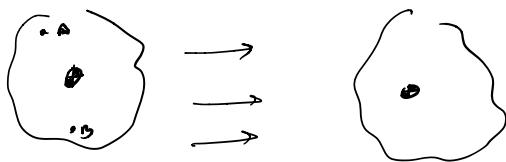


Rigid body kinematics :

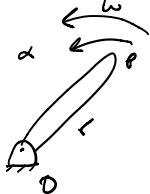
translation



$$v_{cm} = \text{velocity at any point} = \vec{v}_A = \vec{v}_B$$

$$a_{cm} = \vec{a}_A = \vec{a}_B$$

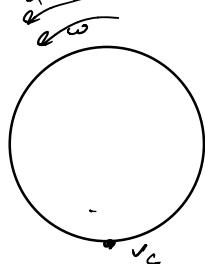
Rotation about fixed axis



$$\begin{aligned} v_p &= w r \\ a_n &= r w^2 = v_p^2/r = v w \\ a_t &= r \alpha \end{aligned}$$

$$\begin{aligned} \vec{v}_p &= \vec{v}_0 + \vec{\omega} \times \vec{r}_{p/0} \\ \vec{a}_p &= \vec{a}_0 + \vec{\alpha} \times \vec{r}_{p/0} - \vec{\omega}^2 \vec{r}_{p/0} \end{aligned}$$

When we have a disk rolling without slipping



v at the point of contact is zero

$$v_{cm} = w r \quad v_c = 0$$



$$\vec{v}_A = \vec{v}_B + \vec{v}_{A,rel} + \vec{\omega} \times \vec{r}_{A/B}$$

$$\vec{a}_n = \vec{a}_B + \vec{a}_{A,rel} + 2 \vec{\omega} \times \vec{v}_{A,rel} + \vec{\alpha} \times \vec{r}_{A/B} - \vec{\omega}^2 \vec{r}_{A/B}$$

Rigid body kinematics:

Newton-Euler

$$\sum \vec{F}_{ext} = \underline{\underline{m}} \vec{a}_{cm} \quad \sum M_{cm} = I_{cm} \vec{\alpha}$$

Rotation about a fixed axis

$$\sum M_0 = \vec{T}_0 \cdot \vec{\alpha}$$

Moment of Inertia of a Bar

(Most used) $I_{cm} = \frac{1}{12} ml^2$

 $I_o = \frac{1}{3} m l^2 \rightarrow (\text{parallel axis theorem})$
 $I_o = I_{cm} + md^2 \rightarrow \text{perpendicular distance}$

Work-Energy:

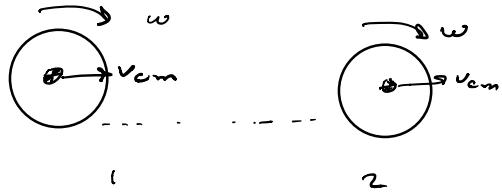
$K_E(\text{translational}) = \frac{1}{2} mv_{cm}^2 \quad (\text{Learned before})$

$K_E(\text{rotational}) = \frac{1}{2} I_{cm} \omega^2$

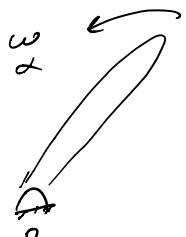
For general planar motion (if something is rotating and translating)

$K_E = K_{EP} + K_{ET}$

$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$



Rotation about a fixed axis



then no K_{ET}

$K_E = \frac{1}{2} I_o \omega^2 \quad I_o = \frac{1}{2} m l^2$

Linear momentum:

$\int_{t_1}^{t_2} \vec{\epsilon F} dt = \vec{mv}_2 - \vec{mv}_1$
 $(t_2 - t_1) \vec{\epsilon F_{avg}} = \vec{mv}_2 - \vec{mv}_1$

When no external forces

$m \vec{v}_{A\ cm} + m \vec{v}_{B\ cm} = \text{constant}$

Angular momentum:

$H_{cm} = I_{cm} \omega$

$H_o = \overbrace{\vec{r} \times m \vec{v}_{cm}} + H_{cm} \rightarrow H_o = I_o \omega$

