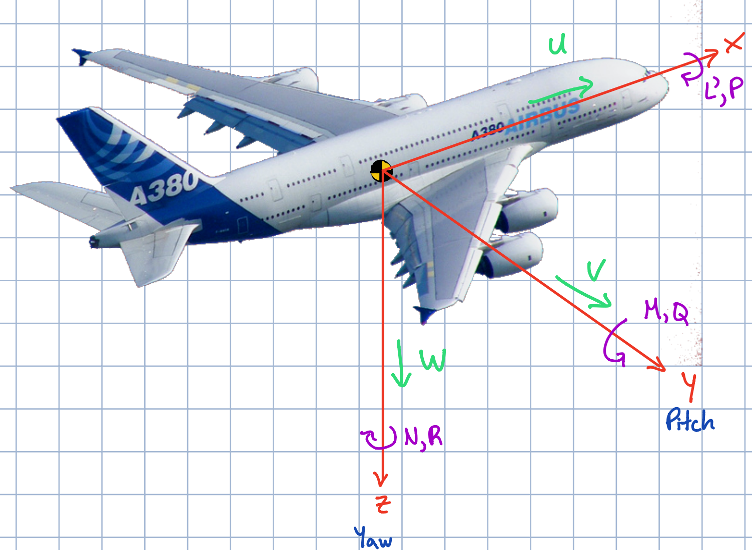
SGT AERO 2201 Final Review - Pre Midterm 2

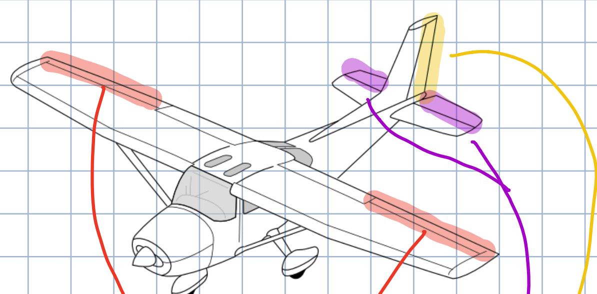
Chap. 7
- → Aircraft coordinate system
- → Basic controls
- → What is stability?
  - → Dynamic
  - → Static
- → L.S.S.
  - → conditions for L.S.S.
  - → N.P.
- → Directional S.S.

Chap. 9
- → Propellors
  - → Shape
  - → Efficiency
    - ↳ Advance Ratio
- → 4-stroke engine
- → Jets
  - → Eq^n
  - → Turbojet, Turbofan, Ramjet
- → Rocket engine
  - → Equations, spec. imp.
  - → Propellant
    - → Liquid
    - → Solid

Aircraft Basics



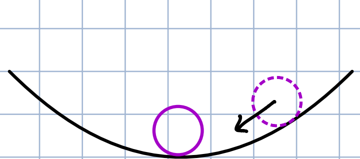
Direction	Translational Velocity	Angular Velocity	Moment
X	U	P	L'
Y	V	Q	M
Z	W	R	N



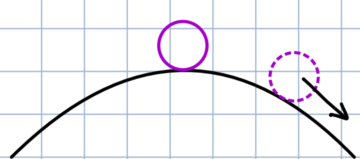
→ Motion about...
- ↳ x-axis is LATERAL (rolling) and is controlled w/ AILERONS
- ↳ y-axis is LONGITUDINAL (pitching) and is controlled w/ ELEVATORS
- ↳ z-axis is DIRECTIONAL (yawing) and is controlled w/ the RUDDER

Stability

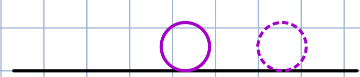
Static Stability: Initial tendency of a body after it is disturbed



Positive S.S.

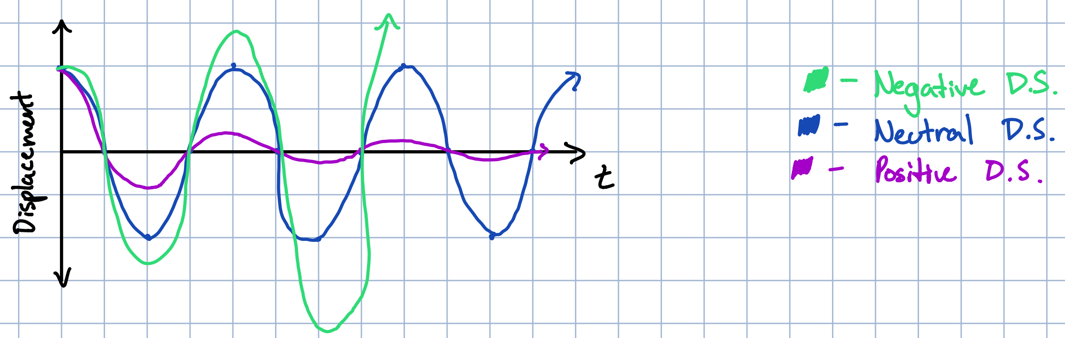


Negative S.S.



Neutral S.S.

