

## 7.1 entropy (nonconserved property)

• Clausius inequality:  $\oint \frac{\delta Q}{T} \leq 0$

cyclic integral  
(integration performed over entire cycle)

note:  $\delta Q$  is differential heat transfer between system and surroundings

- valid for reversible and irreversible cycles

$$\left( \oint \frac{\delta Q}{T} \right)_{\text{int, rev}} = 0$$

(internally reversible)

note:  $T$  is thermodynamic temperature at the boundary

- entropy ( $S$ ) is a thermodynamic property

$$ds = \left( \frac{\delta Q}{T} \right)_{\text{int, rev}} \quad \text{units: kJ/K}$$

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} \Big|_{\text{int, rev}}$$

- entropy is always the same between 2 states

## 7.2 increase of entropy principle

- entropy is always increasing in an isolated system that undergoes irreversible process

↑  
(closed and adiabatic)

$$\hookrightarrow ds \geq \frac{\delta Q}{T}$$

- some entropy is generated during irreversible processes

note: for reversible processes

$$ds = \frac{\delta Q}{T}$$

- entropy generation ( $S_{\text{gen}}$ )

$$\left[ \Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}} \right]$$

- irrev process:  $S_{\text{gen}} > 0$

- rev process:  $S_{\text{gen}} = 0$

↑ always positive or zero (value depends on process - not property of system)

- for an isolated system (closed and adiabatic)

$$\frac{\delta Q}{T} = 0 \rightarrow \Delta S = S_{\text{gen}} \geq 0 \rightarrow S_{\text{gen}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}}$$

increase of entropy principle: the entropy of an isolated system during a process always increases, or in the case of a reversible process remains constant

$$S_{\text{gen}} \begin{cases} > 0 & \text{irreversible} \\ = 0 & \text{reversible} \\ < 0 & \text{impossible} \end{cases}$$

note: larger  $S_{\text{gen}}$  means more irreversible



- entropy of the universe is always increasing

### 7.3 entropy change of pure substances

- entropy change of specified mass (closed sys) during a process

$$\Delta S = m \Delta s = m(s_2 - s_1)$$

- entropy used as "coordinate" on T-S and h-S diagrams

### 7.4 isentropic processes

- entropy remains const in isentropic processes

$$\Delta S = 0 \rightarrow S_2 = S_1$$

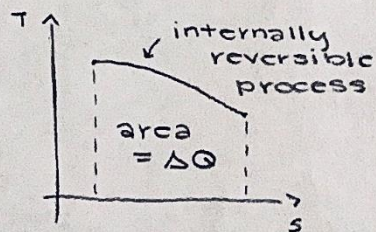
- entropy can be changed by

① heat transfer

② irreversibilities

} isentropic process is reversible and adiabatic

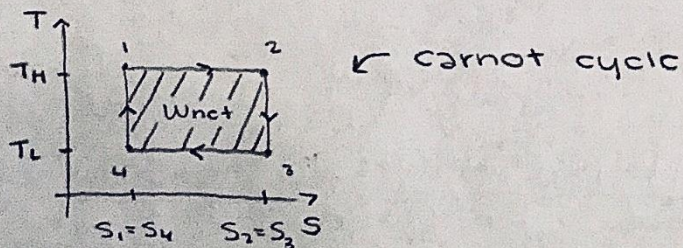
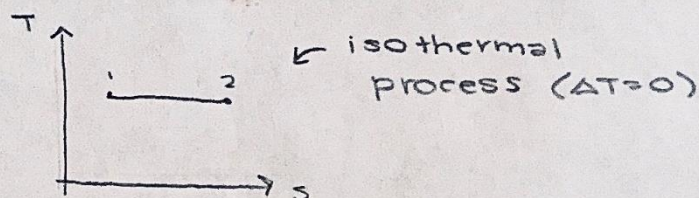
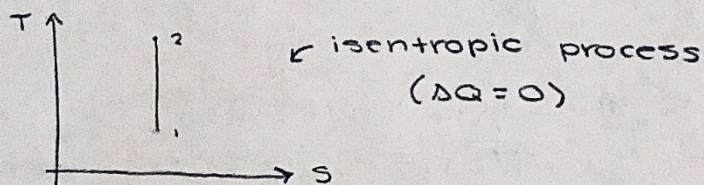
### 7.5 property diagrams involving entropy



note: does not work w irreversible processes

(area under has no meaning for irrev process)

$$\left[ Q_{\text{int, rev}} = \int_1^2 T ds \right] \text{ units: kJ}$$



### 7.6 what is entropy?

- entropy is a measure of molecular disorder/randomness
- more disorder = increased entropy
- entropy is lowest in solid phase and highest in gas phase
- equilibrium = max entropy



Boltzmann relation:  $S = k \ln(W)$   
↑ plank's constant

- 3<sup>rd</sup> Law of Thermodynamics: the entropy of a pure crystalline substance at absolute zero temp is zero
- absolute entropy: entropy determined relative to this point

### 7.7 the Tds relations

1<sup>st</sup> Gibbs equation:  $Tds = du + PdV$   
2<sup>nd</sup> Gibbs equation:  $Tds = dh - VdP$  } valid for both reversible and irreversible processes and closed and open systems

relevant supporting equations:

$$\left. \begin{aligned} du &= C_v dT \\ dh &= C_p dT \\ Pv &= RT \end{aligned} \right\} \text{ideal gas}$$