$\int_{t_1}^{t_2} \vec{\epsilon M} dt = I_o \omega_2 - I_o \omega_1$

$\text{About C.O.M } (t_2 - t_1) \sum M_{avg} = H_{cm2} - H_{cm1}$

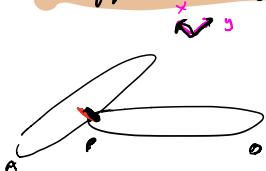
$\text{About fixed point O } (t_2 - t_1) \sum M_{O, avg} = H_{o2} - H_{o1}$

Conservation of angular momentum

(The angular momentum is conserved about the point of impact because in this way we can get rid of the reaction forces)

$$\vec{H}_1 = \vec{H}_2$$

Coefficient of restitution



$$e = \frac{(\vec{v}_{Bc'})_x - (\vec{v}_{Ac'})_x}{(\vec{v}_{Bc})_x - (\vec{v}_{Ac})_x}$$

(Forces are only being exerted in the x direction)

completely Plastic collision if $e=0$ $(\vec{v}_{Bc'})_x = (\vec{v}_{Ac'})_x$

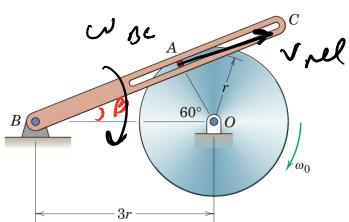
completely Elastic collision if $e=1$ Energy is conserved
(can use conservation of energy) Equate

* Energy is not conserved during an impact (if we have conservation of angular momentum, then we can't use conservation of energy)

Solution strategy for kinematic problems:

- 1) Ref system + type of problem
- 2) FBD
- 3) States (if Energy type problem)
- 4) Kinetics (Momentum, Newton's laws or Energy)
- 5) kinematics

5/178 The disk rotates about a fixed axis through point O with a clockwise angular velocity $\omega_0 = 20 \text{ rad/s}$ and a counterclockwise angular acceleration $\alpha_0 = 5 \text{ rad/s}^2$ at the instant under consideration. The value of r is 200 mm. Pin A is fixed to the disk but slides freely within the slotted member BC. Determine the velocity and acceleration of A relative to slotted member BC and the angular velocity and angular acceleration of BC.



Problem 5/178

1) Ref system: Cartesian

Type of problem: Rigid body kinematics

2) Knowns: $\omega_0 = 20 \text{ rad/s}$
 $\alpha_0 = 5 \text{ rad/s}^2 \text{ CCW}$
 $r = 200 \text{ mm}$ $a_B = 0$ $v_B = 0$

3) Unknowns: v_{rel} , a_{rel} ,
 ω_{BC} , α_{BC}

4) Kinematic Equations

$$\vec{a}_A = \vec{g}_B^0 + \vec{\omega}_{A,B} \times \vec{r}_{A,B} + 2\vec{\omega}_{BC} \times \vec{v}_{A,B} + 2\vec{\omega}_{BC} \times \vec{\omega}_{BC} \vec{r}_{A,B} - \omega_{BC}^2 \vec{r}_{A,B} \quad (1)$$

Need sine rule to calculate AB

$$\frac{AO}{\sin P} = \frac{BA}{\sin 60^\circ}$$

$$\frac{AO}{\sin P} = \frac{BD}{\sin(180^\circ - P - 60^\circ)}$$

$$\frac{r}{\sin P} = \frac{BA}{\sin 60^\circ}$$

$$\frac{r}{\sin P} = \frac{3r}{\sin(120^\circ - P)}$$

$$BA = \frac{r}{\sin P} \sin 60^\circ$$

$$\frac{1}{\sin P} = \frac{3}{0.866 \cos P - 0.5 \sin P}$$

$$= \frac{0.2 \sin 60^\circ}{\sin 19.1^\circ}$$

$$\frac{1}{\sin P} = \frac{3}{0.866 \cos P + 0.5 \sin P} = 3 \csc P$$

$$BA = 0.523 \text{ m}$$

$$0.866 \cos P + 0.5 \sin P = 3 \csc P$$

$$0.866 \frac{\cos P}{\sin P} + 0.5 = 3$$

$$0.866 \cot P = 2.5$$

$$P = 19.1^\circ$$

$$\vec{v}_A = \vec{y}_B^0 + \vec{\omega}_{BC} \times \vec{r}_{A,B} + \vec{v}_{rel}$$