Dynamic Stability: Tendency of body to return to equilibrium after a disturbance <u>over time</u>



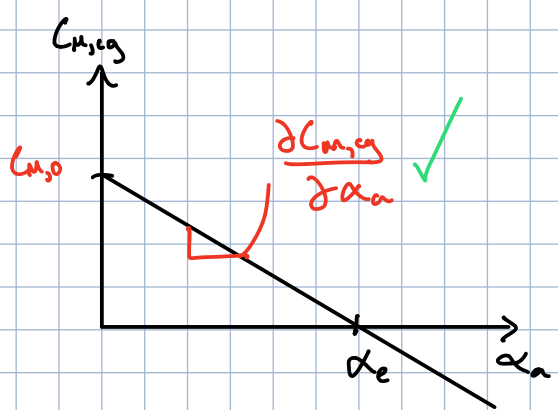
Longitudinal Static Stability

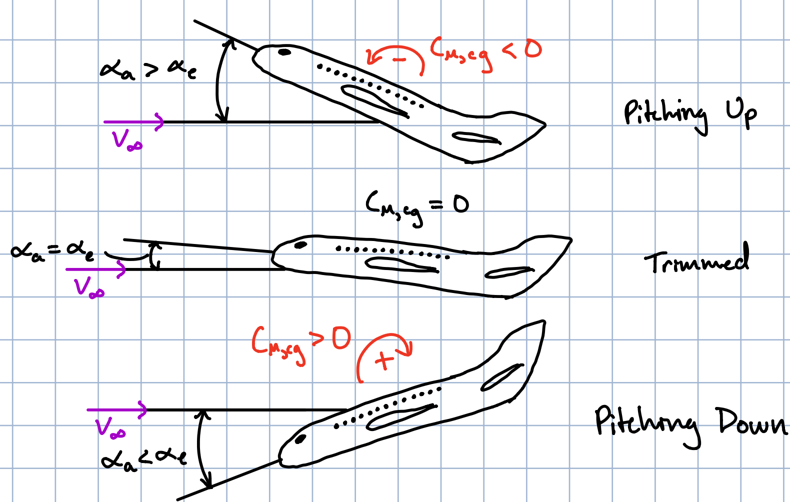
→ Initial tendency of aircraft to return to trim angle of attack after a disturbance

The gist: $\frac{\partial C_{M,cg}}{\partial \alpha_a} < 0$, $C_{M,0} > 0$

$\frac{\partial C_{M,cg}}{\partial \alpha_a} \equiv$ slope of pitching moment coefficient with varying α_a

$C_{M,0} \equiv$ moment coefficient at zero lift ($\alpha_a = 0°$)





Note: Most of the pitching moment is due to lift generated by the tail (or possibly canard)

Important Equations

$$\boxed{C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}\left(h - h_{ac_{wb}}\right) - V_H C_{L,t}} \quad 7.24$$

Or in terms of angle of attack...

$$\boxed{C_{M,cg} = C_{M,ac_{wb}} + a\alpha_a\left[h - h_{ac_{wb}} - V_H \frac{a_t}{a_{wb}}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right] + V_H a_t\left(i_t + \varepsilon_0\right)} \quad 7.26$$

$$\boxed{C_{M,0} \equiv (C_{M,cg})_{L=0} = C_{M,ac_{wb}} + V_H a_t (i_t + \varepsilon_0)} \quad 7.27$$

$$\boxed{\frac{\partial C_{M,cg}}{\partial \alpha_a} = a\left[h - h_{ac_{wb}} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]} \quad 7.28$$

$C_{M,0} > 0$ for L.S.S.:

→ $C_{M,ac_{wb}} < 0$, so $V_H a_t (i_t + \varepsilon_0)$ must be > 0.

↳ V_H & a_t are positive and ε_0 is small so i_t must be > 0

$\frac{\partial C_{M,cg}}{\partial \alpha_a} < 0$ for L.S.S., h & V_H have the largest influence on $\frac{\partial C_{M,cg}}{\partial \alpha_a}$

Neutral Point

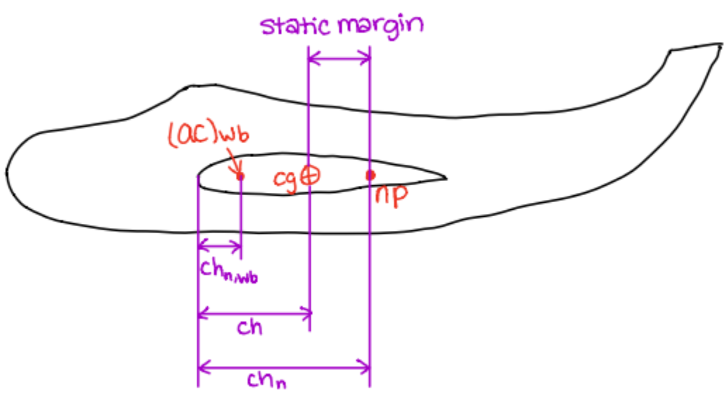
→ The point where $\frac{\partial C_{M,cg}}{\partial \alpha_a} = 0$

↳ Found by: $h = h_n$ & $\frac{\partial C_{M,cg}}{\partial \alpha_a} = 0 \implies \boxed{h_n = h_{ac_{wb}} + V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)}$ 7.30

→ This is a fixed quantity

→ CG should be FORWARD of the NP for positive L.S.S.

Static Margin



* Thank you Chloe for these notes

→ Static margin is defined as $h - h_n$

$$\boxed{\frac{\partial C_{M,cg}}{\partial \alpha_a} = -a(h - h_n) = -a \times \text{static margin}} \quad \text{Eq. 7.33}$$

→ Static margin must be positive for positive longitudinal static stability

Sections 7.12 - 7.14 not covered in this review for brevity...

