2022 SGT Aero 2200 Midterm 1 Review Document – Blank Problem Set

2.1 Fundamental Physical Quantities of Flowing Gas:

2.1.1 Pressure: Pressure is the normal force per unit area exerted on a surface due to the time rate of change of momentum of the gas molecules impacting on that surface.

P = F/A

Units:

- N/m^2
- lb/in^2 (Psi)
- lb/ft^2 (Psf)
- Atm

2.1.2 Density: The density of a substance (including a gas) is the mass of that substance per unit Volume.

$$\rho = \frac{m}{V}$$

Units:

- Kg/m^3
- lbm/ft^3
- Slug/ft^3
- g/cm^3

2.5 Specific Volume: The volume per unit mass equals the inverse of density.

$$\nu = \frac{1}{\rho}$$

Units:

- m^3/kg
- ft^3/slug

2.1.3 Temperature: Is the average kinetic energy of a collection of gas molecules.

Units:

- Kelvin (K)
- Degrees Rankine ($°R$)
- Degrees Celsius ($°C$)
- Degrees Fahrenheit ($°F$)

2.1.4 Flow Velocity and Streamlines:

Velocity: The distance traveled by some object over time.

V = D/T

- Has direction and magnitude
- Point property

Eulerian View: Observing fluid elements move through a point in the flow field
Lagrangian View: Tracking individual fluid element

2.3 Equation of State for Perfect Gas: A perfect gas is one in which intermolecular forces are negligible. A gas in nature in which the particles are widely separated. Air at standard conditions can be readily approximated by a perfect gas.

$$P = \rho RT$$

For Air:
R = 287 $\frac{J}{kg \cdot K}$ = 1716 $\frac{(ft \cdot lb)}{slug \cdot °R}$

Universal:
$R = \frac{\mathcal{R}}{M}$,where M is the molecular mass of the gas
$\mathcal{R}$ = 8314 $\frac{J}{kg \cdot mol \cdot K}$ = 4.97*10^4 $\frac{(ft \cdot lb)}{slug \cdot mol \cdot °R}$

2.4 Discussion of Units: Distinguishing between units and knowing how to convert is very important depending on what the problem is asking for.

Example:

$$\frac{(15\ lb)}{in^2} \cdot \left(\frac{(12\ in)}{1\ ft}\right)^2 = 2160\ \left(\frac{lb}{ft^2}\right)$$

	Metric (SI)	English (lb)
Weight	Newtons (N)	Pounds (lb)
Mass	Kilograms (kg)	Slug

- English units can also express mass as pound mass (lbm)
- On earth's surface a mass of 1 lbm will weigh 1 lbf
- 1 slug = 32.2 lbm

Nautical Mile:

- 1 nmi = 6076.1 ft
- 1 mi = 5280 ft ("statute mile")

Knot:

- 1 knot = nmi/hr

2.6 Anatomy of the Airplane: The fuselage is the center body, where most of the usable volume of the airplane is found. The fuselage carries people, baggage, other payload, instruments, fuel, and anything else that the airplane designer puts there.

The wings are the main lift-producing components of airplanes; the left and right wings are identified as you would see them from inside the airplane, facing forward. The internal volume of the wings can be used for such items as fuel tanks and storage of the main landing gear (the wheels and supporting struts) after the gear is retracted.

The horizontal and vertical stabilizers are located and sized to provide the necessary stability for the airplane in flight.

Flaps and control surfaces are hinged surfaces, usually at the trailing edge (the back edge) of the wings and tail, that can be rotated up or down. The function of a flap is to increase the lift force on the airplane.

The ailerons are control surfaces that control the rolling motion of the airplane around the fuselage.

The elevators are control surfaces that control the nose up-and-down pitching motion.

The rudder is a control surface that can turn the nose of the airplane to the right or left (called yawing).