↳ 0.287

note: Gibbs equations assume only work is boundary work

### 7.8 entropy change of liquids and solids

- can be approximated as incompressible:  $dV \approx 0$

$$\left[ \Delta S = C_{avg} \ln \left( \frac{T_2}{T_1} \right) \right]$$

↑ avg specific heat over given temp interval

if isentropic:

$$\Delta S = 0 = C_{avg} \ln \left( \frac{T_2}{T_1} \right) \rightarrow T_2 = T_1 \text{ then also isothermal}$$

### 7.9 entropy change of ideal gases

const specific heats (approx analysis)

$$\left[ \begin{aligned} \Delta S &= C_{v,avg} \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{V_2}{V_1} \right) \\ \Delta S &= C_{p,avg} \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \end{aligned} \right] \text{ units: kJ/kg} \cdot \text{K}$$

variable specific heats (exact analysis)

$$\left[ \Delta S = S_2^\circ - S_1^\circ - R \ln \left( \frac{P_2}{P_1} \right) \right] \leftarrow \text{DON'T need}$$

↑ ↑  
S° at T<sub>2</sub> and T<sub>1</sub>  
in the property tables



- isentropic processes of ideal gases  
const specific heat (approx analysis)

$$\textcircled{1} \quad \frac{T_2}{T_1} = \left(\frac{e_2}{e_1}\right)^{k-1}$$

$$\textcircled{2} \quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\textcircled{3} \quad \frac{P_2}{P_1} = \left(\frac{e_2}{e_1}\right)^k$$

← isentropic relations ( $k = c_p/c_v$ )

← (use absolute temps)

note: these give  $T_s$  values

note:  $k$  is based on average temps in this equation

variable specific heats (exact analysis)

$$\left[0 = S_2^\circ - S_1^\circ - R \ln\left(\frac{P_2}{P_1}\right)\right] \quad \leftarrow \text{DON'T need}$$

**7-24** During the isothermal heat addition process of a Carnot cycle, 900 kJ of heat is added to the working fluid from a source at 400°C. Determine (a) the entropy change of the working fluid, (b) the entropy change of the source, and (c) the total entropy change for the process.

a) working fluid

$$\Delta S_{\text{fluid}} = \frac{Q_{\text{fluid}}}{T_{\text{fluid}}} = \frac{Q_{\text{in}}}{T_{\text{fluid}}} = \frac{900 \text{ kJ}}{673 \text{ K}} \longrightarrow \boxed{\Delta S_{\text{fluid}} = 1.3373 \frac{\text{kJ}}{\text{K}}}$$

b) source

$$\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = -\frac{Q_{\text{out}}}{T_{\text{source}}} = -\frac{900 \text{ kJ}}{673 \text{ K}} \longrightarrow \boxed{\Delta S_{\text{source}} = -1.3373 \frac{\text{kJ}}{\text{K}}}$$

c) total process

$$\Delta S_{\text{total}} = \Delta S_{\text{fluid}} + \Delta S_{\text{source}} \longrightarrow \boxed{\Delta S_{\text{total}} = 0 \frac{\text{kJ}}{\text{K}}}$$

← knew this bc entropy change of Carnot cycle is zero. ✓



**7-23** A completely reversible heat pump produces heat at a rate of 300 kW to warm a house maintained at 24°C. The exterior air, which is at 7°C, serves as the source. Calculate the rate of entropy change of the two reservoirs and determine if this heat pump satisfies the second law according to the increase of entropy principle.

$$T_H = 24^\circ\text{C} = 297\text{K} \quad \dot{Q}_H = 300\text{kW}$$

$$T_L = 7^\circ = 280\text{K}$$

$$\text{COP}_{\text{HP,rev}} = \frac{T_H}{T_H - T_L} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (280\text{K}/297\text{K})}$$

$$\text{COP}_{\text{HP,rev}} = 17.47$$

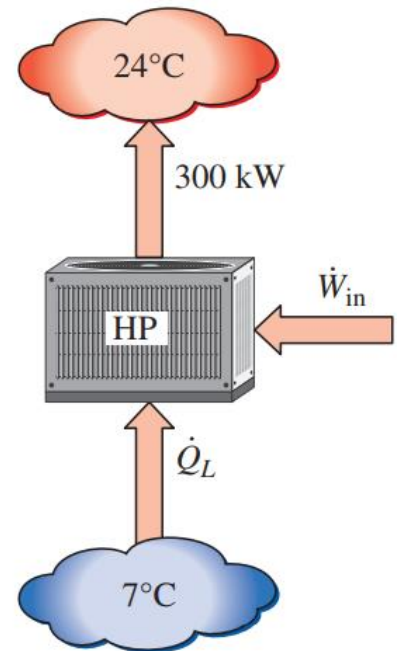
$$\dot{W}_{\text{in}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP}}} = \frac{300\text{kW}}{17.47} \rightarrow \dot{W}_{\text{in}} = 17.17\text{kW}$$

$$\dot{Q}_H = \dot{W}_{\text{in}} + \dot{Q}_L \rightarrow \dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{in}} = 300\text{kW} - 17.17\text{kW} \rightarrow \dot{Q}_L = 282.8\text{kW}$$

$$\Delta\dot{S}_H = \frac{\dot{Q}_H}{T_H} = \frac{300\text{kW}}{297\text{K}} = \boxed{1.01\text{kW/K} = \Delta\dot{S}_H}$$

$$\Delta\dot{S}_L = -\frac{\dot{Q}_L}{T_L} = -\frac{282.8\text{kW}}{280\text{K}} = \boxed{-1.01\text{kW/K} = \Delta\dot{S}_L}$$

$$\Delta\dot{S}_{\text{total}} = \Delta\dot{S}_H + \Delta\dot{S}_L = 1.01\text{kW/K} - 1.01\text{kW/K} = \boxed{0\text{kW/K} = \Delta\dot{S}_{\text{total}}}$$