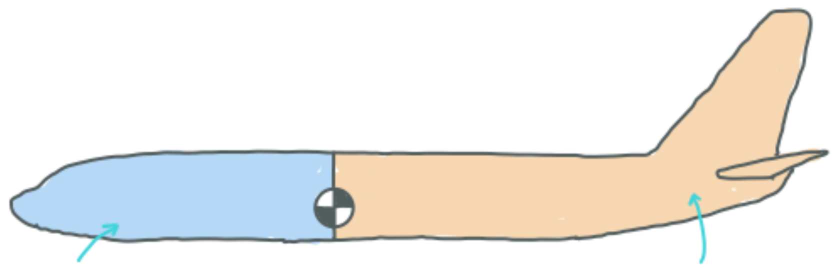
→ The gist is, changing things on the AC such as elevator deflection and CG location allow you alter trim characteristics

Directional Static Stability

★ Thanks Tammy

For directional static stability,

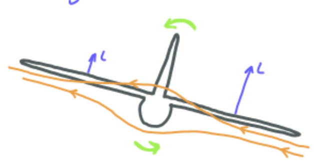
Keel surface forward of the CG < Keel surface aft of the CG



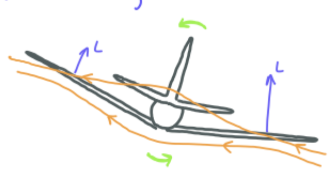
Lateral Static Stability

- Contributions by plane components

Straight, top mounted wing: Positive lateral static stability



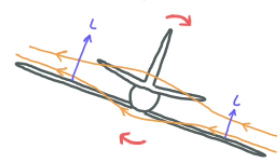
Dihedral wing: Positive lateral static stability



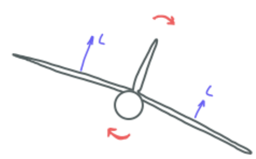
Swept wings: Positive lateral static stability

Right wing will have more lift than left wing → plane rolls left

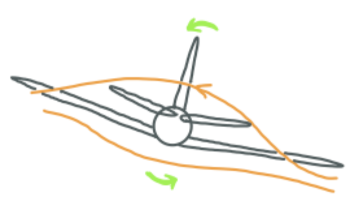
Straight, bottom mounted wing: Negative lateral static stability



Anhedral wing: Negative lateral static stability



Rudder: Positive lateral static stability



Propellers

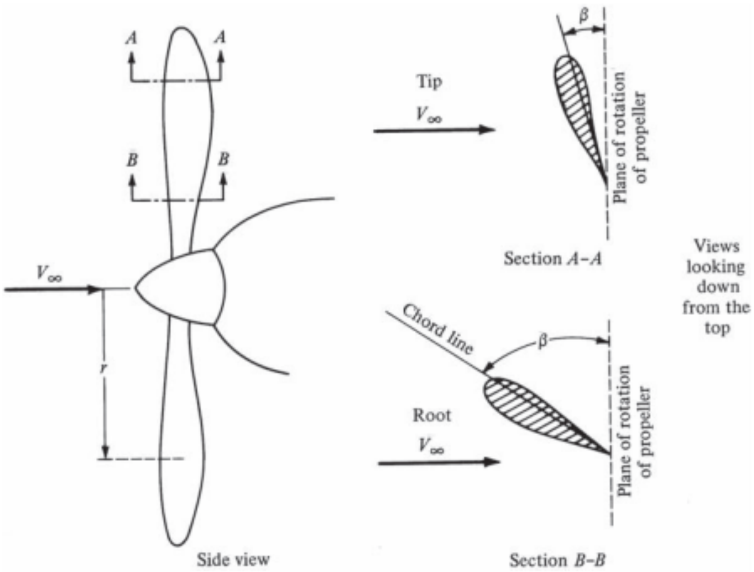


Figure 9.3 Illustration of propeller, showing variation of pitch along the blade.

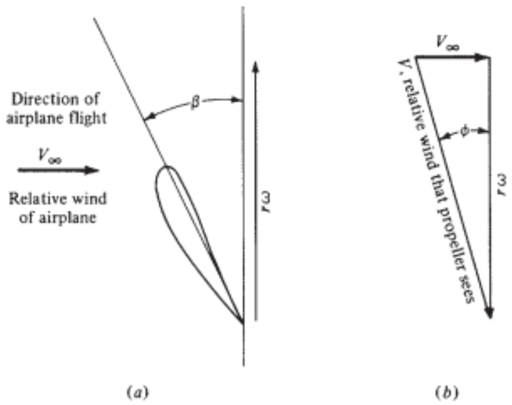


Figure 9.4 Velocity diagram for the flow velocity relative to the propeller.

$\beta \equiv$ pitch angle, often a function of distance from root, r

→ Note twist of blade
↳ blade moving faster near tip, needs smaller β to produce same lift

→ Efficiency: $\eta = \dfrac{P_A}{P}$

Power avail.

shaft brake power (power from engine)

$$\eta = \frac{P_A}{P} = \frac{T_A V_\infty}{P}$$

Thrust available is dependent on β, V_∞, n

rot. / sec.

$T_A(\beta, V_\infty, n)$, this can be nondimensionalized as the ADVANCE RATIO, J, where:

$$\boxed{J \equiv \frac{V_\infty}{nD}}$$

prop. diam.