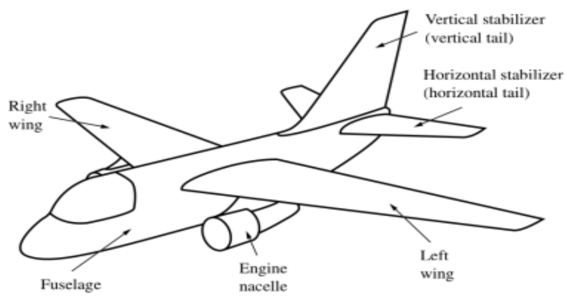


Figure 2.13 Basic components of an airplane.

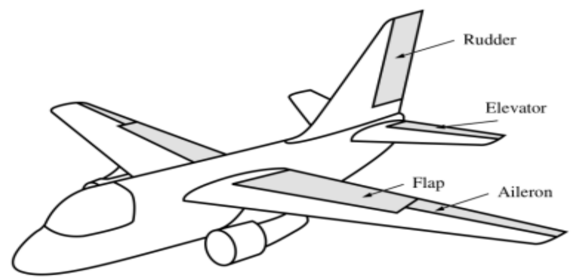


Figure 2.14 Control surfaces and flaps.

3.1 Definition of Altitude:

Altitude Definitions:

- Absolute Altitude (h_a): distance from object to center of earth

r_e = 6.356766*10^6 m at 45° latitude

$$h_a = r_e + h_g$$

- Geometric Altitude (h_g): distance from object to surface of earth
- Geopotential Altitude (h)
- Pressure Altitude
- Temperature Altitude
- Density Altitude

Gravity Variation with Altitude:

$g = g_o \cdot \frac{(r_e)^2}{(r_e + h_g)^2}$, where g_o is the gravitational acceleration at sea level

3.2 Hydrostatic Equation: A force balance on an element of fluid at rest that allows us to calculate variations in pressure, density, and temperature as functions of altitude.

$$dP = -\rho g dh_g$$
$$dP = -\rho g_o dh$$

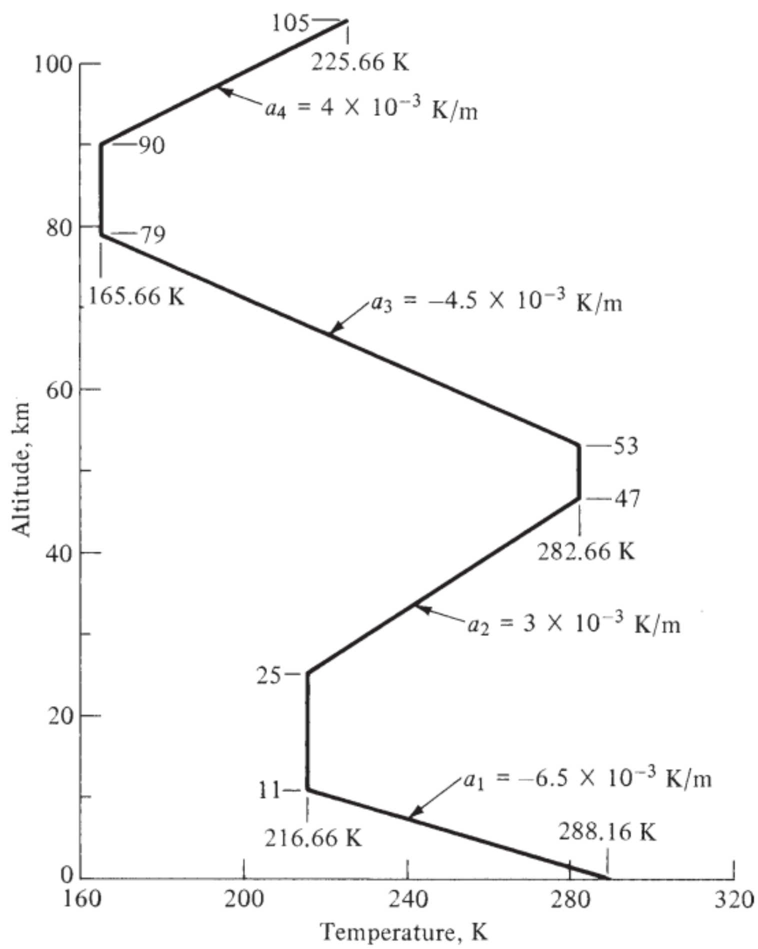
3.3 Relation between Geopotential and Geometric Altitudes: The relationship between h_g and h .