$$\vec{v}_A = \vec{\omega}_{BC} \times \vec{r}_{A,B} + \vec{v}_{rel} \quad (2)$$

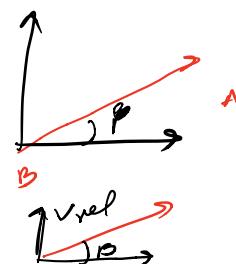
$$\vec{v}_A = \vec{y}_B^0 + \omega_0 \times \vec{r}_{A,O}$$

$$\vec{J}_A = \begin{pmatrix} 1 & 0 & k \\ 0 & 0 & -k \\ 0.2 \cos 60^\circ & 0.2 \sin 60^\circ & 0 \end{pmatrix}$$

$$v_A = 3.46 \uparrow + 2.5$$

(3)

(3) into (2)



$$3.46 \uparrow + 2 \vec{j} = \begin{pmatrix} 1 & 0 & k \\ 0 & 0 & -k \\ 0.523 \cos P & 0.523 \sin P & 0 \end{pmatrix} + v_{rel}$$

$$3.46 \uparrow + 2 \vec{j} = 0.523 \sin 19.1^\circ \omega_{BC} \uparrow - 0.523 \cos 19.1^\circ \omega_{BC} \vec{j} + v_{rel} \cos 19.1^\circ \uparrow + v_{rel} \sin 19.1^\circ \uparrow$$

Equating \uparrow and \uparrow components:

$$x: 3.46 = 0.523 \sin 19.1^\circ w_{BC} + v_{rel} \cos 19.1^\circ \quad (4)$$

$$\mathcal{J}: 2 = -0.523 \cos 19.1^\circ w_{BC} + v_{rel} \sin 19.1^\circ$$

$$v_{rel} = \frac{2 + 0.523 \cos 19.1^\circ w_{BC}}{19.1^\circ} \quad (5)$$

(6) into (4)

$$3.46 = 0.523 \sin 19.1^\circ w_{BC} + \left(\frac{2 + 0.523 \cos 19.1^\circ w_{BC}}{19.1^\circ} \right)$$

$$w_{BC} = -1.429 \text{ rad/s}$$

CCW (opposite to what we assumed)

Then plug w_{BC} into (5)

$$v_{rel} = 3.93 \text{ m/s}$$

$$\vec{a}_A = \vec{\alpha}_B^0 + \vec{\omega}_B \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$$

$$\vec{a}_A = \begin{vmatrix} \uparrow & \uparrow & \uparrow \\ 0 & 0 & 5 \\ -0.2 \cos 60^\circ & 0.2 \sin 60^\circ & 0 \end{vmatrix} - 20^2 (-0.2 \cos 60^\circ + 0.2 \sin 60^\circ)$$

$$a_A = -0.366 \uparrow - 0.5 \uparrow + 40 \uparrow - 69.28 \uparrow$$

$$a_A = 39.134 \uparrow - 69.28 \uparrow$$

Everything back into (3)

$$\vec{a}_A = \vec{\alpha}_B^0 + \vec{\alpha}_{A,B} + 2\vec{\omega}_B \times \vec{v}_{A/B} + \vec{\omega}_B \times \vec{\omega}_B \times \vec{r}_{A/B} - \omega_{BC}^2 \vec{r}_{A/B} \quad (6)$$

$$39.134 \uparrow - 69.28 \uparrow = a_{rel} \cos 19.1^\circ \uparrow + a_{rel} \sin 19.1^\circ + 2 \begin{vmatrix} \uparrow & \uparrow & \uparrow \\ 0 & 0 & -1.429 \end{vmatrix}$$

$$+ \begin{vmatrix} \uparrow & \uparrow & \uparrow \\ 0 & 0 & -1.429 \end{vmatrix} = (-1.429)^2 \cdot (0.523 \cos \theta - 0.523 \sin \theta)$$