→ $\eta < 1$, $P_A < P$: some power is always lost

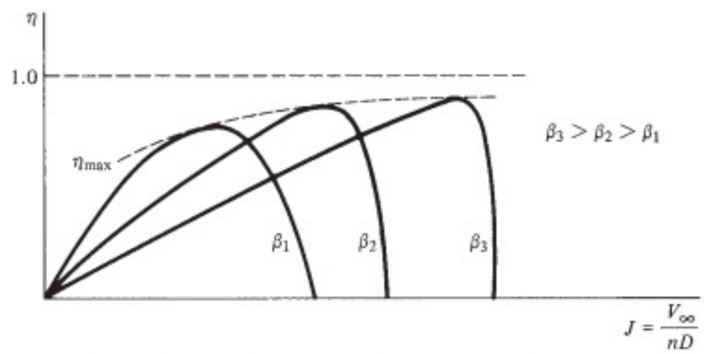
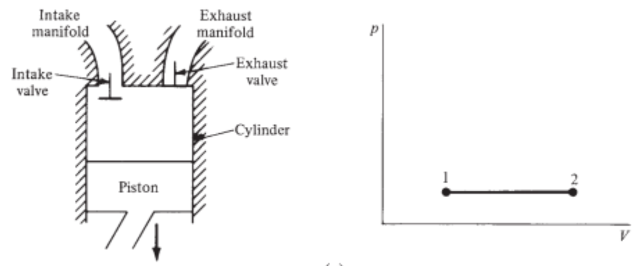


Figure 9.6 Propeller efficiency versus advance ratio. Note that D denotes propeller diameter.

→ At higher speed, relative wind angle increases, low β props start falling off in performance
↳ Typical η: $0.83 < \eta < 0.90$

→ Variable-pitch props are more efficient over wider range

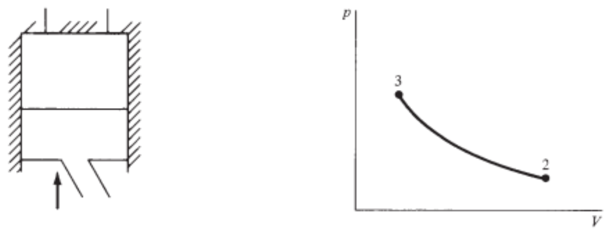
4-Stroke Engine



(a)

Intake

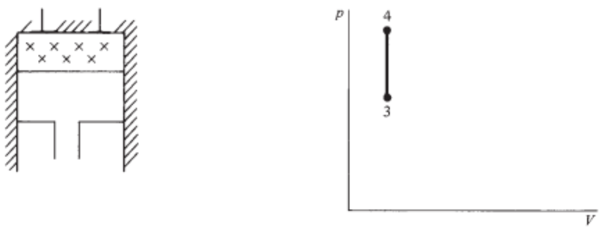
- → Taking in fresh air & fuel
- → Volume increases, pressure remains nearly unchanged



(b)

Compression

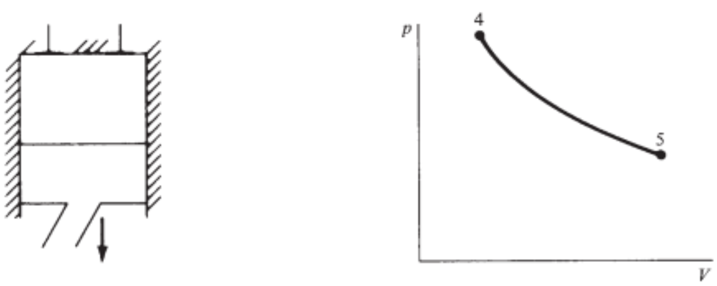
- → Uses momentum from crankshaft to SQUEEZE the fresh air & fuel
- → V↓, P↑, isentropic



(c)

Combustion

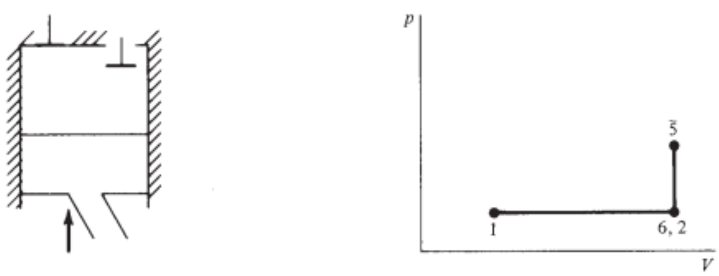
- → BANG! blows up the fuel (method depends on fuel)
- → P↑, Volume remains the same



(d)

Power

- → Air-fuel mixture expands isentropically and pushes cylinder down
- → P↓, V↑

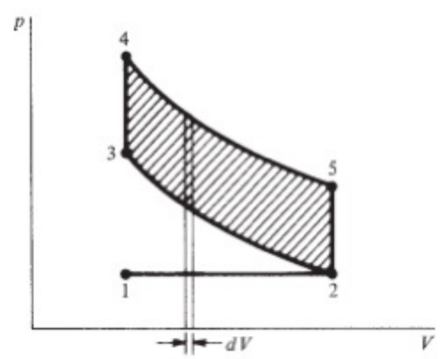


(e)

Exhaust

- → Exhaust valve opens, pressure returns to ambient, piston pushes out exhaust
- → P↓, V↓

Overall Process →



Jets!