$$h = \frac{r_e}{(r_e + h_g)} \cdot h_g$$

3.4 Definition of the Standard Atmosphere: A defined variation of temperature with altitude that allows us to obtain density, pressure, and temperature as functions of h for the standard atmosphere.

Standard S.L Values:

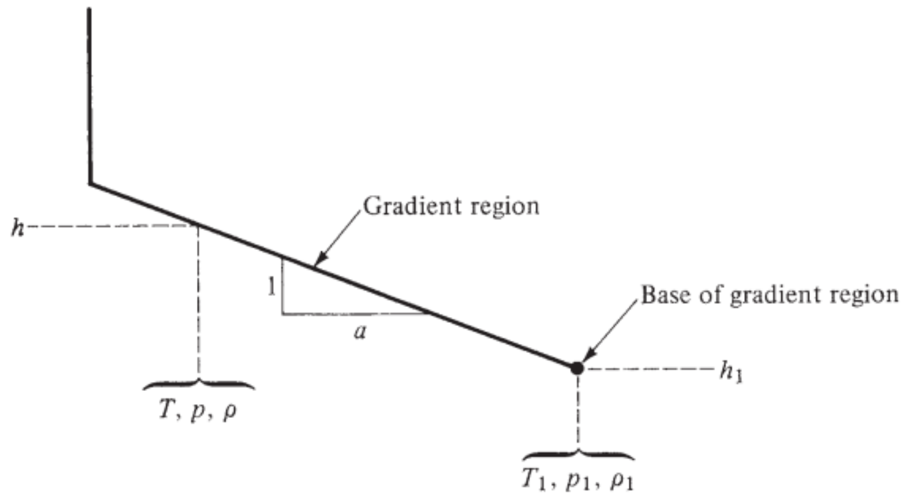
$$P_{SL} = 101{,}325\ \frac{N}{m^2} = 2116.2\ \frac{lb}{ft^2}$$
$$\rho_{SL} = 1.225\ \frac{kg}{m^3} = 0.0023769\ \frac{slug}{ft^3}$$
$$T_{SL} = 288.16\ K = 518.69\ °R$$



Temperature Gradient Regions:

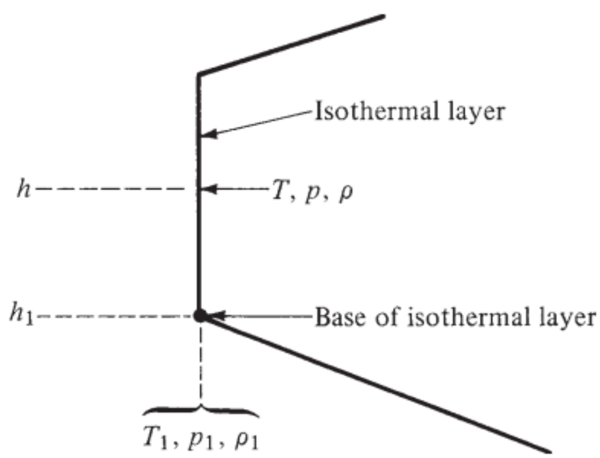
$$a = \frac{dT}{dh} = lapse\ rate$$
$$T = T_1 + a(h - h_1)$$
$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_o}{aR}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left[\frac{g_o}{aR}+1\right]}$$



<u>Isothermal Region:</u>

$$T = const.$$
$$\frac{P}{P_1} = e^{\left[-\frac{g_o}{RT}\cdot(h-h_1)\right]}$$
$$\frac{\rho}{\rho_1} = e^{\left[-\frac{g_o}{RT}\cdot(h-h_1)\right]}$$



3.5 Pressure, Density, and Temperature Altitudes: Using Appendix A and B we can determine the altitudes that correspond to the respective pressure, density, and temperatures at a certain altitude.

