w$$

Total Drag = Skin Friction Drag+ Pressure Drag + Wave Drag (tran, supersonic)

$$\therefore \quad c_d = c_{d,f} + c_{d,p} + c_{d,w}$$

$c_{d,f} + c_{d,p}$ is known as Profile Drag

** c_d is relatively constant at subsonic speeds, once it reaches drag divergence it skyrockets until M_∞, and then it decreases again at a rate of approximately $1/\sqrt{M_\infty^2 - 1}$. This region between drag divergence and M_∞ is know as the *sound barrier*

5.13 Finite Wings:

The lift and drag coefficients for a finite wing are different than the lift and drag coefficients for an airfoil section. This is because the end effects are removed when testing in a wind tunnel.

In a finite wing, air leaks around the wing tips from the high-pressure side to the low-pressure side. This creates a circular motion of air that is called a vortex. Wing tip vortices downstream of the wing induce a small downward component of velocity near the wing called *downwash.* The effect of downwash is to produce a "local" relative wind that is slanted downwards from the original direction of the freestream velocity.

Downwash on wings has several consequences

1. The effective angle of attack of the wing is reduced in comparison to the angle of attack referenced to the freestream flow velocity
2. There is an increase in drag that is called the *induced drag*

Therefore, the lift coefficient for a finite wing is less than the lift coefficient for an airfoil section, while the drag coefficient for a finite wing is greater than the drag coefficient of an airfoil section.

Remember that capital letters L and D denote finite wing aerodynamic coefficients while lowercase letters denote airfoil aerodynamic coefficients.

5.14 Calculation of Induced Drag:

With downwash, the induced angle of attack, defined as α_i, is the difference between the local flow direction and the free-stream direction. The difference between the geometric angle of attack, α, and the induced angle of attack is the effective angle of attack α_{eff}. The effective angle of attack is what the wing sees when downwash occurs.

$$D_i = L \sin\alpha_i$$

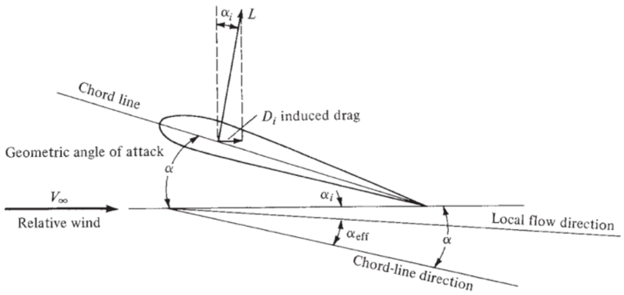


Figure 5.47 The origin of induced drag.

The lift vector remains perpendicular to the local relative wind and is therefore tilted back through angle α_i; however, drag is still measured parallel to the free stream direction. Because of the tilted lift vector, there is an induced drag component D_i. Assuming small angles, the induced drag component is calculated by

$$D_i = \mathrm{L}\alpha_i$$

Where the angle is in radians. The calculation of the induced angle of attack is beyond this course, but for an elliptical lift distribution, it can be defined as

$$\alpha_i = \frac{C_L}{\pi AR}$$

Plugging the lift equation and induced angle of attack into the induced drag equation results in the induced coefficient of drag

$$C_{D,i} = \frac{D_i}{q_\infty S} = \frac{C_L^2}{\pi eAR}$$

Here, the factor e is added to account for non-elliptical lift distributions, and it is defined as a span efficiency factor. For elliptical planforms, e = 1; for all other planforms, e < 1. Thus, $C_{D,i}$, i and hence induced drag are a minimum for an elliptical planform.