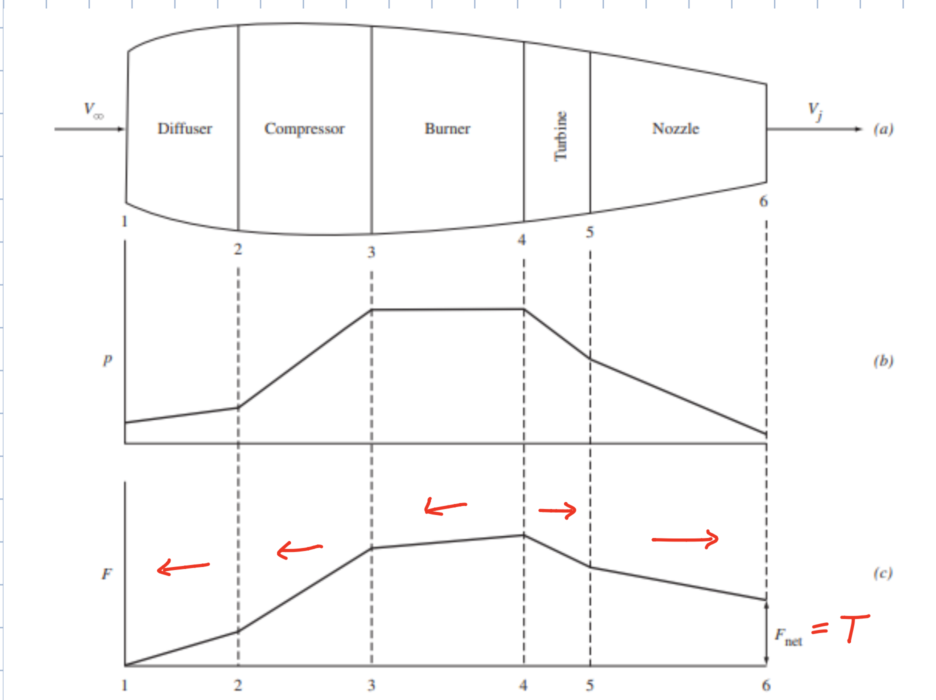
→ "Suck, Squeeze, Bang, Blow"

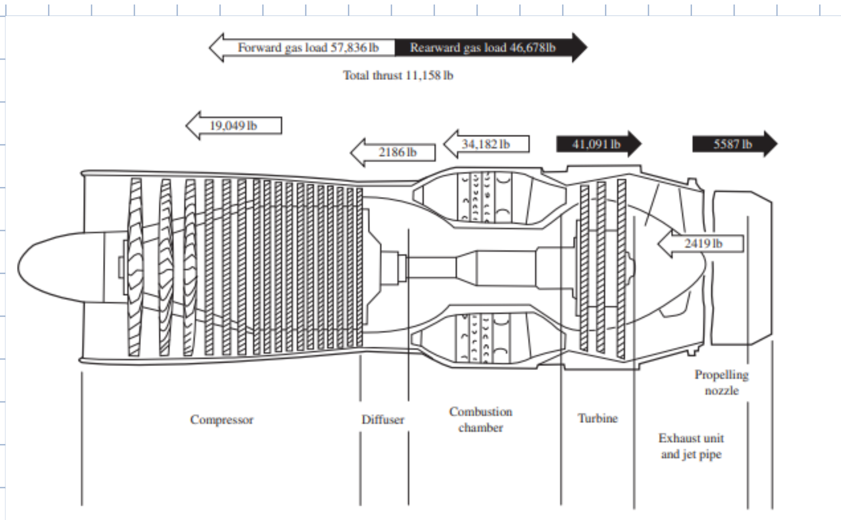
→ Thrust Equation:

$$\boxed{T = (\dot{m}_{air} + \dot{m}_{fuel})V_e - \dot{m}_{air}V_\infty + (P_e - P_\infty)A_e} \quad 9.24$$

usually $\frac{\dot{m}_{fuel}}{\dot{m}_{air}} \ll 1$, can be neglected...

$$\boxed{T = \dot{m}_{air}(V_e - V_\infty) + (P_e - P_\infty)A_e} \quad 9.25$$





Types of Jet Engines:

Turbojet
- ↳ Shown above
- ↳ Typical TSFC of 1.0 lb fuel per lb thrust per hour
- ↳ Used on fighter jets: compact, powerful, not efficient, loud

Turbo fan
- ↳ Bypasses large ammount of air around compressor
- ↳ Typical TSFC of 0.6 lb fuel per lb thrust per hour
- ↳ Used on commercial airliners: giant, powerful, more efficient, quieter

Other notable engines
- ↳ Ramjets
  - ↳ Use nozzle geometry to compress air
- ↳ Turbo prop
  - ↳ Uses jet engine to power propeller
- ↳ Unducted fan
  - ↳ More efficient than turbo fan, weird

Rocket Engines

→ Important Equations ⇒ Thrust: $T = \dot{m} V_e + (P_e - P_a) A_e$

Exit Velocity: $V_e = \left\{ \frac{2\gamma R T_0}{\gamma - 1} \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma-1}{\gamma}} \right\}^{1/2}$

Specific Impulse: $I_{sp} = \frac{V_e}{g_0} = \frac{1}{g_0} \left\{ \frac{2\gamma R T_0}{\gamma - 1} \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma-1}{\gamma}} \right\}^{1/2}, \quad R = \frac{\bar{R}}{\bar{M}}$

Mass Flow Rate: $\dot{m} = \frac{P_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$

→ Most important performance characteristic is I_{sp}, can be raised by:

$\left.\begin{array}{l} \longrightarrow \text{high combust. temp., } T_0 \\ \longrightarrow \text{low molecular weight, } M \end{array}\right\}$ based on fuel chemistry

→ To increase $\dot{m}$: $\uparrow P_0$ and $\uparrow A^*$, both limited by nozzle construction

Rocket Propellants

Liquid Fuels (fuel + oxidizer)