Example:

$$h_g = 4{,}100\ m$$
$$P_\infty = 6.16 \cdot 10^4 \left(\frac{N}{m^2}\right) \rightarrow\ 4{,}000\ m$$
$$T_\infty = 265.4\ K\ \rightarrow\ 3{,}500\ m$$

Chapter 4

4.1 Continuity Equation:

Physical Principle: Mass can neither be created nor destroyed

Continuity Equation:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

4.2 Incompressible and Compressible Flow:

Compressible Flow: Flow in which the density of the fluid elements can change from to point

Incompressible Flow: Flow in which the density of the fluid elements is always constant

$$\rho_1 = \rho_2$$
$$A_1 V_1 = A_2 V_2$$

Incompressible Flow does not exist! But flow is nearly incompressible at low speeds (less than 100 m/s or 300 ft/s)

4.3 Momentum Equation:

Newton's Second Law:

$$F = ma$$

Force is a combination of these three phenomena:

1. Pressure acting in the normal direction on all faces of the element
2. Frictional shear acting tangentially on all faces of the element
3. Gravity acting on the mass inside the element

Euler's Equation:

$$dp\ = -\rho V\, dV$$

Bernoulli's Equations:

$$p_2 + \rho\frac{V_2^2}{2} = p_1 + \rho\frac{V_1^2}{2}$$
$$p + \rho\frac{V^2}{2} = const.$$

Notes:

1. Bernoulli's Equations hold true only for inviscid, incompressible flow
2. Bernoulli's Equations relate different points along a streamline
3. For a compressible flow, Bernoulli's Equations cannot be used, and if you were to use Euler's Equation density would be treated as a variable
4. Euler's Equation and Bernoulli's Equations essentially apply Newton's Law to fluid dynamics

4.4 A Comment:

Distinctions to keep in mind:

- The equation of state relates pressure, temperature, and density to one another at the *same* point
- The flow equations relate density and velocity in the Continuity Equations and pressure and velocity in the Bernoulli Equations from one point in the flow to another point in the flow

4.6 Isentropic Flow:

Adiabatic Process: A process in which no heat is added or taken away

Reversible Process: A process in which no frictional or other dissipative effects occur

Isentropic Process: A process that is both adiabatic and reversible

Isentropic Equations:

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

4.7 Energy Equation:

Physical Principle: Energy can neither be created nor destroyed, it can only change form.

First Law of Thermodynamics applied to fluid dynamics:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h + \frac{V^2}{2} = const.$$

Taking into account frictionless, adiabatic flow:

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2$$

$$c_p T + \frac{1}{2}V^2 = const.$$

4.10 Low Speed Subsonic Wind Tunnels:

Velocity in a Subsonic Wind Tunnel:

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}}$$

Manometer: Used for measuring the pressure difference (a U-tube is a type of manometer)

$$p_1 - p_2 = w\Delta h$$

4.11 Measurement of Airspeed:

Static Pressure: Pressure that is the consequence of randomly flowing molecules

Total Pressure: The pressure at a given point in the flow that would exist if the flow

were slowed down isentropically to zero velocity (also known as "Stagnation Pressure")

Pitot Tube: An aerodynamic instrument that measures the total pressure at a point in the flow

Stagnation Point: Any point of a flow where velocity is zero

4.11.1 Incompressible Flow:

Can assume incompressible flow if the Mach number is less than 0.3

Dynamic Pressure:

$$q \equiv \frac{1}{2}\rho V^2$$

This representation is only true for incompressible flow

True Airspeed:

$$V_{true} = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

Equivalent Airspeed:

$$V_e = \sqrt{\frac{2(p_0 - p)}{\rho_s}}$$

4.11.2 Subsonic Compressible Flow:

If the Mach number is greater than 0.3, we must assume compressible flow

Specific Heat:

$$c_p = \frac{\gamma R}{\gamma - 1}$$

Solving for the Mach number using Isentropic Equations:

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{1}{\gamma-1}}$$

Actual Flight Velocity:

$$V_1^2 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right]$$

4.11.3 Supersonic Flow:

Supersonic Flow refers to flow with a Mach number greater than 1

As a fluid element flows through a shock wave:

- Mach number decreases
- Static Pressure increases
- Static Temperature increases
- Flow Velocity decreases
- Total Pressure decreases
- Total Temperature stays the same for a perfect gas

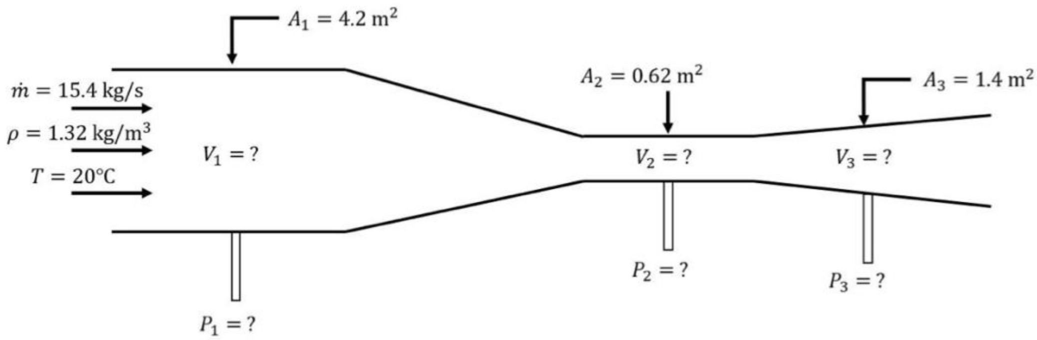


Figure 1: Wind tunnel sketch

1. A low-speed subsonic wind tunnel (Fig.1) is operating with a mass flow rate of 15.4 kg/s, a density of 1.32 kg/m^3, and a temperature of 20 °C at its settling chamber. Determine the velocities [m/s] and (static)pressures [Pa] in the wind tunnel at:

   (a) Location 1
   (b) Location 2 & location 3

$$\left.\begin{aligned} \dot{m} &= 15.4\ \mathrm{kg/s} \\ \rho &= 1.32\ \mathrm{kg/m^3} \\ T &= 20°C + 273 = 293\ °K \\ A &= 4.2\ \mathrm{m^2} \end{aligned}\right\} @1$$

$V_1, P\,?$

$P = \rho R T \qquad \dot{m} = \rho A V$

$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$

1) $\dot{m} = \rho A V$

$$V = \frac{\dot{m}}{\rho A} = \frac{15.4}{1.32\,(4.2)} = \underline{2.778\ \mathrm{m/s}\ V_1}$$

$$P = \rho R T$$

$$P = 1.32(287)(293) = \underline{111000.12 \text{ N/m}^2} \; P_1$$

$$A_1 V_1 = A_2 V_2$$

$$(4.2)(2.778) = (0.62)(V_2)$$

$$V_2 = 18.819 \text{ m/s}$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 = P_1 + \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2$$

$$= (111000.12) + \frac{1}{2}(1.32)(2.778)^2 - \frac{1}{2}(1.32)(18.819)^2$$

$$\underline{P_2 = 110771.47 \text{ N/m}^2}$$

$$A_1 V_1 = A_3 V_3$$

$$(4.2)(2.778) = (1.4) V_3$$

$$\underline{V_3 = 8.334 \text{ m/s}}$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_3 + \frac{1}{2}\rho V_3^2$$

$$(111000.12) + \frac{1}{2}(1.32)(2.778)^2 = P_3 + \frac{1}{2}(1.32)(8.334)^2$$

$$\underline{P_3 = 110959.37 \text{ N/m}^2}$$

Figure 2: P-51 Mustang

2. When the P-51 shown in *Fig. 2* (specifications given in a handout posted on Carmen) is flying at maximum speed at an altitude of 25,000 ft at its operating empty weight plus 2,000 lb of payload,

   (a) What is its lift coefficient?