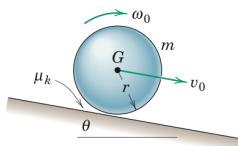
$$0.52362 \text{ rad} \quad 0.523 \text{ rad/s}$$

Equate τ and γ components to get a_{nl} and α_{bc}

$$a_{nl} = 15.22 \text{ m/s}^2$$

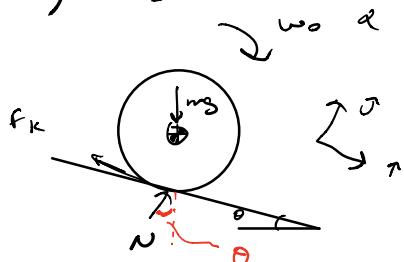
$$\alpha_{bc} = 17^\circ \text{ rad/s}$$

- 6/188** The homogeneous sphere of mass m and radius r is projected along the incline of angle θ with an initial speed v_0 and no angular velocity ($\omega_0 = 0$). If the coefficient of kinetic friction is μ_k , determine the time duration t of the period of slipping. In addition, state the velocity v of the mass center G and the angular velocity ω at the end of the period of slipping.



1) Ref system: Cartesian
type of problem: Momentum

a) FBD



b) Momentum equations:

$$\int_{t_1}^{t_2} \epsilon \vec{F} dt = \vec{m v}_2 - \vec{m v}_1 \quad \text{Linear momentum}$$

$$(t_2 - t_1) \epsilon \vec{F}_{av} = \vec{m v}_2 - \vec{m v}_1$$

$$\epsilon F_y = m v_{y2} - m v_{y1} \quad (\text{not moving in the } y \text{ direction})$$

$$N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$\epsilon F_x = m v_{x2} - m v_{x1}$$

$$\epsilon (-F_k + mg \sin \theta) = m v_{x2} - m v_{x1}$$

$$F_k = \mu_k N$$

$$t(-\mu_k g \cos \theta) v_{x0} + t g \sin \theta = m v_x - \mu_k v_{x0} \quad f_{ik} = \mu_k g \cos \theta$$

$$t(-g \cos \theta) v_{x0} + g \sin \theta = v_x - v_{x0}$$

$$v = (g \sin \theta - \mu_k g \cos \theta) t + v_0 \quad (1)$$

$$\int_{t_1}^{t_2} \{M dt = I\omega_2 - I\omega_1\}$$

$$(I\omega_2 - I\omega_1)^2 M_G = I\omega_2 - I\omega_1^2$$

$$-\mu_k mg \cos \theta \cdot r t = -I\omega \rightarrow \omega \text{ is rotating in the clockwise direction so } (-)$$

$$\mu_k mg \cos \theta \cdot r t = I\omega$$

$$\mu_k mg \cos \theta \cdot r t = \frac{2}{5} m R^2 \omega \quad I \text{ for sphere} = \frac{2}{5} m R^2$$

$$\mu_k g \cos \theta t = \frac{2}{5} R \omega$$

$$\text{For no slip} \quad V_{cm} = \omega r$$

$$\omega = \frac{v}{r}$$

$$\mu_k g \cos \theta t = \frac{2}{5} \cdot \cancel{F} \cdot \cancel{x}$$

$$\mu_k g \cos \theta t = \frac{2}{5} v \quad (2)$$

(1) into (2)

$$\mu_k g \cos \theta t = \frac{2}{5} [(g \sin \theta - \mu_k g \cos \theta) t + v_0]$$

$$\frac{2}{5} \mu_k g \cos \theta t = \frac{2}{5} g \sin \theta t + \frac{2}{5} v_0$$

$$(\frac{2}{5} \mu_k g \cos \theta - \frac{2}{5} g \sin \theta) t = \frac{2}{5} v_0$$