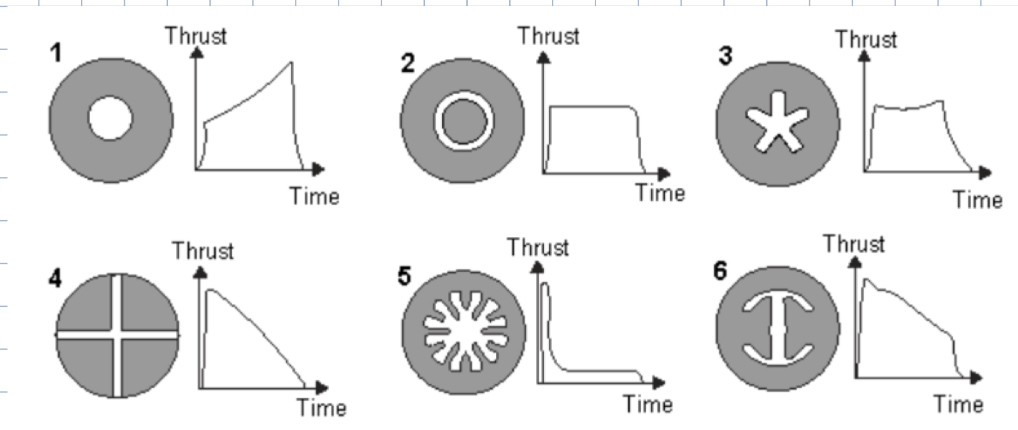
→ Pressure fed
- ↳ HIGH pressure tanks (thick-walled, heavy)
- ↳ Simpler

→ Pump fed
- ↳ Use pumps to push fuel + oxidizer into comb. chamber
- ↳ Lighter, more powerful, but mechanically complex

→ See previous review for notes on specific fuels

Solid Propellants

→ Larger surface areas burn faster & produce more thrust
   ↳ Changing internal bore changes thrust curve



Pros: → Simpler, Safer, more Reliable than liquid-fueled rockets
→ Easier to store, stable for a looonnnggg time
→ Denser, more bang for your buck

Cons: → Much lower I_{sp}, typically 200-300s (compare to 453s for Rocketdyne RS-25)
→ Once ignited, you can't "turn them off" or control the thrust

Rocket Eqⁿ:

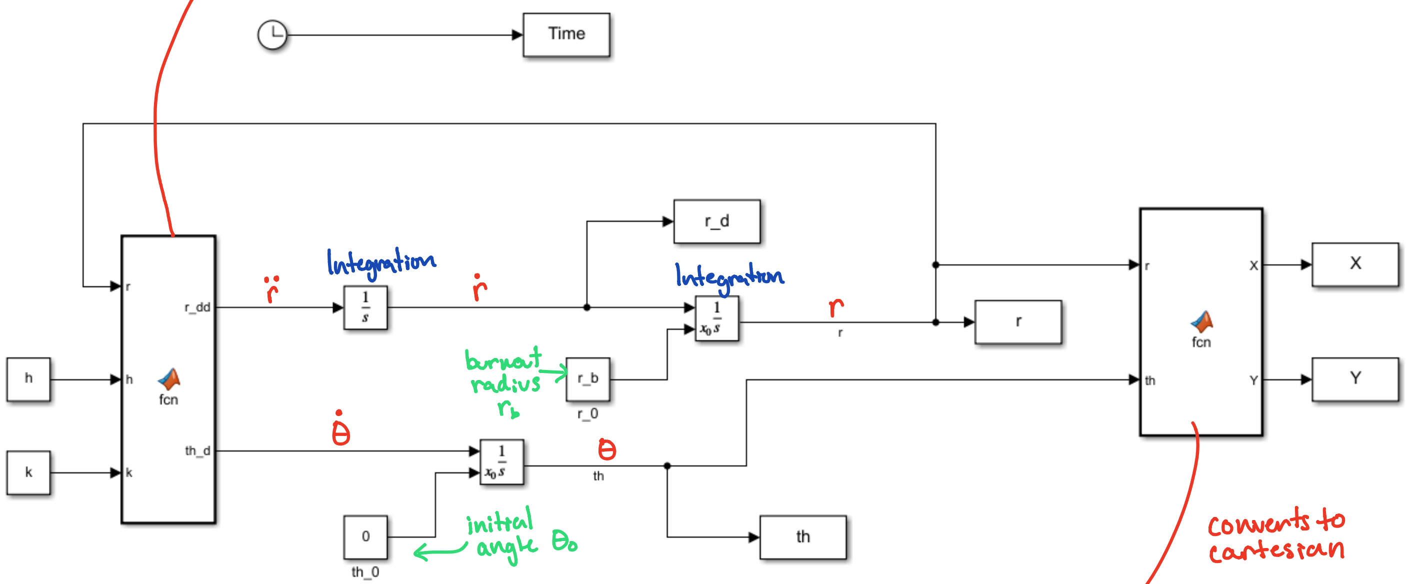
$$\boxed{V_b = g_0 I_{sp} \ln\left(\frac{M_i}{M_f}\right)} \qquad \boxed{\frac{M_i}{M_f} = \exp\left(\frac{V_b}{g_0 I_{sp}}\right)}$$

Brief Simulink Overview:

Governing Eq^{n}s: $\dot{\theta} = \frac{h}{r^2}$, $\ddot{r} = \frac{h^2}{r^3} - \frac{k^2}{r^2}$, $h \equiv r^2\dot{\theta}$, $k^2 \equiv GM$

Plug info into gov. eq^{n}s

```
function [r_dd,th_d] = fcn(r, h, k)
th_d = h/(r^2);
r_dd = (h^2)/(r^3)-(k^2)/(r^2);
```




```
function [X,Y] = fcn(r, th)

X = r*cos(th);
Y = r*sin(th);
```

8.2 DIFFERENTIAL EQUATIONS:

$\frac{dr}{dt}$ RATE OF CHANGE OF r WITH RESPECT TO t.

