Chapter 4:

4.1: Continuity Equation

Physical Principle: Mass can be neither created nor destroyed. $\dot{m}_1 = \dot{m}_2$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ FOR CONSTANT ρ

$\rightarrow A_1 V_1 = A_2 V_2$

4.2 Incompressible and Compressible Flow

Incompressible Flow: Does not actually exist, but is an ASSUMPTION, where density 1 is equal to density 2. Velocities less than 100 m/s or 300 ft/s can be assumed incompressible.

Hose Nozzle: Converging nozzle so A2 is less than A1 the velocity of the water will increase.

4.3 Momentum Equation

Newton's Second Law: F = ma

1. Pressure acting in a normal direction on all six faces of the element.
2. Frictional shear acting tangentially on all six faces of the element.
3. Gravity acting on the mass inside the element.

Euler Equation: $dp = -\rho V dV$

Assumptions: Inviscid (frictionless), Steady (constant with time)

Bernoulli's Equation: $P_2 + \rho \frac{V_2^2}{2} = P_1 + \rho \frac{V_1^2}{2}$ $P + \rho \frac{V^2}{2} = $ CONSTANT

Assumptions: Inviscid, Incompressible Flow, Points along a streamline,

Bernoulli's Equation is a simplification of Euler Equation for incompressible flow, if your flow is compressible you would have to use Euler's Equation

4.5 Elementary Thermodynamics : FIRST LAW OF THERMO

INTERNAL ENERGY = HEAT ADDED TO & WORK DONE TO THE SYSTEMS

$\delta e = \delta q + \delta w$

WITH CONSTANT PRESSURE : $\delta w = -P dV$

WITH CONSTANT VOLUME : $\delta q = dh - V dP$

h ENTHALPY: $h = C_p T$, $e = C_v T$

CONSTANT VOLUME PROCESS: $\delta q = C_v dT$

CONSTANT PRESSURE PROCESS: $\delta q = C_p dT$

4.6 Isentropic Flow

Adiabatic Process: one in which no heat is added or taken away.

Reversible Process: One in which no frictional or other dissipative effects occur.

Isentropic Process: one that is both adiabatic and reversible. (Entropy is constant in isentropic flow)

ISENTROPIC RELATIONS:

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}, \quad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}, \quad \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$$

4.7 Energy Equation

Physical Principle: Energy can be neither created nor destroyed. It can only change form.

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \rightarrow h + \frac{V^2}{2} = \text{CONST.}$$

$$C_p T_1 + \frac{1}{2}V_1^2 = C_p T_2 + \frac{1}{2}V_2^2 \rightarrow C_p T + \frac{1}{2}V^2 = \text{CONST.}$$

4.9 Speed of Sound

The speed of sound in a perfect gas depends only on the temperature of the gas

$$a = \sqrt{\gamma \frac{P}{\rho}} \qquad a = \sqrt{\gamma R T}$$

Mach Number: $M = \frac{V}{a} \rightarrow \frac{V}{\sqrt{\gamma R T}}$

1. Subsonic flow: M < 1
2. Sonic flow: M = 1
3. Supersonic flow: M > 1
4. Transonic flow: 0.8 < M < 1.2
5. Hypersonic flow: M > 5

4.10 Low-Speed Subsonic Wind Tunnels

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho[1 - (A_2/A_1)^2]}}$$

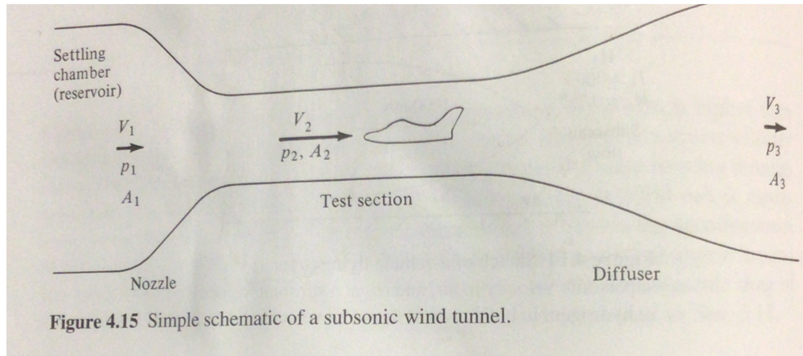


Figure 4.15 Simple schematic of a subsonic wind tunnel.