Now we can write the total drag coefficient for a finite wing as the sum of the induced drag and the “profile drag”. The “profile drag” depends on the profile of the wing, or in other words, the shape of the airfoil. The induced drag comes from the lift of a finite wing. For infinite wings, the aspect ratio is infinite, so the induced drag is zero. Keep in mind that the “profile drag” itself is made up of skin friction drag and pressure drag.

$$C_D = c_d + \frac{C_L^2}{\pi eAR}$$

5.15 Change in the Lift Slope:

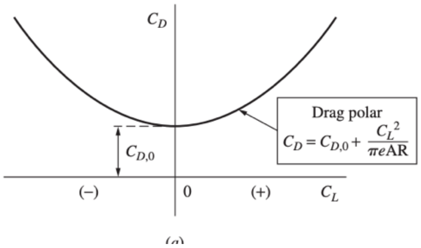
In the section above, we looked at how the coefficient of drag changes for a finite and infinite wing. Another difference is that the finite wing lift curve slope is smaller than that of an infinite wing for the same airfoil section. The flow over a finite wing at an angle of attack α is essentially the same as the flow over an infinite wing at an angle of attack α_{eff}. The finite wing lift curve slope can be calculated as

$$a = \frac{a_0}{1 + \frac{57.3a_0}{\pi ARe_1}}$$

Where e_1 is the span effectiveness factor and is practically the same as the span efficiency factor for a given wing. While the lift curve slope changes, the angle of attack for zero lift stays the same. This is because when there is no lift, the induced angle of attack is zero, so the effective angle of attack and the geometric angle of attack are the same.

6.1 Introduction: The Drag Polar:

By plotting the variation of the drag coefficient with the lift coefficient for an entire aircraft, the drag polar can be sketched.



$C_{D,0}$ is the parasite drag coefficient at zero lift, and e is now defined as the Oswald efficiency factor.

6.2 Equations of Motion:

When an aircraft is in flight, there are four physical forces acting on it.

1. Lift L , which is perpendicular to the flight path direction.
2. Drag D , which is parallel to the flight path direction.
3. Weight W , which acts vertically toward the center of the earth (and hence is inclined at angle θ with respect to the lift direction).
4. Thrust T, which in general is inclined at the angle αT with respect to the fl ight path direction.

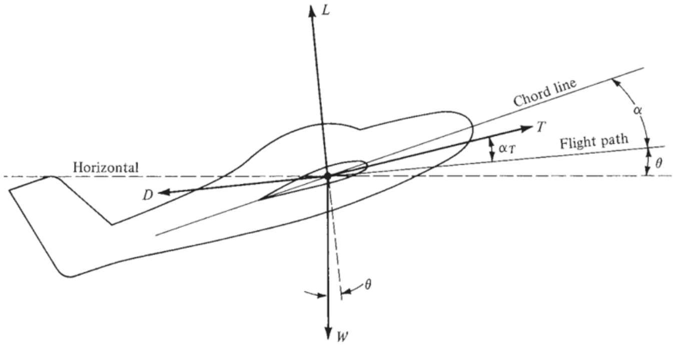


Figure 6.5 Force diagram for an airplane in flight.

By summing force parallel and perpendicular to the flight path, we get for level, unaccelerated flight

$$\boxed{T\cos\alpha_T = D}$$

$$\boxed{L + T\sin\alpha_T = W}$$

Assuming α_T is small, we get

$$\boxed{T = D}$$

$$\boxed{L = W}$$

6.3 Thrust Required for Level, Unaccelerated Flight:

For level, unaccelerated flight, the thrust required can be found by using the lift and drag equation, along with the relations from the previous section. The thrust required to fly at a given velocity is given by

$$T_R = \frac{W}{\frac{C_L}{C_D}} = \frac{W}{\frac{L}{D}}$$

The thrust required curve is a plot of the variation of the thrust required with freestream velocity at a specific altitude.

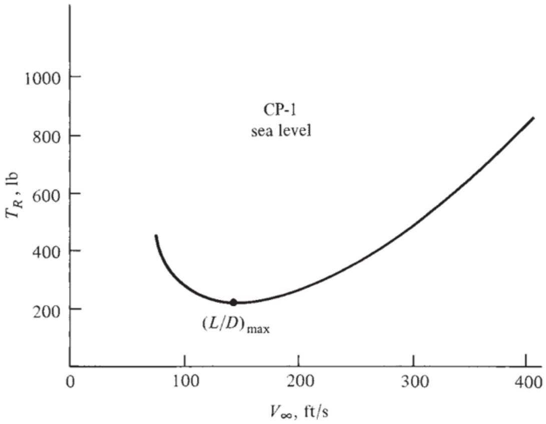


Figure 6.6 Thrust-required curve. The results on this and subsequent figures correspond to answers for some of the sample problems in this chapter.