   (b) What is the slowest speed (stall speed) at which the P-51 can fly in straight and level flight at sea level if its maximum lift coefficient is 1.35?

Operating Empty Weight = 7125 lb +2000 lb = 9125 lb

Max Speed = 640.93 ft/sec

Max Wing Loading = 49.785 psf

Max Weight = 11600 lbs

Max C_L = 1.35

$$W = L = \frac{1}{2}\rho V^2 S C_L$$

@25,000 ft : $\rho = 1.0663 \times 10^{-3}$ slug/ft³

$$\frac{W_{max}}{S} = 49.785 \text{ psf} \qquad S = \frac{11600 \text{ lb}}{49.785 \text{ lb/ft}^2}$$

$$S = 233 \text{ ft}^2$$

$$C_L = \frac{2W}{\rho V^2 S} = \frac{2(9125\,lb)}{(1.0663\times10^{-3})(640.93)(233)}$$

a)

$$\boxed{C_L = 0.179}$$

b) $L = W = \frac{1}{2}\rho_{SL} V_{min}^2 S\, C_{L_{max}}$

$\rho_{SL} = 2.3679\times10^{-3}\ slug/ft^3$

$C_{L_{max}} = 1.35$

$$V_{min} = \sqrt{\frac{2W}{\rho_{SL} S\, C_{L_{max}}}}$$

$$V_{min} = \sqrt{\frac{2(9125)}{(2.3679\times10^{-3})(233)(1.35)}}$$

$V_{min} = 156\ ft/s$

Figure 3: Balloon Launched Hang Glider

3. When a balloon dropped hang glider (Fig. 3) that is cruising at steady level flight at $16.7\ \frac{m}{s}$ at a geometric altitude of 11,850 m, what is its lift coefficient? The hang glider (+ the pilot) has a total mass of 88 kg, and a planform area of $15.3\ \text{m}^2$. *Note: geometric altitude is defined as the height measured from sea level.*

given: $V_\infty = 16.7\ m/s$ find: $C_L = ?$

$h_G = 11{,}850\ m$

$m = 88\ kg$

$S = 15.3\ m^2$

? ✓ known

equations: $L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$?

known

Lin. Int

$$y = y_1 + (x+x_1)\frac{(y_2-y_1)}{(x_2-x_1)}$$

from Apx A.

	h_{G}, m	ρ_∞
①	11,800	3.2189×10^{-1}
x	11,850	y
②	11,900	3.1687×10^{-1}

$$y = (3.2189 \times 10^{-1}) + (11{,}850 + 11{,}800)\frac{(3.1687 \times 10^{-1} - 3.2189 \times 10^{-1})}{(11{,}900 - 11{,}800)}$$

$$y = 0.31938 \text{ kg/m}^3 = \rho_\infty$$

$$L = W$$

$$L = W = mg = (88 \text{ kg})(9.81 \text{ m/s}^2) =$$

$$L = 862.4 \text{ kg}\cdot\text{m/s}^2$$

$$L = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L \Rightarrow$$

$$C_L = \frac{L}{\tfrac{1}{2}\rho_\infty V_\infty^2 S}$$

$$C_L = \frac{2L}{\rho_\infty V_\infty^2 S}$$

$$C_L = \frac{2(862.4 \text{ kg}\cdot\text{m/s}^2)}{(0.31938 \text{ kg/m}^3)(16.7 \text{ m/s})(15.3 \text{m}^2)}$$