4.11 Measurement of Airspeed

Pitot-Static Tube:

Static Pressure: pressure you would feel is we were moving along with the flow at that point. (P)

Total Pressure: at a given point in a flow is the pressure that would exist if the flow were slowed down isentropically to zero velocity. (P_0)

Pitot Tube: Measures the total pressure at a point in the flow.

- If we assume incompressible the Bernoulli's equation can be used, which relates dynamic pressure, total pressure, and static pressure

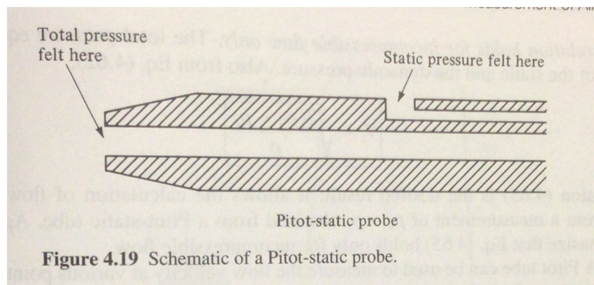


Figure 4.19 Schematic of a Pitot-static probe.

4.11.1 Incompressible Flow

Dynamic pressure is always defined as q for subsonic to hypersonic flows: q

$$q = \frac{1}{2}\rho V_1^2$$

This relation only holds for incompressible flow: 4.65

True Airspeed, with proper surrounding density value this will give you the actual speed of the aircraft: 4.66

$$V_{TRUE} = \sqrt{\frac{2(P_0 - P)}{\rho}}$$

Equivalent Airspeed: is the airspeed measured by an airspeed indicator and deals with sea level density.

$$V_e = \sqrt{\frac{2(P_0 - P)}{\rho_{SL}}}$$

If the incompressible assumption cannot be made, then one way to find the true velocity is solve for it using Mach number = v/a (Be careful not to use T0 when finding Mach number, remember velocity is zero at the stagnation point, therefore M=0!) We can find Mach number using the isentropic Mach relations

4.11.2 Subsonic Compressible Flow

Subsonic Wind tunnels:

Most of the times can assume incompressible because we are dealing with low speed.

- Velocity increases as the area decreases through the convergent nozzle, and the opposite occurs for the divergent part Different types of pressures

CAN SOLVE FOR MACH USING ISENTROPIC RELATIONS

$$\frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2 \quad , \quad \frac{P_0}{P_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\gamma/(\gamma-1)} \quad , \quad \frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{1/(\gamma-1)}$$

Supersonic wind tunnels -For the velocity to increase the area must increase

Supersonic wind

Rocket nozzle

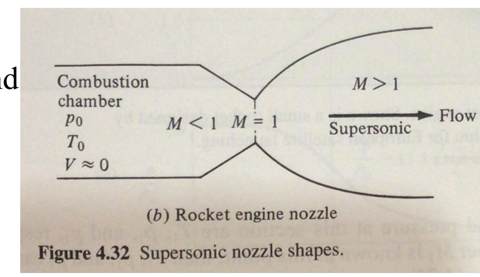
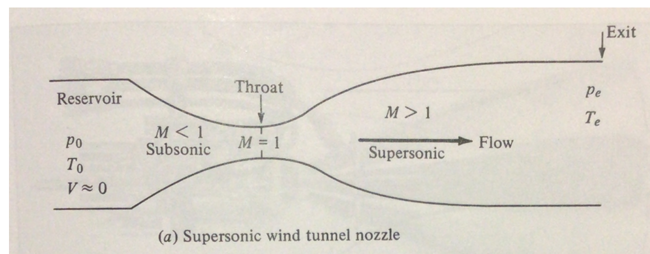


Figure 4.32 Supersonic nozzle shapes.