To calculate the thrust required at a given velocity

1. Choose a velocity
2. Calculate C_L at this velocity from the lift equation using the weight of the aircraft
3. Calculate C_D from the known drag polar
4. Find the lift to drag ratio and use the equation above

We can see from the curve that the minimum thrust required occurs at the maximum lift to drag ratio. There exists a minimum because at higher flight velocities, the coefficient of lift is smaller due to the high dynamic pressure. The high dynamic pressure in the drag equation also causes drag to be high. As velocity decreases, dynamic pressure decreases so the thrust required goes down. However, C_L must increase. At low speeds, the induced drag increases much faster than the dynamic pressure can decrease so drag increases again at low speeds, and thrust required increases.

Thrust required can be broken down into two components: the thrust required to balance out the zero-lift drag and the thrust required to balance out the induced drag.

$$T_R = \underbrace{q_\infty S C_{D,0}}_{\text{Zero-lift } T_R} + \underbrace{q_\infty S \frac{C_L^2}{\pi e \mathrm{AR}}}_{\text{Lift-induced } T_R}$$

Plotting the separate curves, the minimum thrust required occurs at the intersection of these curves. In other words, the velocity where the zero-lift drag equals the induced drag.

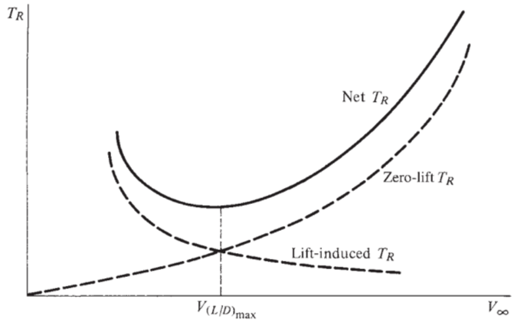
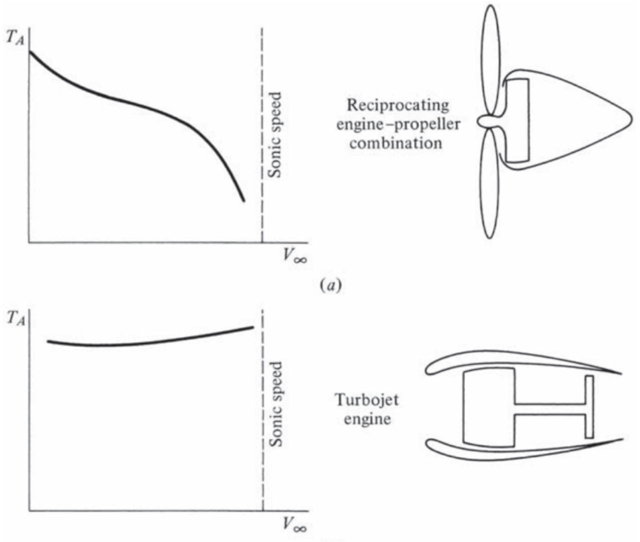


Figure 6.9 Comparison of lift-induced and zero-lift thrust required.

6.4 Thrust Available and Maximum Velocity:

Thrust required was dictated by the aerodynamics and weight of the airplane but thrust available is dictated by the engine of the airplane. For propellors, thrust available decreases with increasing velocity, and drops off near sonic speeds. Stays relatively constant for turbojet engines.



The intersection of the thrust required curve and the maximum thrust available curve defines the maximum velocity of the aircraft at a specific altitude.

6.5 Power Required for Level, Unaccelerated Flight:

Power is energy per unit time. It can be calculated by multiplying the force by the velocity of the object.

Power required at a given velocity and altitude can be calculated by multiplying the thrust required at these conditions by the velocity. For power required

$$P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}}$$

As seen from the equation above, power required varies inversely with $C_L^{\frac{3}{2}}/C_D$, while thrust required varied inversely with C_L/C_D.

