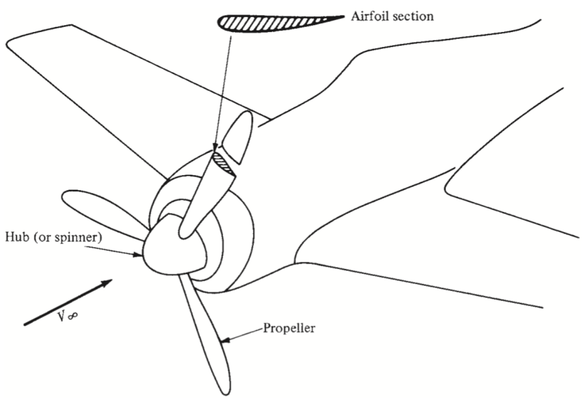
Propellers

Airplane wings and propellers are both made up of airfoil sections designed to generate an aerodynamic force. The wing force provides lift to sustain the airplane in the air; the propeller force provides thrust to push the airplane through the air.



Unlike a wing, where the chord lines of the airfoil sections are essentially all in the same direction, a propeller is twisted so that the chord line changes from being almost parallel to V∞ at the root to almost perpendicular at the tip.

9.2 Propeller

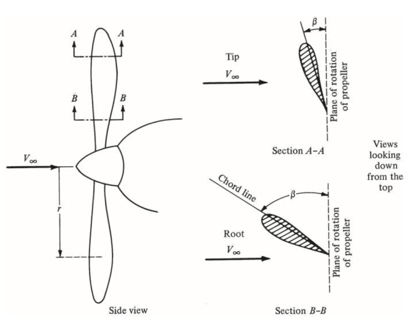


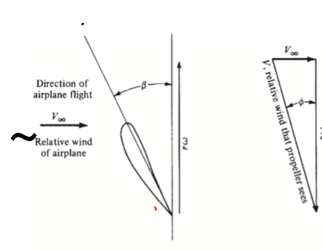
Figure 9.3 Illustration of propeller, showing variation of pitch along the blade.

The angle between the chord line and the propeller's plane of rotation is defined as the pitch angle β. The distance from the root to a given section is r.

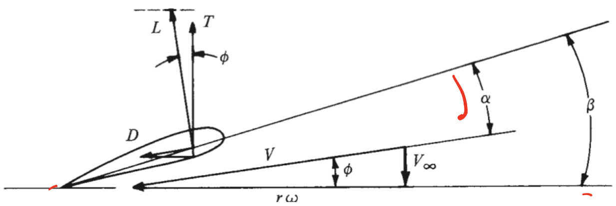
The airflow seen by a given propeller section is a combination of the airplane's forward motion and the rotation of the propeller itself , where the airplane's relative wind is V∞ and the speed of the blade section due to rotation of the propeller is rω. Here ω denotes the angular velocity of the propeller in radians per second. Hence, the relative wind seen by the propeller section is the vector sum of V∞ and rω. If the chord line of the airfoil section is at an angle of attack α with respect to the local relative wind V, then lift and drag (perpendicular and parallel to V, respectively) are generated. In turn, the components of L and D in the direction of V∞ produce a net thrust

$T = L\cos\phi - D\sin\phi$

where $\phi = \beta - \alpha$.



This thrust, when summed over the entire length of the propeller blades, yields the net thrust available , which drives the airplane forward.



Propeller efficiency

The propeller efficiency is defined as:

η = PA/P

where P is the shaft brake power (the power delivered to the propeller by the shaft of the engine) and PA is the power available from the propeller.