$$\boxed{\begin{aligned} t &= \frac{2v_0}{\mu_k g \cos \theta - 2g \sin \theta} \\ &\Rightarrow \mu_k g \cos \theta - 2g \sin \theta \end{aligned}}$$

$$v = \frac{2}{5} (g \sin \theta - \mu_k g \cos \theta) \frac{2v_0}{(\mu_k g \cos \theta - 2g \sin \theta)} + v_0$$

$$\boxed{v = \frac{5v_0 \mu_k}{\mu_k - 2 \tan \theta}}$$

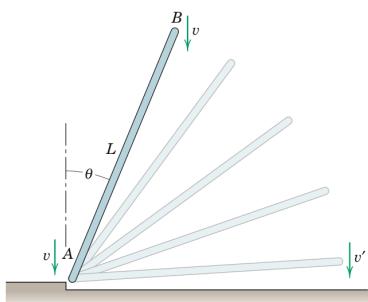
$$\boxed{w = -\dots \mu_k}$$

$$\frac{v_0}{r(\mu_k - 2 + \tan\theta)}$$

6/196 A uniform pole of length L , inclined at an angle θ with the vertical, is dropped and both ends have a velocity v as end A hits the ground. If end A pivots about its contact point during the remainder of the motion, determine the velocity v' with which end B hits the ground.

1) Ref system: Cartesian

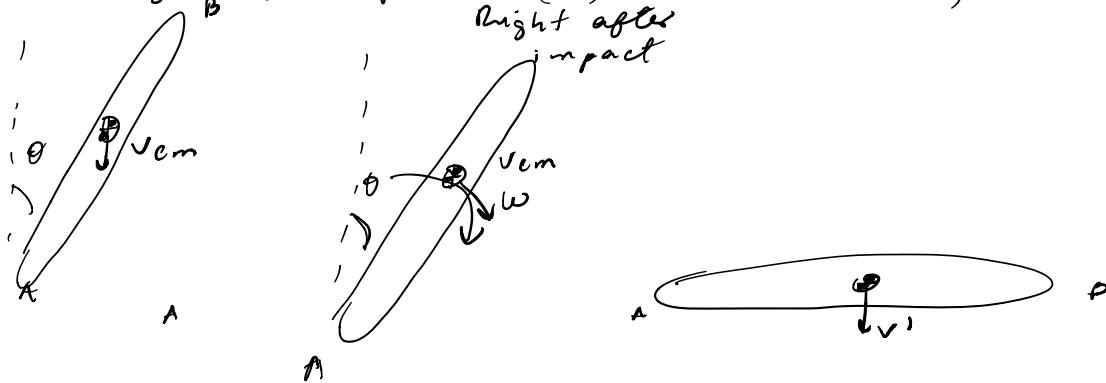
Type of problem: Momentum - Impact



Problem 6/196

2) FBD

(1) Right before impact (2) Right after impact (3)



3) Momentum Equations (conservation of angular momentum)

$$(H_o)_1 = (H_o)_2$$

$$m \times v_{cm} r + I_{cm} \omega = I_B \omega \rightarrow \text{rotation about a fixed point}$$

only translating initially (no rotation)

$$m \cdot v_{cm} \cancel{\frac{1}{2} \sin \theta} = \frac{1}{3} \cancel{\pi L^2} \omega$$

$\cancel{\frac{1}{2} \sin \theta}$
perpendicular
distance

$$\omega = \frac{3}{2} \frac{v_{\min} \theta}{L}$$

a) Energy conservation

s takes

$\xrightarrow{2}$ no translation

$$K_E = K_E(\text{translational}) + K_E(\text{rotational})$$

$$K_E = \frac{1}{2} I_a \omega^2$$

$$PE = mg \frac{L}{2} \cos \theta$$

$$K_E_2 + PE_2 = K_E_3 + PE_3$$

$$\frac{1}{2} I_a \omega^2 + mg \frac{L}{2} \cos \theta = \frac{1}{2} I_a \omega'^2$$

$$\frac{1}{2} \cdot \frac{1}{3} m L^2 \omega^2 + mg \frac{L}{2} \cos \theta = \frac{1}{2} \cdot \frac{1}{3} m L^2 \omega'^2$$