Reservoir: P0, T0, rho0 (flow going into the wind tunnel)

Test section: P_exit, T_exit, rho_exit (flow going out).

Area Mach relations I we are given a Mach number or an area ratio, we can get either of those from the table.

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{(\gamma+1)/(\gamma-1)}$$

- Be careful when using the table, since your result will depend on if the flow is subsonic or supersonic

- It's important to know that a throat is the point where the smallest area of a wind tunnel or rocket nozzle can be found, but having a throat does not necessarily mean that you have a choke point where M=1.

- A* which is the area where M=1 can be thought as a property of the flow like P0, T0 and rho0. Even if we do not physically have it, we can still solve for it, and use this value to find other variables.

- You will see indications that wind tunnel has a physical choke point (e.g. if you are told that the flow goes from subsonic to supersonic)

- Even if there is throat, if the M is not 1 at this point, the flow can stay subsonic or supersonic.

Example 1:
A pilot is flying at an altitude of 27,000 ft and the airspeed indicator reads 325 knots. A pitot tube on the aircraft measures 1250 lb/ft^2.
a) what is the Mach number
b) what is the true airspeed? Give your answer in knots.

@ 27,000 FEET USING TABLES.

$\rho_\infty = 9.931 \times 10^{-4}$ SLUGS/ft³

$P_\infty = 720.26$ lb/ft²

$T_\infty = 422.53$ K

$P_0 = 1080$ lb/ft²

a)

$$\frac{P_0}{P_\infty} = (1 + 0.2M^2)^{3.5} \longrightarrow \frac{1080}{720.26} = (1 + 0.2M^2)^{3.5}$$

$$\rightarrow \left(\frac{1080}{720.26}\right)^{1/3.5} - 1 = 0.2M^2$$

$$\left(\frac{\left[\left(\frac{1080}{720.26}\right)^{1/3.5} - 1\right]}{0.2}\right)^{1/2} = M$$

$$\boxed{M = 0.78}$$

b) $V_{TRUE} = a_\infty M_\infty$

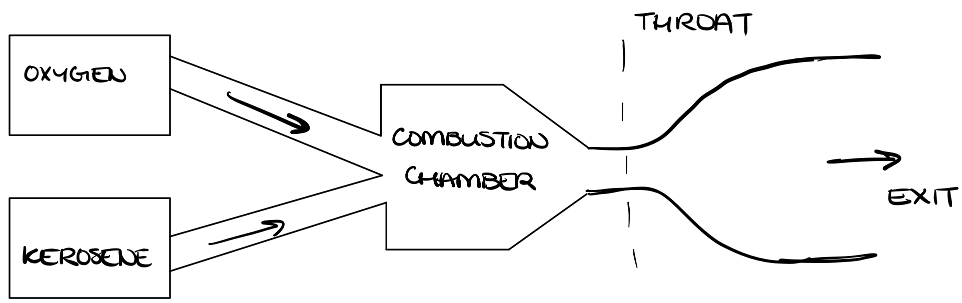
CALCULATE SPEED OF SOUND $a_\infty = \sqrt{\gamma R T}$

$$a_\infty = \sqrt{(1.4)(1716)(422.53)} = 1007.51 \text{ ft/s}$$

$$V_{TRUE} = (0.78)(1007.51) = 785.86 \text{ ft/s}$$

$$785.86 \frac{\text{ft}}{\text{s}} \left(\frac{1\text{mi}}{5280\text{ft}}\right)\left(\frac{3600\text{s}}{1.15\text{mi}}\right) = \boxed{465.9 \text{ KNOTS}}$$

Example 2:
Consider the rocket engine drawn below. Kerosene and oxygen are mixed and burned in the combustion chamber resulting in the following properties,
T0 = 4179 K
p0 = 27 atm
R = 380 J/kgK
Gamma = 1.32
The pressure at the exit is 1 atm. Assuming isentropic flow the rocket nozzle, determine the following.
a) The temperature at the exit.
b) the velocity at the exit.