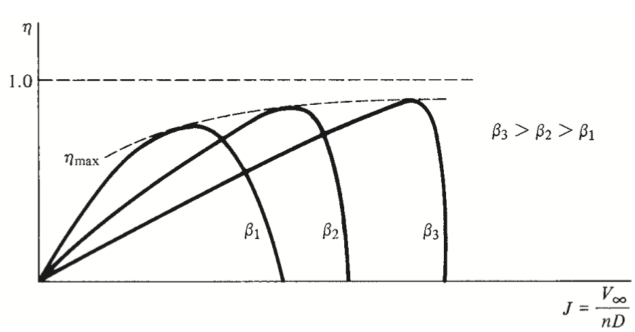
PA = TAV∞

η = TA*V∞ /P

TA is basically an aerodynamic phenomenon that is dependent on the angle of attack α, which is dictated by the pitch angle β and ϕ, where ϕ itself depends on the magnitudes of V∞ and rω. The angular velocity ω = 2π n, where n is the number of propeller revolutions per second. Consequently, TA must be a function of at least β, V∞, and n. Finally, the thrust must also depend on the size of the propeller, characterized by the propeller diameter D. In turn, the propeller efficiency must depend on β, V∞, η, and D.

For a fixed pitch angle β, η is a function of the dimensionless quantity advanced ratio

J =V∞ /n*D



A typical variation of η with J for a fixed β three curves are shown corresponding to three different values of pitch. Note that $\eta < 1$; this is because some of the power delivered by the shaft to the propeller is always lost, and hence $PA < P$

These losses occur because of several different effects:

•Slipstream from the propeller; that is, the air is set into both translational and rotational motion by the passage of the propeller. Consequently, you observe some translational and rotational kinetic energy of the air where before there was none. This kinetic energy has come from part of the power delivered by the shaft to the propeller; it does no useful work and hence robs the propeller of some available power.

•Frictional loss due to the skin friction and pressure drag (profile drag) on the propeller. Friction of any sort always reduces power.

•A third source is compressibility loss. The fastest-moving part of the propeller is the tip. For many high-performance engines, the propeller tip speeds result in a near-sonic relative wind. When this occurs, the same type of shock wave and boundary layer separation losses that cause the drag-divergence increase for wings now rob the propeller of available power. If the propeller tip speed is supersonic, η drops dramatically. This is the primary reason why propellers have not been used for transonic and supersonic airplanes.

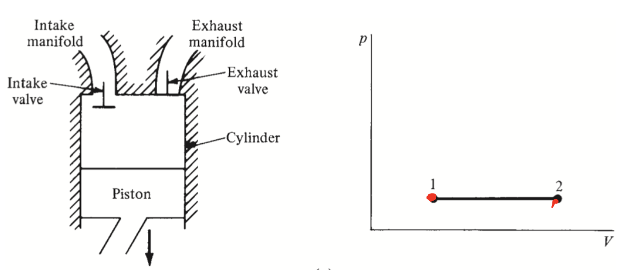
There is also another type of propellers which are called variable pitch propellers and are more efficient than fixed pitch propellers

Reciprocating engines

The basic operation of these engines is a piston moving back and forth (reciprocating) inside a cylinder, with valves that open and close appropriately to let fresh fuel–air mixture in and burned exhaust gases out. The piston is connected to a shaft via a connecting rod that converts the reciprocating motion of the piston to rotational motion of the shaft.

Intake:

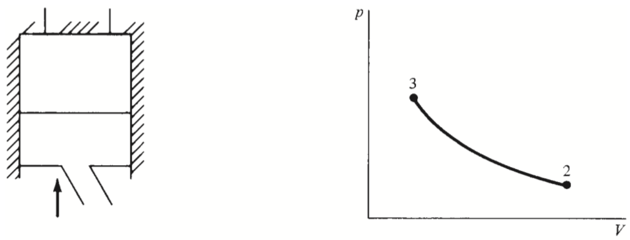
During the intake stroke, the piston moves downward, the intake valve is open, and a fresh charge of gasoline–air mixture is drawn into the cylinder. This process is sketched on the p–V diagram (a plot of pressure versus volume) Here point 1 corresponds to the beginning of the stroke (where the piston is at the top, called top dead center), and point 2 corresponds to the end of the stroke (where the piston is at the bottom, called bottom dead center). The volume V is the total mixture volume between the top of the cylinder and the face of the piston. The intake stroke takes place at essentially constant pressure, and the total mass of fuel–air mixture inside the cylinder increases throughout the stroke.