IF r IS DISTANCE AND t IS TIME THEN THE RATE OF CHANGE OF DISTANCE WITH RESPECT TO TIME IS - VELOCITY -

THE SECOND DERIVATIVE:

$$\frac{d(dr/dt)}{dt} = \frac{d^2r}{dt^2}$$

IF r IS DISTANCE AND t IS TIME THEN THE RATE OF CHANGE OF VELOCITY WITH RESPECT TO TIME IS — ACCELERATION —

SOME DIFFY Q: $\frac{d^2r}{dt^2} + r\frac{dr}{dt} - 2t^3 = 2$

THEN IF GIVEN SOME FUNCTIONAL RELATION $r = f(t)$ THAT SATISFIES THE EQUATION WE CAN THEN FIND A SOLUTION FOR THE DIFFY Q.

SO IF

$$r = t^2$$
$$\frac{dr}{dt} = 2t$$
$$\frac{dr^2}{dt^2} = 2$$

PLUG BACK INTO DIFFY Q:

$$\frac{d^2r}{dt^2} + r\frac{dr}{dt} - 2t^3 = 2$$

$$(2) + t^2(2t) - 2t^3 = 2$$
$$2 + \cancel{2t^3} - \cancel{2t^3} = 2$$
$$2 = 2 \checkmark$$

SATISFIES EQUATION, $r(t)$ IS A SOLUTION.

NOTATION: $\dot{r} \equiv \frac{dr}{dt}$, $\ddot{r} \equiv \frac{d^2r}{dt^2}$

8.3 LAGRANGE EQUATION

STATICS : BODY IS MOTIONLESS

DYNAMICS : BODY IS MOVING

↳ USES NEWTON'S SECOND LAW $F = ma$

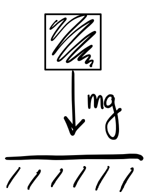
IN STUDY OF SPACE VEHICLE ORBITS & TRAJECTORIES LAGRANGE'S EQUATION SIMPLIFIES ANALYSIS.

LANGRAGIAN ISN'T COMPLICATED IT'S JUST TAKING THE DIFFERENCE IN ENERGY : (KINETIC − POTENTIAL) AT SOME POINT AND MEASURING HOW IT CHANGES W.R.T. SOME OTHER VARIABLE. FOR EXAMPLE THE LOCATION OF THE POINT OR THE VELOCITY OF THE POINT.

FROM BOOK EXAMPLE : T : KINETIC ENERGY , Φ : POTENTIAL ENERGY

LAGRANGIAN : $B = T - \Phi$

EXAMPLE OF LAGRANGIAN TO DERIVE EQUATION OF MOTION OF A FALLING BODY.



$$B = \frac{1}{2} m(\dot{x})^2 - mgx$$

$$\frac{d}{dt}\left(\frac{\partial B}{\partial \dot{x}}\right) - \frac{\partial B}{\partial x} = 0$$

THE DIFFERENCE OF

THE RATE OF THE RATE OF CHANGE OF THE ENERGY DIFFERENCE W.R.T. VELOCITY W.R.T. TIME.

AND THE

RATE OF CHANGE OF THE ENERGY DIFFERENCE W.R.T. DISTANCE.

IS EQUAL TO ZERO.

$$\frac{\partial B}{\partial \dot{x}} = m\dot{x}$$

$$\frac{\partial B}{\partial x} = -mg$$

PLUGGING IN,

$$\frac{d}{dt}(m\dot{x}) - (-mg) = 0$$

$$m\frac{d}{dt}(\dot{x}) + mg = 0$$

$$m\ddot{x} + mg = 0$$

$$m\ddot{x} = mg$$

$$\ddot{x} = g$$

POLAR COORDINATES:

POSITION : $r = ru_r$

INSTANTENOUS VELOCITY : $\bar{v} = \dot{r}\bar{u}_r + r\dot{\bar{u}}_r$

FOR SMALL ANGLES : $\dot{\bar{u}}_r = \dot{\theta}\bar{u}_\theta$

VELOCITY : $v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

ACCELERATION: $\dot{\bar{u}}_\theta = -\dot{\theta}\bar{u}_r$

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$$

8.4 ORBIT EQUATIONS:

EQUATIONS THAT DESCRIBE THE PATH OF OUR SATELLITE, PROBE, ETC.

8.4.1 FORCE & ENERGY:

LAW OF UNIVERSAL GRAVITATION : $F = \frac{Gmm}{r^2}$

$$G = 6.67 \times 10^{-11}\ m^3/kg(s)^2$$

$$\rightarrow d\Phi = Fdr = \frac{GMm}{r^2}\,dr$$

$$\int_0^{\Phi} d\Phi = \int_{\infty}^{r} \frac{GMm}{r^2}\,dr$$

- DERIVATIONS OF ALL FOLLOWING EQUATIONS ARE IN BOOK.