# Thermo Final

## 7.10 Reversible, steady flow work

- $W_{\text{rev, in}} = \int_1^2 \nu dP$

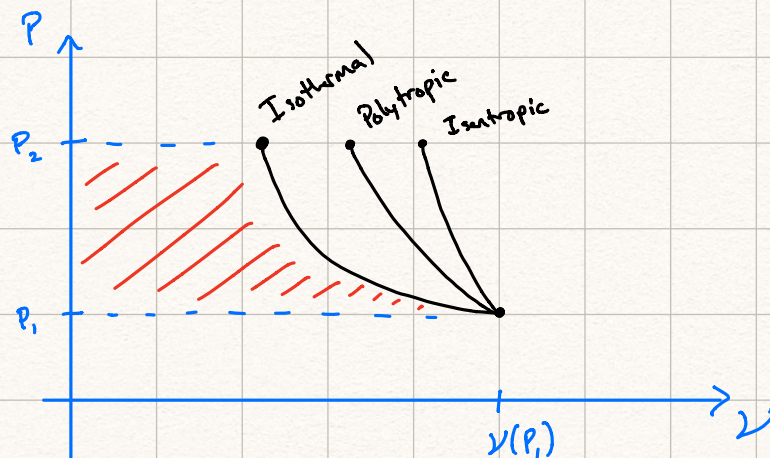
- minimize work input  $\rightarrow$  minimize  $\nu$

- cool the fluid

- 1 Isentropic (no cooling)

- 2 Polytropic (some cooling)

- 3 Isothermal (most cooling)





## Isentropic Efficiency for steady flow devices

### Turbine

$$\eta_T = \frac{\text{Isentropic turbine work}}{\text{Turbine work}} = \frac{w_a}{w_s} \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

### Compressor

$$\eta_c = \frac{\text{Isentropic compressor work}}{\text{Compressor work}} = \frac{w_s}{w_a} \approx \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

### Nozzle

$$\eta_n = \frac{\text{KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{(V_{2a})^2}{(V_{2s})^2} \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

## 7.13 Entropy Balance

$$S_{in} - S_{out} + S_{gen} = \Delta S_{system}$$

$$\bullet \text{ Entropy by heat transfer } S = \frac{Q}{T}$$

$$\bullet \text{ Entropy by mass flow } \dot{S} = \dot{m} s$$

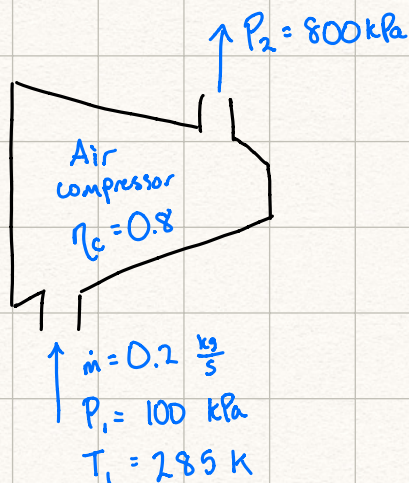


Ex.  
7-15

### EXAMPLE 7-15

### Effect of Efficiency on Compressor Power Input

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine (a) the exit temperature of air and (b) the required power input to the compressor.



a Find  $T_2$

$$\eta_c = \frac{\text{Isentropic Work Input}}{\text{Actual Work Input}} = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_c = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_c = \frac{c_p T_{2s} - c_p T_1}{c_p T_{2a} - c_p T_1}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_{2a} - T_1}$$

$$\Rightarrow T_{2,a} = T_1 + \frac{T_{2s} - T_1}{\eta_c}$$

Need to find  $T_{2s}$



$$\bullet \quad \frac{P_2}{P_1} = \left( \frac{T_{2s}}{T_1} \right)^{\frac{\kappa}{\kappa-1}}$$

$$\bullet \quad T_{2s} = \left( \frac{P_2}{P_1} \right)^{\frac{\kappa-1}{\kappa}} T_1$$

$$= \left( 8^{\frac{1}{1.4}} \right) (285 \text{ K})$$

$$T_{2s} = 516.26 \text{ K}$$

$$\bullet \quad T_{2,a} = T_1 + \frac{T_{2s} - T_1}{\eta_c}$$

$$= 285 \text{ K} + \frac{516.26 \text{ K} - 285 \text{ K}}{0.8}$$

$$\boxed{T_{2,a} = 574.1 \text{ K}}$$

$\bullet$  Find power input  
 $\Rightarrow$  Energy Balance

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m} \theta_{in} = \dot{m} \theta_{out}$$