(a)

Compression :

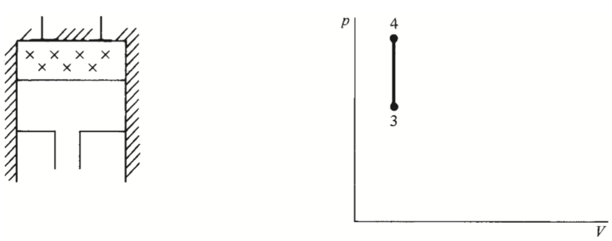
At the bottom of the intake stroke, the intake valve closes, and the compression stroke begins. Here the piston compresses the now-constant mass of gas from a low pressure p2 to a higher pressure p3, as shown in the p–V diagram. If frictional effects are ignored, the compression takes place isentropically because no heat is added or taken away.



(b)

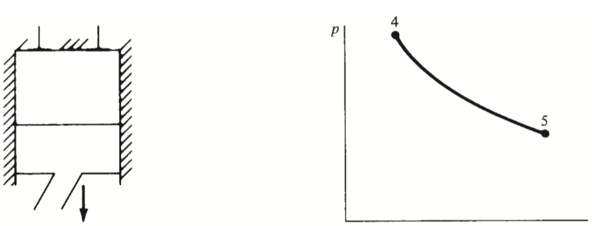
Combustion:

At the top of the compression stroke, the mixture is ignited, usually by an electric spark. Combustion takes place rapidly before the piston has moved any meaningful distance. Hence, for all practical purposes, the combustion process is one of constant volume. Because energy is released, the temperature increases remarkedly; in turn, because the volume is constant, the equation of state, dictates that pressure increases from p3 to p4. This high pressure exerted over the face of the piston generates a strong force that drives the piston downward on the power stroke.



Power:

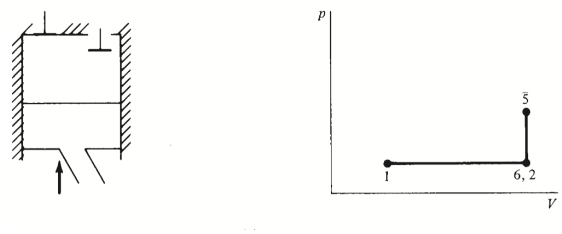
Again, assuming that frictional and heat transfer effects are negligible, the gas inside the cylinder expands isentropically to the pressure p5. At the bottom of the power stroke, the exhaust valve opens.



(d)

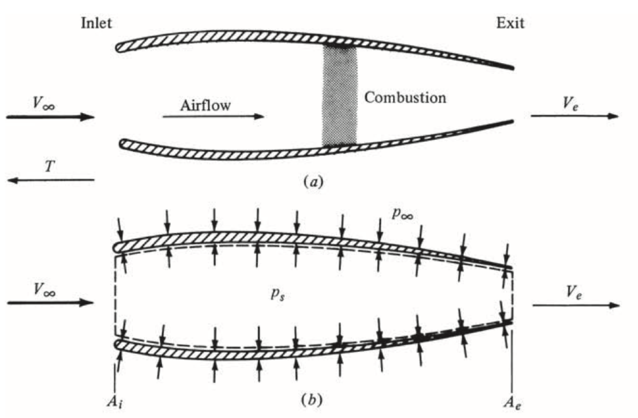
Exhaust:

The pressure inside the cylinder instantly adjusts to the exhaust manifold pressure p6, which is usually about the same value as p2. Then, during the exhaust stroke, the piston pushes the burned gases out of the cylinder, returning to conditions at point 1.