$$\frac{1}{3} \cancel{\omega^2} \left(\frac{3}{2} \frac{v_{\min} \theta}{L} \right)^2 + g L \cos \theta = \frac{1}{3} L^2 \omega'^2$$

$$\frac{3}{4} v^2 \sin^2 \theta + g L \cos \theta = \frac{1}{3} L^2 \omega'^2$$

$$\omega' = \left[\frac{3}{L^2} \left(\frac{3}{4} v^2 \sin^2 \theta + g L \cos \theta \right) \right]^{1/2}$$

$$v = \omega r$$

$$v' = \omega' r$$

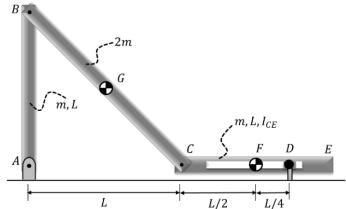
$$v' = \left[\frac{3}{L^2} \left(\frac{3}{4} v^2 \sin^2 \theta + g L \cos \theta \right) \right]^{1/2} \cdot r$$

$$v' = \sqrt{\frac{9}{4} v^2 \sin^2 \theta + 3 g L \cos \theta}$$

from Hw 7

Problem 5 (Practice – Exam type question)

The system starts from rest when a moment M is applied at A in the position shown, point D is pinned to the ground and slides in the smooth slot within bar CE .

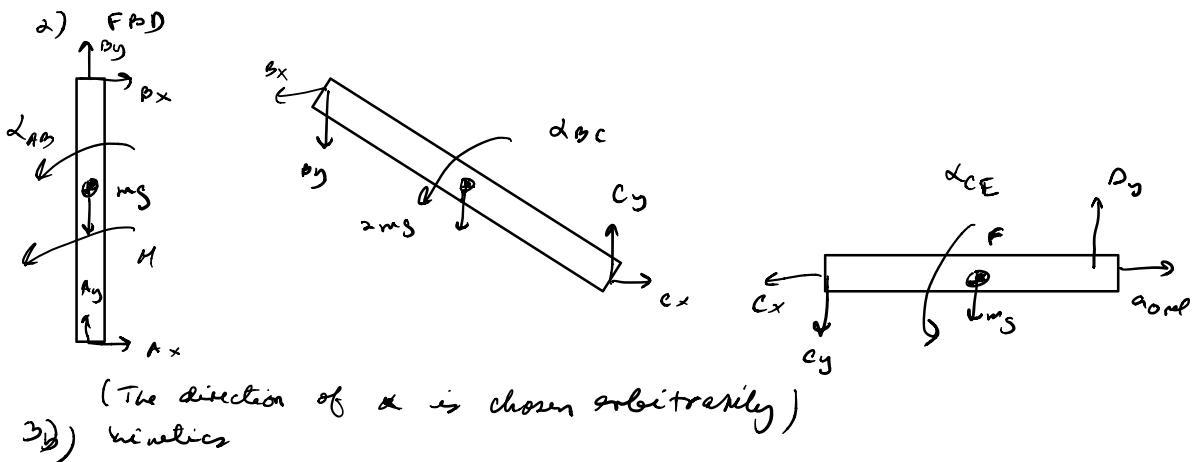


2

ME 2030: Problem Set 7

- (a) Draw a free body diagram of each bar (include a reference system and the directions of the angular accelerations of each bar and the relative acceleration at point D).
 (b) Find the forces at pins B , C and D at the instant the moment is applied (note that at this instant the center of mass of bar CE is at point F , and I_{CE} is the moment of inertia of bar CE about its center of mass).