$$\dot{W}_{in} = \dot{m} (\theta_{out} - \theta_{in})$$



$$\dot{W}_{in} = \dot{m} (h_2 - h_1)$$

$$\dot{W}_{in} = \dot{m} c_p (T_2 - T_1)$$

$$\dot{W}_{in} = (0.2 \frac{\text{kg}}{\text{s}}) (1.011 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) (574.1 \text{ K} - 285 \text{ K})$$

$$\dot{W}_{in} = 58.5 \text{ kW}$$



## Thermo final review

### Section 9.1 Basic considerations in the analysis of power cycles

An ideal cycle is when the actual cycle is stripped of all the internal irreversibility and complexities, we end up with a cycle that resembles the actual cycle closely but is made up totally of internally reversible processes. The conclusions reached from the analysis of ideal cycles are also applicable to actual cycles, however, the numerical values are not necessarily representative of the actual cycles.

Heat engines are designed for the purpose of converting thermal energy to work, and their performance is expressed in terms of the thermal efficiency, which is the ratio of the net work produced by the engine to the total heat input:

$$\eta_{th} = \frac{W_{net}}{Q_{in}} \quad \text{or} \quad \eta_{th} = \frac{w_{net}}{q_{in}}$$

Heat engines that operate on a totally reversible cycle, such as the Carnot cycle, have the highest thermal efficiency of all heat engines operating between the same temperature levels. That is, nobody can develop a cycle more efficient than the Carnot cycle.

The idealizations and simplifications commonly employed in the analysis of power cycles can be summarized as follows:

1. The cycle does not involve any friction. Therefore, the working fluid does not experience any pressure drop as it flows in pipes or devices such as heat exchangers.
2. All expansion and compression processes take place in a quasiequilibrium manner.
3. The pipes connecting the various components of a system are well insulated, and heat transfer through them is negligible.
4. Neglecting the changes in kinetic and potential energies of the working fluid is a reasonable assumption except where the changes in kinetic energy are significant for nozzles and diffusers, which are specifically designed to create large changes in velocity.

### Carnot cycle: Ideal heat engine. Section 9.2

-Composed of four totally reversible processes:

-Isothermal heat addition, isentropic expansion, isothermal heat rejection, and isentropic compression.

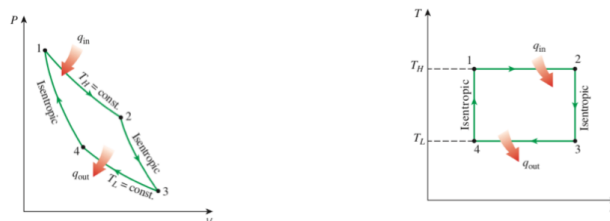
- Isothermal heat addition (**Reversible Isothermal Expansion**) (process 1-2,  $TH = \text{constant}$ ). Initially (state 1), the temperature of the gas is  $TH$  and the cylinder head is in close contact with a source at temperature  $TH$ . The gas is allowed to expand slowly, doing work on the surroundings. As the gas expands, the temperature of the gas tends to decrease. But as soon as the temperature drops by an infinitesimal amount  $dT$ , some heat is transferred from the



reservoir into the gas, raising the gas temperature to  $T_H$ . Thus, the gas temperature is kept constant at  $T_H$ . Since the temperature difference between the gas and the reservoir never exceeds a differential amount  $dT$ , this is a reversible heat transfer process. It continues until the piston reaches position 2. The amount of total heat transferred to the gas during this process is  $Q_H$ .

- Isentropic expansion (**Reversible Adiabatic Expansion**) (process 2-3, temperature drops from  $T_H$  to  $T_L$ ). At state 2, the reservoir that was in contact with the cylinder head is removed and replaced by insulation so that the system becomes adiabatic. The gas continues to expand slowly, doing work on the surroundings until its temperature drops from  $T_H$  to  $T_L$  (state 3). The piston is assumed to be frictionless and the process to be quasi-equilibrium, so the process is reversible as well as adiabatic.
- Isothermal heat rejection (**Reversible Isothermal Compression**) (process 3-4,  $T_L = \text{constant}$ ). At state 3, the insulation at the cylinder head is removed, and the cylinder is brought into contact with a sink at temperature  $T_L$ . Now the piston is pushed inward by an external force, doing work on the gas. As the gas is compressed, its temperature tends to rise. But as soon as it rises by an infinitesimal amount  $dT$ , heat is transferred from the gas to the sink, causing the gas temperature to drop to  $T_L$ . Thus, the gas temperature remains constant at  $T_L$ . Since the temperature difference between the gas and the sink never exceeds a differential amount  $dT$ , this is a reversible heat transfer process. It continues until the piston reaches state 4. The amount of heat rejected from the gas during this process is  $Q_L$ .
- Isentropic compression (**Reversible Adiabatic Compression**) (process 4-1, temperature rises from  $T_L$  to  $T_H$ ). State 4 is such that when the low-temperature reservoir is removed, the insulation is put back on the cylinder head, and the gas is compressed in a reversible manner, so the gas returns to its initial state (state 1). The temperature rises from  $T_L$  to  $T_H$  during this reversible adiabatic compression process, which completes the cycle.

-The P-v and T-s diagrams of a Carnot cycle



-The Carnot cycle can be executed in a closed system (a piston–cylinder device) or a steady-flow system (utilizing two turbines and two compressors) and either a gas or a vapor can be utilized as the working fluid.