Jet propulsion-Thrust equation:

The jet engine is a device that takes in air at essentially the free-stream velocity V∞, heats it by combustion of fuel inside the duct, and then blasts the hot mixture of air and combustion products out the back end at a much higher velocity Ve. In contrast to a propeller, the jet engine creates a change in momentum of the gas by taking a small mass of air and giving it a large increase in velocity (hundreds of meters per second). The true fundamental source of the thrust of a jet engine is the net force produced by the pressure and shear stress distributions exerted over the surface of the engine.



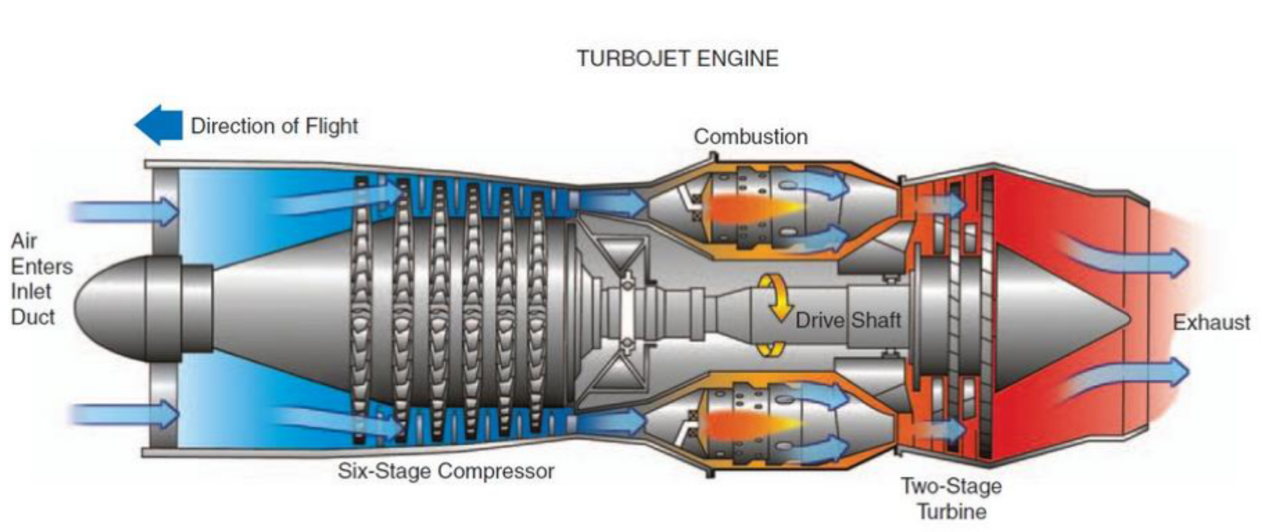
This illustrates the distribution of pressure ps over the internal surface of the engine duct, and the ambient pressure, essentially p∞, over the external engine surface. Shear stress, which is generally secondary in comparison to the magnitude of the pressures, is ignored here.

$$\boxed{T = (\dot{m}_{\text{air}} + \dot{m}_{\text{fuel}})V_e - \dot{m}_{\text{air}}V_\infty + (p_e - p_\infty)A_e}$$

Turbojet Engine (9.5)

→ Thrust Eq:

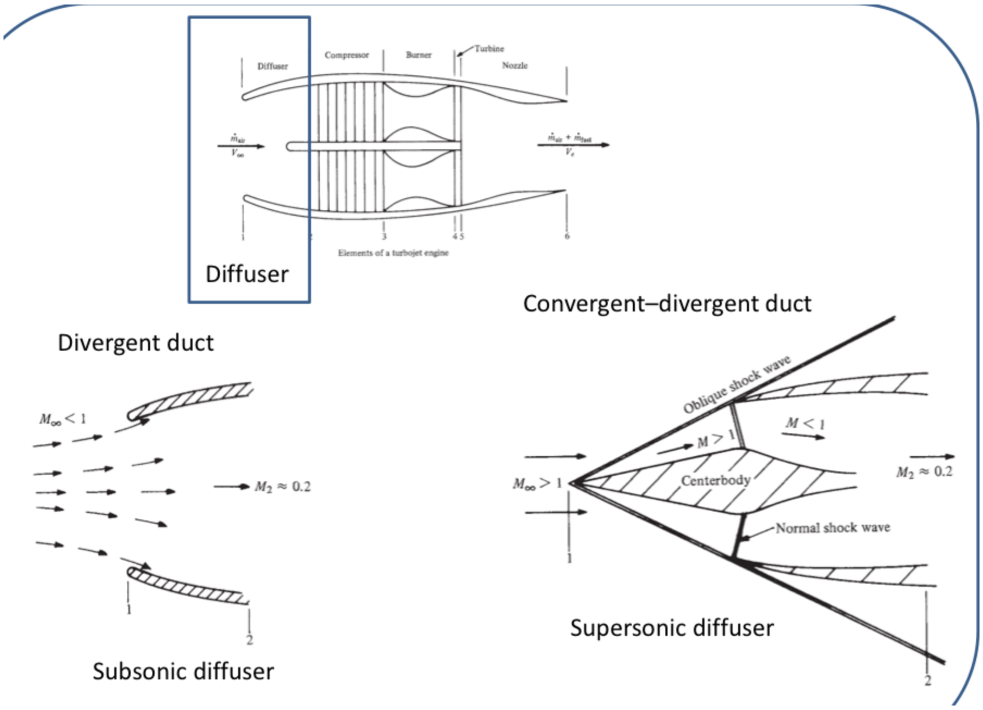
$$T = \dot{m}_{air}(V_e - V_\infty) + (p_e - p_\infty)A_e \quad (9.25)$$



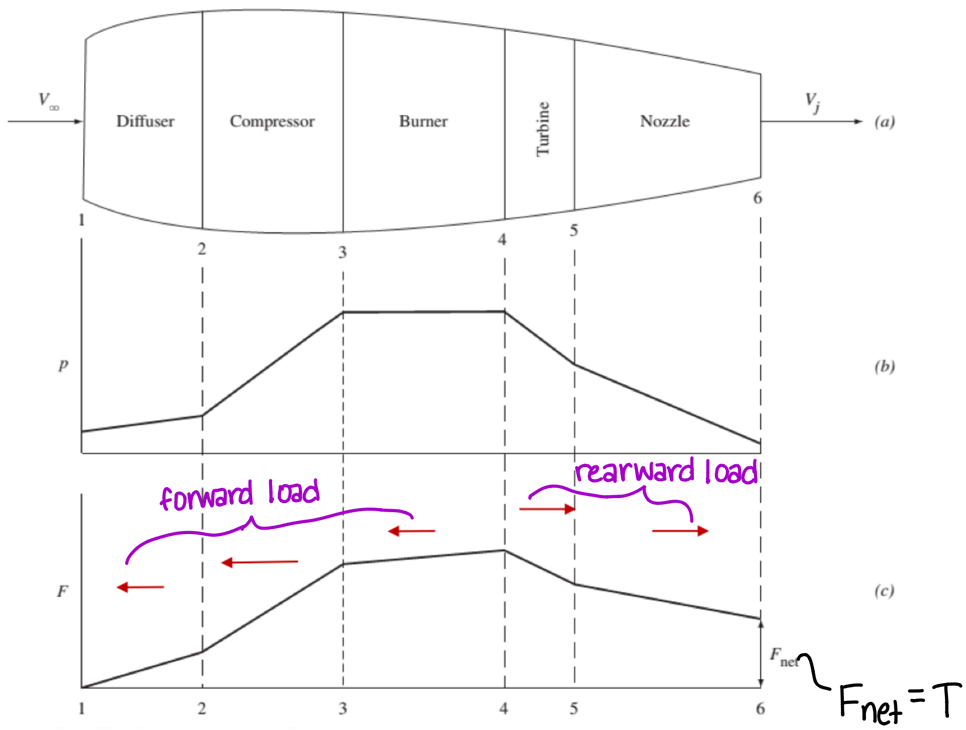
Function: exhaust the gas out the back end faster than it comes in through the front end