As before, we can split the power required into the two components of drag.

$$P_R = \underbrace{q_\infty S C_{D,0} V_\infty}_{\substack{\text{Zero-lift power}\\ \text{required}}} + \underbrace{q_\infty S V_\infty \frac{C_L^2}{\pi e AR}}_{\substack{\text{Lift-induced power}\\ \text{required}}}$$

Looking at the two curves, we see that they don't intersect at the minimum power required. The new condition is that at minimum power required,

$$C_{D,0} = \frac{1}{3} C_{D,i}$$

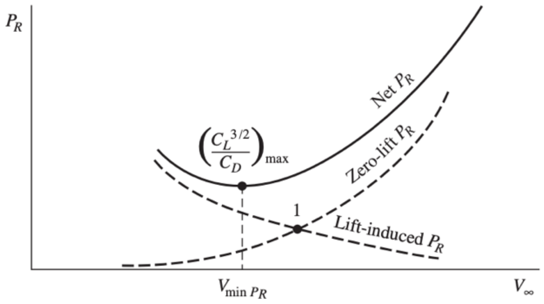


Figure 6.16 Comparison of lift-induced, zero-lift, and net power required.

To find the velocity for minimum thrust required on the power required curve, draw a tangent line from the origin to the power required curve.

6.6 Power Available and Maximum Velocity:

For propellors, the power available is given by

$$P_A = \eta P$$

Since not all power is available to drive the airplane, the power available, P_A, is a fraction of the power P ,the power delivered to the propellor from the crank shaft. Here η is the propellor efficiency.

For units of power, horsepower is commonly used. The conversion for horsepower is hp = 550 ft lb/s = 746 W.

The power available for jet engines varies linearly with velocity since thrust available is relatively constant at all velocities.

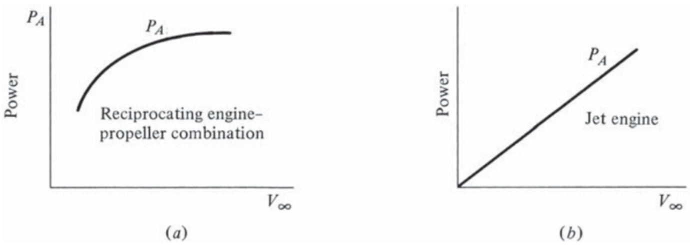


Figure 6.20 Power available for (*a*) a piston engine–propeller combination and (*b*) a jet engine.

1. Consider an airplane patterned after the twin-engine Beechcraft Queen Air executive transport. The airplane weight is 38,222 N, wing area is 27.6 m^2, aspect ratio is 7.8, Oswald efficiency factor is 0.9, and zero-lift drag coefficient is $C_{D,0} = 0.03$. Calculate the thrust (in N) required to fly at a velocity of 350 km/h at:

   (a) Standard sea level

   (b) An altitude of 4.5 km

2. Consider a 1 ft long flat plate placed horizontally inside the test section of a wind tunnel (*Fig. 1*) The flow passing over the plate has a velocity of 0.35 ft/s. If the plate is experiencing standard atmospheric conditions equal to those at 9,000 ft (assume laminar flow):
   (a) What is Reynolds Number for the flow over the plate?

   (b) What is the wall shear stress in lb/ft^2 for the full length of the plate ($x = L$)?

   (c) At half the length of the plate ($x = L/2$), what is the boundary layer thickness in m, and the skin friction coefficient?

3. Consider a hot, high-pressure gas (R = 378 J/kg K, $\gamma = 1.26$) entering a rocket engine from the combustion chamber. The flow is proceeding from a subsonic combustion chamber, through a throat into a nozzle. The nozzle exit velocity is 2500 m/s and the mass flow rate is 126 kg/s, with exit conditions of $T_e = 1348\ K\ and\ p_e = 1\ atm$. Assume isentropic flow.

   (a) What is the stagnation temperature in K?

   (b) What is the stagnation density in kg/m^3?

   (c) What is the required throat area in m^2?

4. Consider a model airplane wing with an NACA 1412 airfoil section chord length of 1 ft. The wing has an angle of attack of 6° in an airflow velocity of 50 ft/s at standard sea-level conditions. For the wing, the normal and axial force with respect to the chord are 20 lbs and 0.5 lbs respectively. Calculate the:
   (a) Max camber of the wing in and its location in ft

   (b) Lift in lb

   (c) Drag in lb