SOLUTION:

$T_o = 4179\,K$

$P_o = 27\text{ atm} \rightarrow 2735775\text{ Pa}$

$R = 380\ J/kgK$

$\gamma = 1.32$

$P_e = 1\text{ atm} \rightarrow 101{,}325\text{ Pa}$

a) $$\frac{P_e}{P_o} = \left(\frac{T_e}{T_o}\right)^{\gamma/\gamma-1} \Rightarrow T_e = (4179\,K)\left(\frac{101325}{2735775}\right)^{\frac{1.32-1}{1.32}}$$

$$\boxed{T_e = 1879.64\,K}$$

b) $$a_e = \sqrt{\gamma R T} = \sqrt{(1.32)(287)(1879.64)} = 843.85\ m/s$$

$$\frac{T_o}{T_e} = 1 + \frac{\gamma-1}{2}M_e^2 \rightarrow M_e = \left(\frac{2}{\gamma-1}\left(\frac{T_o}{T_e}-1\right)\right)^{1/2}$$

$$M_e = \left(\frac{2}{1.32-1}\left(\frac{4179}{1879.75}-1\right)\right)^{1/2}$$

$$M_e = 2.765$$

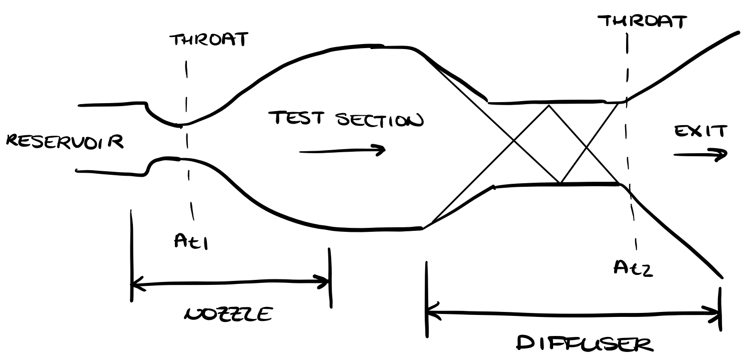
$$M_e = \frac{a_e}{V_e} \rightarrow V_e = M_e a_e = 2.765(843.85)$$

$$\boxed{V_e = 2333.18 \text{ m/s}}$$

Example 3:
The total pressure in area of the throat in the super sonic when tunnel in figure 2 are related by the following expression:

$$\frac{A_{t2}}{A_{t1}} = \frac{P_{01}}{P_{02}}$$

The area of the first throat is 300 cm² and the reservoir pressure is given at 3 atm the static pressure measured at the pressure tap in the wall of the second throat is 0.79 atm. The local mark number recorded at the second throat is MT2 equals one. Determine the area of the second throat AT2.



$A_{t1} = 300 \text{ cm}^2 \rightarrow$

$P_0 = 3 \text{ atm} \rightarrow 303975 \text{ Pa}$

$M_{t2} = 1$

$P_2 = 0.79 \text{ atm}$

$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma - 1}{2} M_{t2}^2\right)^{\gamma/\gamma-1} = \left(1 + 0.2(1)^2\right)^{3.5} = 1.893$$

$$P_{02} = (1.893)(0.79 \text{ atm}) = 1.49547 \text{ atm}$$

$$\frac{A_{t2}}{A_{t1}} = \frac{P_{01}}{P_{02}} \implies A_{t2} = A_{t1}\left(\frac{P_{01}}{P_{02}}\right) = (300\,cm^2)\left(\frac{3\,atm}{1.49547\,atm}\right)$$

$$\boxed{A_{t2} = 601.82\ cm^2}$$