How it performs:

1. inducts a mass of air through the inlet
2. flow is reduced to a low Mach # ($M \approx 0.2$) in a diffuser
    a. subsonic V_∞ → diffuser is a divergent duct
    b. supersonic V_∞ → diffuser is a convergent-divergent duct
        i. decrease in flow velocity accomplished partly through shock waves



Thrust Buildup for a Turbojet Engine



b. mean pressure distribution

c. accumulated thrust

Turbofan Engine (9.6)

→ Large ducted fan mounted on the shaft ahead of the compressor

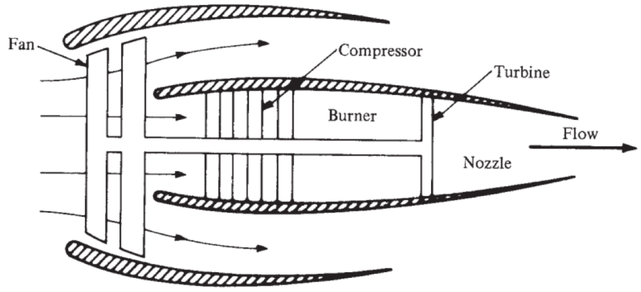


Figure 9.20 A turbofan engine.

→ turbojets create large thrust, but efficiency is low due to high exhaust velocities

→ piston engine-propeller combo is more efficient than a turbojet (turbofan is also more efficient)

→ turbofan engine combines the concepts of a pure turbojet & a propeller

→ ducted fan accels. a large mass of air that flows btwn. inner & outer shrouds

- → unburned air mixes w/ jet exhaust downstream
- → thrust is a combo of thrust produced by fan blades & jet from exhaust nozzle

→ efficiency denoted by thrust-specific fuel consumption (TSFC)

turbojet : TSFC = 1.0 lb. of fuel per lb. of thrust per hour

turbofan : TSFC = 0.6 lb of fuel per lb. of thrust per hour

→ turboprop : 85% of thrust comes from propellers & 15% comes from jet exhaust (range of 300-500 mph)

Ramjet Engine (9.7)

- no more rotating machinery
  - air inducted through inlet @ velocity V_∞
  - 1 to 2: air decelerated in diffuser
  - 2 to 3: air burned in region where fuel is injected
  - 3 to 4: air blasted out exhaust nozzle @ very high velocity

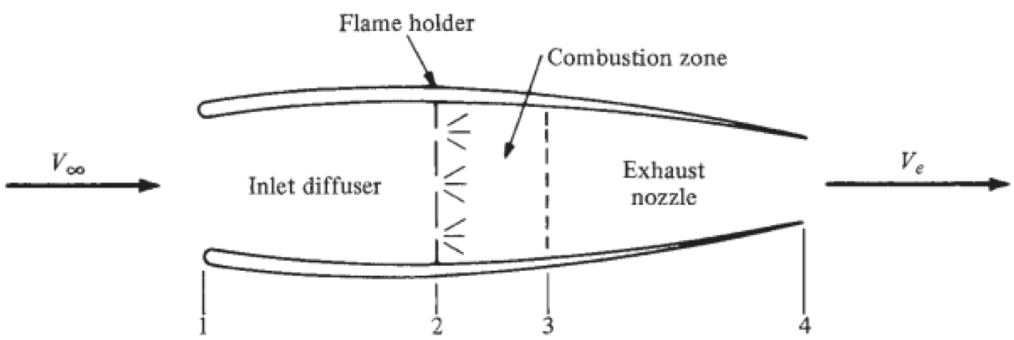


Figure 9.22 Ramjet engine.