$$\boxed{C_L = 1.27}$$

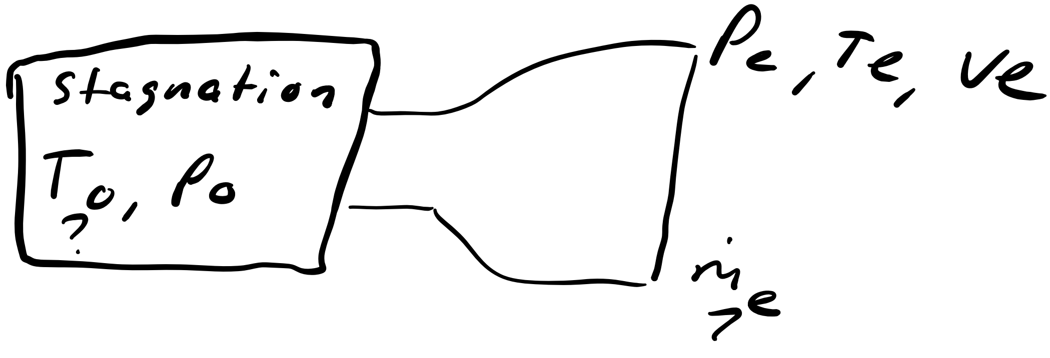
4. The nozzle (exit) of a rocket engine is connected to the combustion chamber of the engine, where fuel and oxidizer are burned and then expand to high velocities through the nozzle. Downstream of the combustion chamber, through the nozzle, the flow is isentropic. Consider a case where the pressure of the gas exiting the nozzle is 0.8 atm, with a temperature of 2100 K. (Assume $\gamma = 1.12$)

(a) If the pressure of the burned gas in the combustion chamber is 22 atm, what is its temperature?

(b) Given that $c_p = 1008 \frac{J}{kg\,K}$, the exit velocity of the gas is 4 km/s, and the cross-sectional area of the nozzle exit is 4 m^2, what is the mass flow rate through the nozzle?
(Recall: $R = c_p - c_v$)

Given: • isentropic, $P_e = 0.8$ atm, $T_e = 2100$K, $\gamma = 1.12$, $P_o = 22$ atm, $c_p = 1008 \frac{J}{kgK}$, $V_e = 4$ km/s, $A_e = 4 m^2$

Find: a) T_o b) $\dot{m}_e$



Eqns: $$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\dot{m} = \rho A V, \quad R = c_p - c_v$$

$$\gamma = c_p / c_v$$

a) $T_0 = ?$ $\quad \frac{P_0}{P_e} = \left(\frac{T_0}{T_e}\right)^{\gamma/\gamma-1}$

$$\rightarrow T_0 = \left(\frac{P_0}{P_e}\right)^{\frac{\gamma-1}{\gamma}} \cdot T_e$$

$$= \left(\frac{22\ atm}{0.8\ atm}\right)^{\frac{1.12-1}{1.12}} \cdot 2100\,K$$

$$\rightarrow \underline{T_0 = 2995\ K}$$

b) $\dot{m}_e = ? = A_e V_e \rho_e$ $\quad P_e = \rho_e R T_e$

$\quad\quad\quad\quad\quad\quad \checkmark \ \ \checkmark \ \ ?$ $\quad \rightarrow \rho_e = \frac{P_e}{R T_e}$

$R = C_p - C_v$

$\underbrace{\gamma = \frac{C_p}{C_v}}$, $\gamma = 1.12$, $C_p = 1008\ \frac{J}{kg\,K}$

$\rightarrow C_v = 900\ \frac{J}{kg\,K}$ $\quad \rightarrow R = 108\ \frac{J}{kg\,K}$

$$\rho_e = \frac{(0.8\,atm)\left(101325\ \frac{Pa}{atm}\right)}{\left(108\ \frac{J}{kg\,K}\right)(2100\,K)}$$

$$\rightarrow \rho_e = 0.357\ kg/m^3$$

$$\dot{m}_e = A_e v_e \rho_e = (4\,m^2)(4000\,m/s)(0.357\,\frac{kg}{m^3})$$

$$\rightarrow \dot{m}_e = 5718.5\ kg/s$$