-Is the most efficient cycle that can be executed between a heat source at temperature  $T_H$  and a sink at temperature  $T_L$

- Its thermal efficiency is expressed as

$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$



- The real value of the Carnot cycle comes from its being a standard against which the actual or the ideal cycles can be compared. (Reversible isothermal heat transfer is very difficult to achieve because it would require very large heat exchangers and it would take a very long time. Therefore, it is not practical to build an engine that would operate on a cycle that closely approximates the Carnot cycle)

-The thermal efficiency of the Carnot cycle is a function of the sink and source temperatures only.

-The thermal efficiency relation for the Carnot cycle conveys an important message that is equally applicable to both ideal and actual cycles: Thermal efficiency increases with an increase in the average temperature at which heat is supplied to the system or with a decrease in the average temperature at which heat is rejected from the system.

The highest temperature in the cycle is limited by the maximum temperature that the components of the heat engine, such as the piston or the turbine blades, can withstand. The lowest temperature is limited by the temperature of the cooling medium utilized in the cycle such as a lake, a river, or the atmospheric air.



## 9.8 Brayton Cycle

- ideal cycle for gas-turbine engines
- made up of 4 internally reversible processes

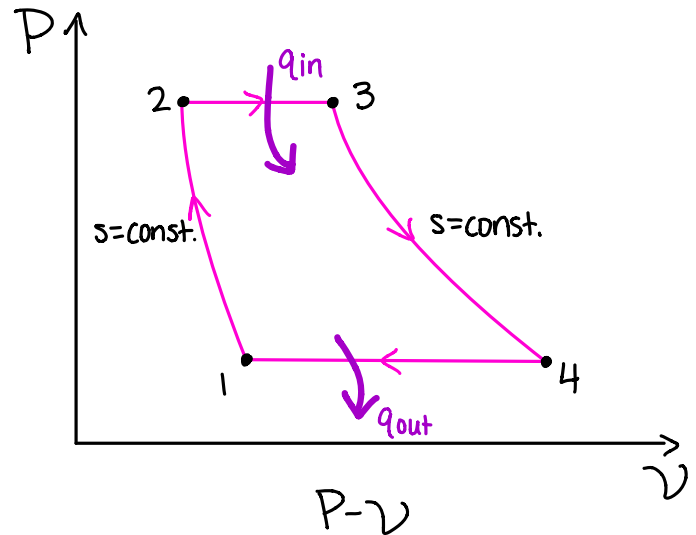
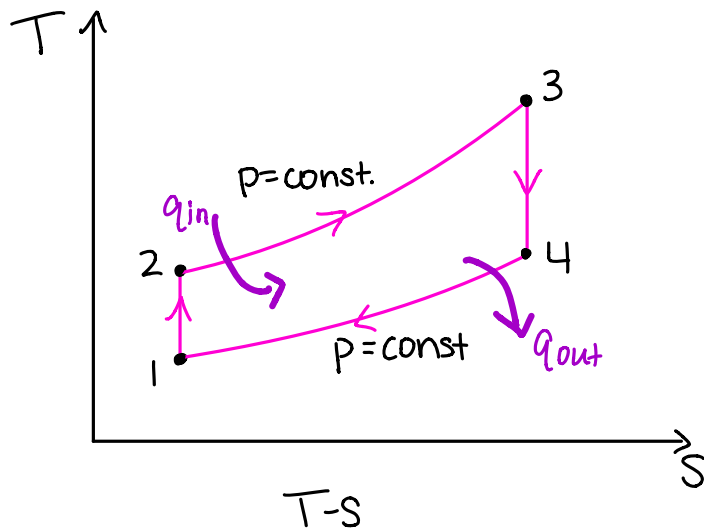
### Cycle Processes:

1-2: isentropic compression (in a compressor)

2-3: constant-pressure heat addition

3-4: isentropic expansion (in a turbine)

4-1: constant-pressure heat rejection



### Thermal Efficiency:

$$\eta_{th, \text{Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}}, \quad r_p \equiv \frac{P_2}{P_1}$$

$r_p$ : pressure ratio

$k$ : specific heat ratio

### Backwork Ratio:

$$r_{bw} = \frac{W_{comp}}{W_{turb}} = \frac{W_{in}}{W_{out}}$$

$$\eta_{th, B} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$



## Ex. 9-5

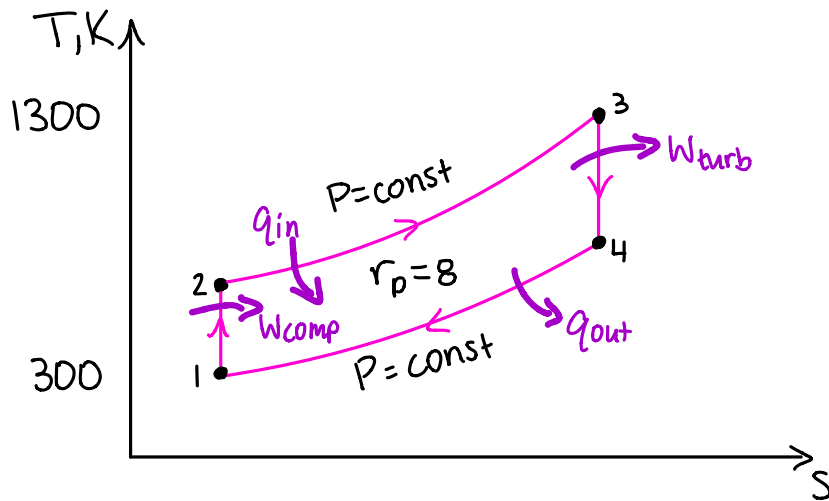
### EXAMPLE 9-5 The Simple Ideal Brayton Cycle

A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Utilizing the air-standard assumptions, determine (a) the gas temperature at the exits of the compressor and the turbine, (b) the back work ratio, and (c) the thermal efficiency.