1) Ref system: Cartesian
 type of problem: Newton/Euler, kinematics



3b) kinetics

Bar AB

$$\Sigma M_A : M - B_x \cdot L = I_A \alpha_{AB}$$

$$M - B_x \cdot L = \frac{1}{3} m L^2 \alpha_{AB} \quad (1)$$

Bar BC

$$\Sigma M_C : \frac{L}{2} (B_x + B_y) + \frac{L}{2} (C_x + C_y) = I_{BC} \alpha_{BC}$$

$$\frac{L}{2} (C_x + B_x + C_y + B_y) = \frac{1}{12} \cdot 2m (\sqrt{2}L)^2 \alpha_{BC} \quad (2)$$

$$\Sigma F_{Gx} : C_x - B_x = 2m a_{Gx} \quad (3)$$

$$\Sigma F_{Gy} : C_y - B_y - 2mg = 2m a_{Gy} \quad (4)$$

Bar CE

$$\sum M_F = \alpha_B \frac{L}{2} + \alpha_y \frac{L}{4} = I_{CE} \text{ des } (5)$$

$$\sum F_x : -C_x = m_a F_x \quad (6)$$

$$\sum F_y : \alpha_y - C_y - m_g = m_a F_y \quad (7)$$

4) kinematics:

$$\vec{a}_B = \vec{a}_A + \alpha_{AB} \times \vec{r}_{BA} \quad (\text{A can't move})$$

$$= \alpha_{AB} \hat{k} \times \vec{r}$$

$$= -L \alpha_{AB} \hat{k}$$

$$\vec{a}_B = \vec{a}_G + \vec{a}_{BC} \times \vec{r}_{BG}$$

$$= \alpha_{Gx} \hat{i} + \alpha_{Gy} \hat{j} + \alpha_{Gz} \hat{k} \times \left(-\frac{L}{2} \hat{i} + \frac{L}{2} \hat{j} \right)$$

Equalize \hat{i} and \hat{j}

$$\hat{i} : -L \alpha_{AB} = \alpha_{Gx} - \alpha_{BC} \frac{L}{2}$$

$$\hat{j} : 0 = \alpha_{Gy} - \alpha_{BC} \frac{L}{2}$$

$$\vec{a}_C = \vec{a}_B + \vec{a}_{BC} \times \vec{r}_{CB}$$

$$= -L \alpha_{AB} \hat{i} + \alpha_{BC} \hat{k} \times (L \hat{i} - L \hat{j})$$

$$L(\alpha_{BC} - \alpha_{AB}) \hat{i} + L \alpha_{BC} \hat{j} \quad (A)$$

$$\vec{a}_C = \vec{a}_F + \vec{a}_{CF} \times \vec{r}_{CF}$$

$$= \alpha_{Fx} \hat{i} + \alpha_{Fy} \hat{j} + \alpha_{Ce} \hat{k} \times -\frac{L}{2} \hat{i} \\ \alpha_{Fx} \hat{i} + \alpha_{Fy} \hat{j} - \alpha_{Ce} \frac{L}{2} \hat{j} \quad (B)$$

$$\vec{a}_D = \vec{a}_C + \vec{a}_{CD} + \vec{a}_{CF} \times \vec{r}_{DC} \quad (\text{D can't move})$$

$$\vec{a}_C = -\alpha_{CD} \hat{i} - \alpha_{Ce} \hat{k} \times \frac{3L}{4} \hat{i}$$

$$\vec{a}_C = -\alpha_{CD} \hat{i} - \alpha_{Ce} \cdot \frac{3L}{4} \hat{j} \quad (C)$$

Equating A = B and A = C

$$\hat{i} : L(\alpha_{BC} - \alpha_{AB}) = -\alpha_{Fx} \quad (10)$$

$$\hat{j} : \alpha_{BC} = \alpha_{Fy} - \frac{L}{2} \alpha_{Ce} \quad (11)$$

$$\hat{i} : L(\alpha_{BC} - \alpha_{AB}) = -\alpha_{CD} \quad (12)$$

$$\hat{j} : L \alpha_{BC} = -\alpha_{Ce} \frac{3L}{4} \quad (13)$$

(13) Equations 1-13 and 13 unknowns

$\alpha_{BC}, \alpha_{AB}, \alpha_{Ce}, \alpha_{Ce}, \alpha_{Fx}, \alpha_{Gy}, \alpha_{Fy}, \alpha_{Fy}, \alpha_{Gx}, b_x, b_y, c_x, c_y, \rho_y, \alpha_{CD}$