- ideal ramjet process:
  - 1 to 2: compression in diffuser
    - $\frac{P_2}{P_1}$ is a function of flight Mach #
  - 2 to 3: combustion @ end of diffuser (const. p)
  - 3 to 4: expansion through exhaust nozzle

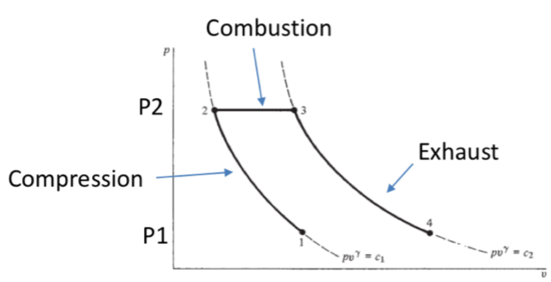
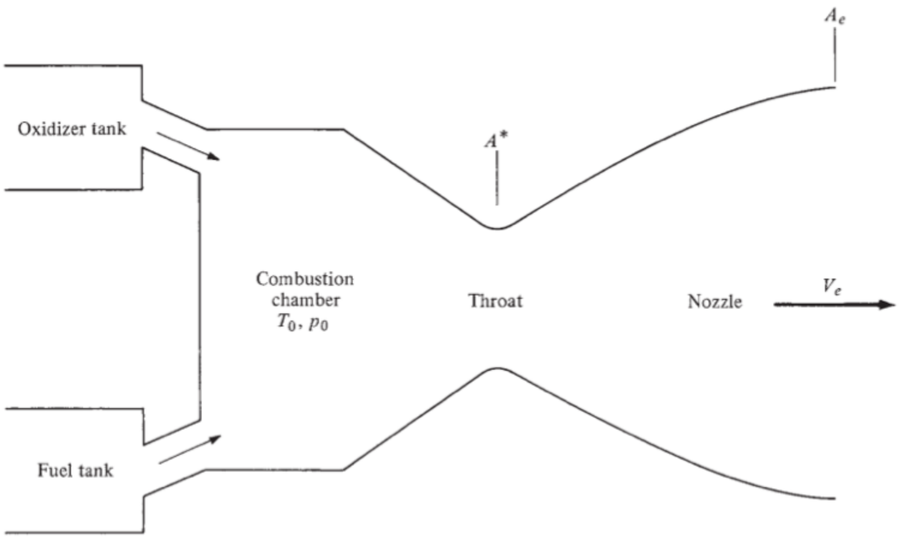


Figure 9.24 Pressure-specific volume diagram for an ideal ramjet.

Rocket Engine (9.8)

→ carries both its fuel & oxidizer

→ completely independent of the atmosphere for its combustion



→ Thrust of a rocket engine:

$$T = \dot{m}V_e + (p_e - p_\infty)A_e \qquad (9.28)$$

solving for V_e:

energy eq:

$$h_0 = h_e + \frac{V_e^2}{2}$$

$$c_pT_0 = c_pT_e + \frac{V_e^2}{2}$$

$$V_e^2 = 2c_p(T_0 - T_e) = 2c_pT_0\left(1 - \frac{T_e}{T_0}\right)$$

isentropic expansion, so $\frac{T_e}{T_0} = \left(\frac{p_e}{p_0}\right)^{\gamma-1/\gamma}$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$V_e = \left\{\frac{2\gamma RT_0}{\gamma-1}\left[1 - \left(\frac{p_e}{p_0}\right)^{\gamma-1/\gamma}\right]\right\}^{1/2} \qquad (9.32)$$

→ Efficiency

specific impulse; $I_{sp} \equiv \frac{T}{\dot{W}}$ [sec], $\dot{W} = \dot{m}g_0$

assume $P_e = P_\infty$:

$$I_{sp} = \frac{T}{\dot{w}} = \frac{T}{g_0 \dot{m}} = \frac{\dot{m} V_e}{g_0 \dot{m}} = \frac{V_e}{g_0}$$

$$I_{sp} = \frac{1}{g_0}\left\{\frac{2\gamma R T_0}{\gamma-1}\left[1-\left(\frac{p_e}{p_0}\right)^{\gamma-1/\gamma}\right]\right\}^{1/2}, \quad R = \bar{R}/\bar{M}$$