$r_{bw}$

$T_2$

$T_4$



Assumptions:

1. Steady operating conditions
2. Air-standard assumptions applicable
3.  $\Delta KE$  &  $\Delta PE$  negligible
4. Specific heat variations w/ temp to be considered

a) 1-2 : isentropic comp.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{K-1/K} \quad @ 300 K : K = 1.4, C_p = 1.005 \text{ kJ/kgK}$$

$$\frac{T_2}{300} = (8)^{1.4-1/1.4} \Rightarrow \boxed{T_2 = 543.43 K}$$



3-4: isentropic exp.

$$\frac{P_2}{P_1} = \frac{P_3}{P_4}$$

$$\frac{T_3}{T_4} = \left( \frac{P_3}{P_4} \right)^{K-1/K} \quad @ 1000K: K=1.336$$

$$\frac{1300}{T_4} = (8)^{1.336-1/1.336} \Rightarrow \boxed{T_4 = 770.6 K}$$

b)  $r_{bw} = \frac{W_{comp}}{W_{turb}}$

$$W_{comp} = h_2 - h_1 = C_p(T_2 - T_1) \quad @ T_{avg} = 421.7 K: C_p = 1.0165 \text{ kJ/kgK}$$
$$= 1.0165(543.43 - 300)$$

$$W_{comp} = 247.45 \text{ kJ}$$

$$W_{turb} = h_3 - h_4 = C_p(T_3 - T_4) \quad @ T_{avg} = 1035.3 K: C_p = 1.142 \text{ kJ/kgK}$$
$$= 1.142(1300 - 770.6)$$
$$= 604.57 \text{ kJ}$$

$\approx 1000 K$

$$\boxed{r_{bw} = \frac{247.45}{604.57} = 0.409} \quad (\text{book has } 0.403)$$

c)  $\eta_{th} = \frac{W_{net}}{Q_{in}}$

$$Q_{in} = h_3 - h_2 = C_p(T_3 - T_2) \quad @ T_{avg} \approx 900 K: C_p = 1.121 \text{ kJ/kgK}$$
$$= 1.121(1300 - 543.43)$$
$$= 848.11 \text{ kJ}$$

$C_p = 1.121 \text{ J/kgK}$



$$W_{\text{net}} = W_{\text{turb}} - W_{\text{comp}} = 604.57 - 247.45 \\ = 357.12 \text{ kJ}$$

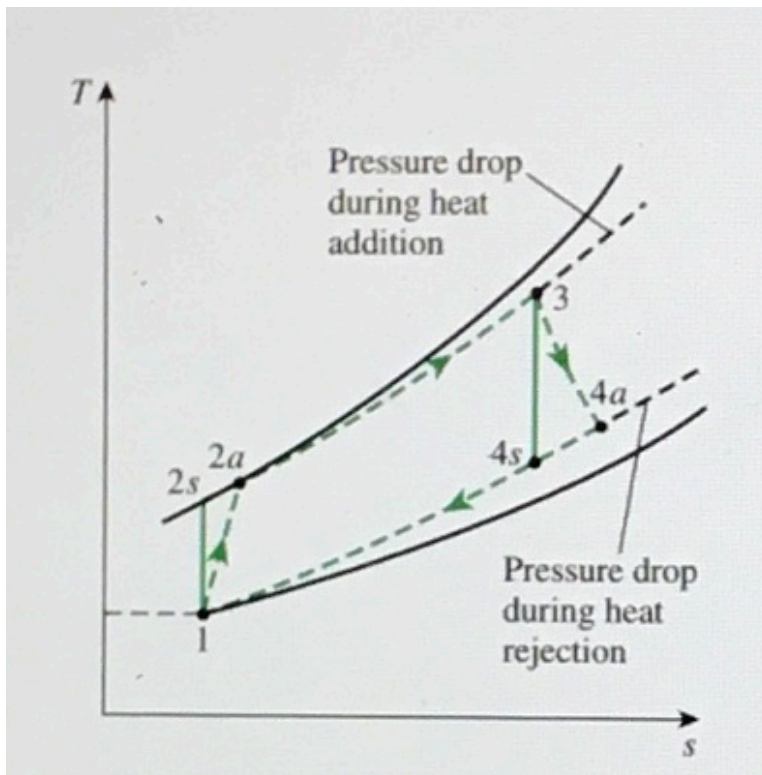
$$\eta_{\text{th}} = \frac{357.12}{848.11} = 0.421 \quad (\text{book has } 0.426)$$