POTENTIAL ENERGY OF SPACE VEHICLE :

$$\boxed{\Phi = -\frac{GmM}{r}}$$

KINETIC ENERGY OF SPACE VEHICLE :

$$\boxed{T = \frac{1}{2}\left[\dot{r}^2 + (r\dot{\theta})^2\right]m}$$

8.4.2 EQUATIONS OF MOTION:

EQUATION OF MOTION FOR SPACE VEHICLE TRAVELING IN DIRECTION θ :

$$\boxed{mr^2\dot{\theta} = \text{CONST.} = C_1}$$

ANGULAR MOMENT OF THE SPACE VEHICLE :

$$\boxed{mr^2\dot{\theta} = \text{ANGULAR MOMENT} = \text{CONST.}}$$

EQUATION OF MOTION FOR SPACE VEHICLE IN THE DIRECTION r.

$$\boxed{\ddot{r} - \frac{h^2}{r^3} + \frac{k^2}{r^2} = 0}$$

DESIRED EQUATION OF THE PATH (ORBIT, TRAJECTORY) OF SPACE VEHICLE :

$$\boxed{r = \frac{(h^2/k^2)}{1 + A(h^2/k^2)\cos(\theta - C)}}$$

* CONSTANTS h^2, A, C ARE FIXED BY CONDITIONS AT THE INSTANCE OF BURNOUT OF THE ROCKET BOOSTER.

* EQUATION APPLIES TO THE TRAJECTORY OF A SPACE VEHICLE ESCAPING FROM THE GRAVITATIONAL FIELD OF EARTH AS WELL AS ARTIFICIAL SATELLITE ORBIT ABOUT THE EARTH.

$$r = \frac{p}{1+e\cos(\theta - C)} \qquad (8.44)$$

$e=0$, circle

$e<1$, ellipse

$e=1$, parabola

$e>1$, hyperbola

$$e = \sqrt{1 + \frac{2h^2H}{mk^4}}$$

T - potential energy

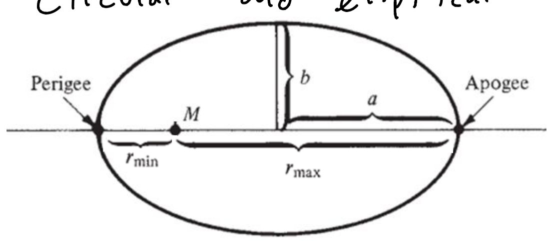
$$V = \sqrt{\frac{k^2}{r}} \quad \text{circular velocity}$$

$$V = \sqrt{\frac{2k^2}{r}} \quad \text{parabolic velocity}$$

Kepler's Law

1. All planets move about the Sun in elliptical orbits, having the Sun as one of the foci.
2. A radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time.
3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

circular and eliptical orbits



When $\cos(\theta - C) = \begin{cases} 1; \; r_{min} = \frac{h^2/k^2}{1+e} \\ -1; r_{max} = \frac{h^2/k^2}{1-e} \end{cases}$

$$a = \frac{h^2}{k^2(1-e^2)}$$

vis-visa

$$V = \sqrt{k^2\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$\tau^2 = \frac{4\pi^2}{k^2} a^3 = (\text{const})\, a^3$$

third law

Problem 1:

At the end of a rocket launch of a space vehicle from earth, the burnout velocity is 13 km/s in a direction due south and 10° above the local horizontal. The burnout point is directly over the equator at an altitude of 400 mi above sea level. Calculate the trajectory of the space vehicle.

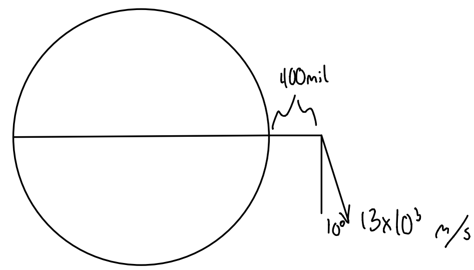
$V = 13000\ m/s$

$\gamma = 10^\circ$

$h_G = 400\ mil \rightarrow 6.43738 \times 10^5\ m$

$\lambda = 0^\circ$

$r_e = 6.4 \times 10^6\ m$



$r_b = r_e + h_G = 6.4 \times 10^6\ m + 6.43738 \times 10^5\ m = 7.04374 \times 10^6$

$V_\theta = V\cos(\gamma) = 13\times10^3 \cos(10^\circ) = 12802.5\ m/s$

$V_r = V\sin(\gamma) = 13\times10^3 \sin(10^\circ) = 2257.43\ m/s$

$K^2 \equiv GM = 3.986\times10^{14}\ m^3/s^2$ Gravitational Constant

$h = r_b V_\theta = 7.04374\times10^6 (12802.5) = 9.02\times10^{10}\ m^2/s \Rightarrow h^2 = 8.13\times10^{21}\ m^4/s^2$

$p = h^2/K^2 = \frac{8.13\times10^{21}}{3.986\times10^{14}} = 2.04\times10^7\ m$

$H/m = \frac{T - |\Phi|}{m} = \frac{T}{m} - \frac{\Phi}{m} = \frac{V^2}{2} - \frac{K^2}{r_b} = \frac{(13\times10^3)^2}{2} - \frac{3.986\times10^{14}}{7.04374\times10^6} = 2.79\times10^7\ m^2/s^2$

$e = \sqrt{1 + \frac{2h^2 H}{m K^4}} = \left(1 + \left(\frac{2h^2}{K^4}\right)\left(\frac{H}{m}\right)\right)^{1/2} = \left(1 + \frac{2(8.13\times10^{21})}{(3.986\times10^{14})^2}\left(2.79\times10^7\right)\right)^{1/2} = 1.96$

Hyperbolic orbit because $e > 1$

$r = \frac{p}{1 + e\cos(\theta - c)} \longrightarrow 7.04374\times10^6 = \frac{2.04\times10^7}{1 + 1.96\cos(0 - c)} \rightarrow c^\circ = 15^\circ$

$$\boxed{r = \frac{2.04\times10^7}{1 + 1.96\cos(\theta - 15)}}$$