## Deviation of Actual from Idealized

$$\eta_c = \frac{W_s}{W_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

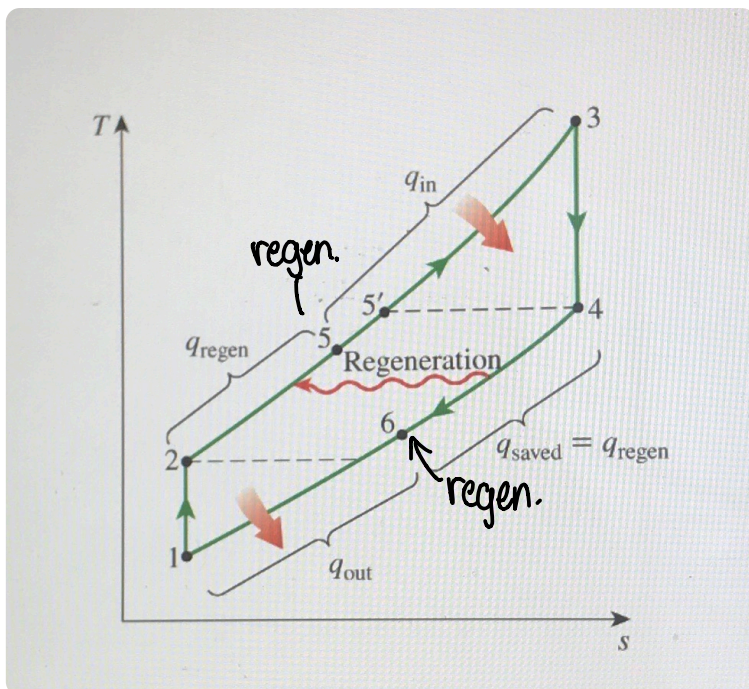
$$\eta_t = \frac{W_a}{W_s} = \frac{h_{4a} - h_3}{h_{4s} - h_3}$$

} isentropic  $\eta$





## 9.9 Brayton Cycle w/ Regeneration



highest temp. in  
regenerator:  $T_4$

actual heat transfer:  $q_{regen,act} = h_5 - h_2$

max. heat transfer:  $q_{regen,max} = h_{5'} - h_2 = h_4 - h_2$

effectiveness: extent to which a regenerator approaches  
an ideal regenerator

equation:

$$\epsilon = \frac{q_{regen,act}}{q_{regen,max}} = \frac{h_5 - h_2}{h_4 - h_2}$$

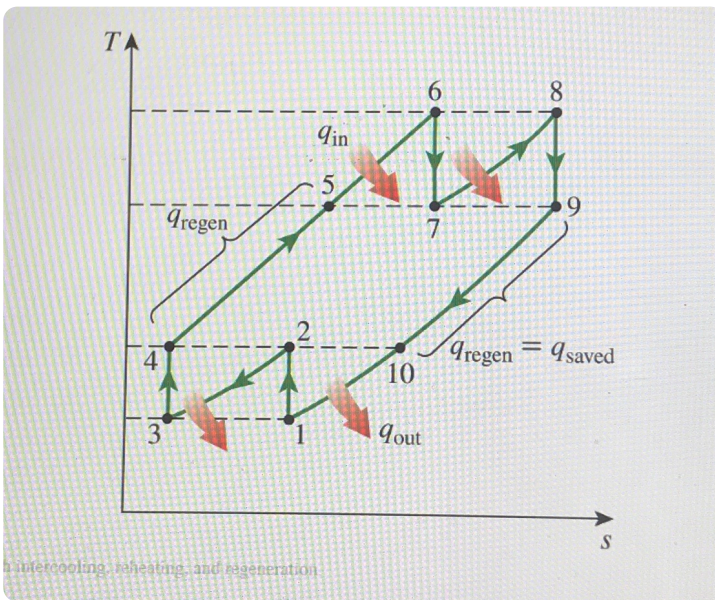
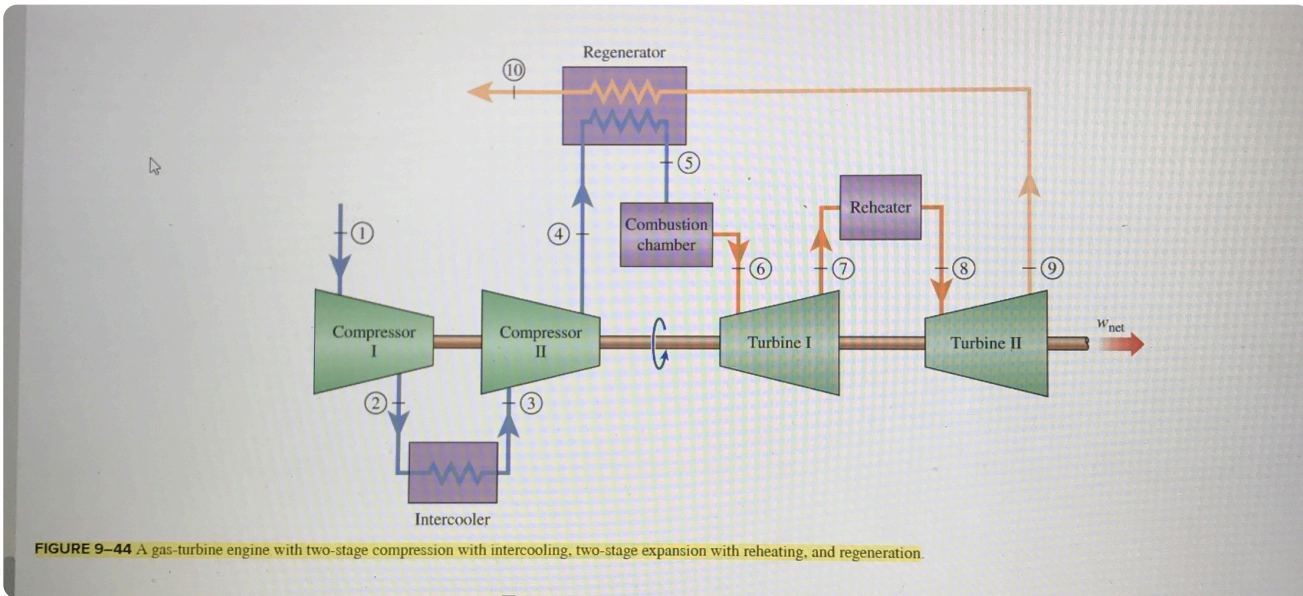
cold-air assumption:

$$\epsilon \cong \frac{T_5 - T_2}{T_4 - T_2}$$

thermal efficiency:

$$\eta_{th,regen} = 1 - \left( \frac{T_1}{T_3} \right) (r_p)^{k-1/k}$$

## 9.10 Brayton Cycle w/ Intercooling, Reheating, & Regen.



1→2: Compressor 1

2→3: intercooler

3→4: Compressor 2

4→5: regenerator

5→6: combustion chamber

6→7: turbine 1

7→8: reheater

8→9: turbine 2

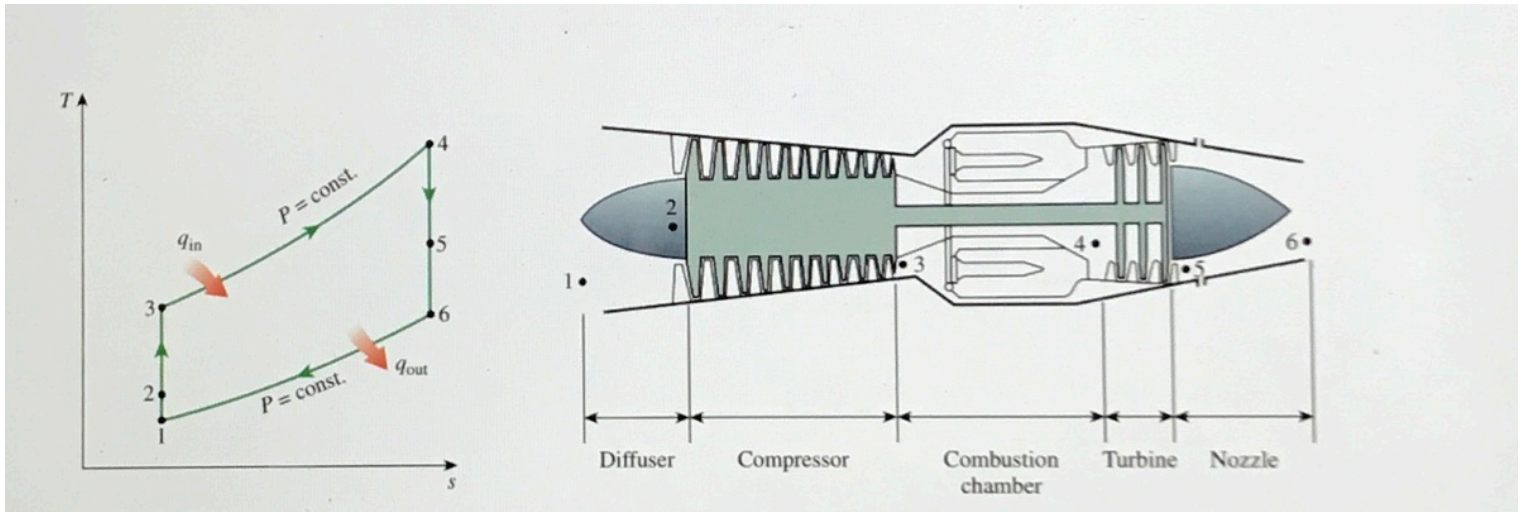
9→10: regenerator

key ideas:  $\frac{P_2}{P_1} = \frac{P_4}{P_3}$  &  $\frac{P_6}{P_7} = \frac{P_8}{P_9}$



## 9.11 Ideal Jet-Propulsion Cycles

- open cycle aircraft gas turbines operate on
- difference between jet-propulsion cycle & Brayton Cycle :
  - gases are expanded to the ambient pressure in the turbine (Brayton)
  - gases are expanded to a pressure such that the power produced by the turbine is just sufficient to drive the compressor & the auxiliary equipment (jet-propulsion)



net thrust developed by engine:

$$F = (\dot{m}V)_{\text{exit}} - (\dot{m}V)_{\text{inlet}} = \dot{m}(V_{\text{exit}} - V_{\text{inlet}})$$

propulsive power:

$$\dot{W}_p = F V_{\text{aircraft}} = \dot{m}(V_{\text{exit}} - V_{\text{inlet}}) V_{\text{aircraft}}$$

propulsive efficiency:

$$\eta_p = \frac{\dot{W}_p}{\dot{Q}_{\text{in}}}$$