- How to get a high I_{sp} :
  - high combustion temp. T_0
  - low molecular weight M
- mass flow of the propellants & area of nozzle throat govern p_0

$$\dot{m} = \frac{p_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

  - $\dot{m}$ is directly proportional to p_0 & A^*

9.9 Rocket Propellants

Liquid Propellants

→ Fuel & Oxidizer are stored separately and are pressurized and injected into the combustion chamber

Two Main Types of Liquid Propellant Systems:

1) Pressure-fed
   ↳ Uses HIGH pressure tanks for fuel & oxidizer
      Pro: Simple
      Con: Heavy (tank walls must be thick)
   → Usually used as attitude control, not main thrusters (relatively low thrust)

2) Pump-fed
   ↳ Fuel & oxidizer stored in LOW pressure tanks pressurized by pumps
      Pro: Lighter & Generally more powerful
      Con: Mechanically complex
   → Often have multiple pump stages powered by separate combustion

Types of Liquid Fuel:

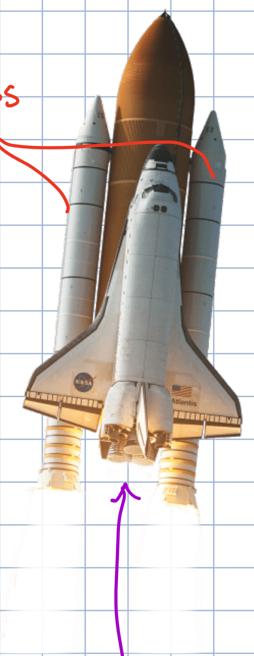
Cryogenic Propellants:
→ Most commonly liquid oxygen (LOX) and hydrogen (LH_2), kerosene (RP-1), or methane
→ Stored at extremely low temps
→ High Isp, requires spark

Bipropellants & Monopropellants
→ Most rockets use a bipropellant system
→ Monopropellant
   ↳ Simpler, reduces weight
   ↳ Lower Isp
   ↳ Used for attitude control
   ↳ Usually use Hydrazine (N_2H_4)

Hypergolic Propellants
→ fuels that ignite upon contact
→ dangerous but simpler (no ignition system)
→ Typically use monomethylhydrazine (MMH) and nitrogen tetroxide (N_2O_4)
→ Lower Isp than Lox and LH_2, used in space shuttle for RCS and orbital injection

Solid Propellants

SRBs



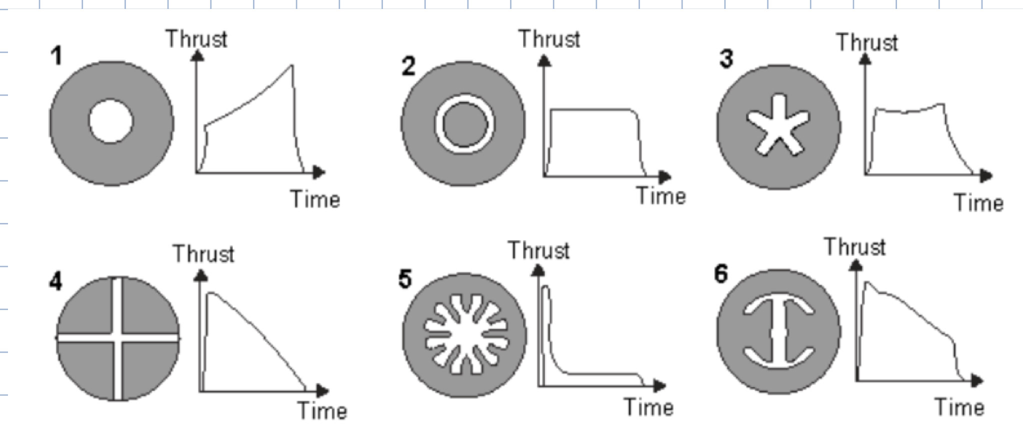
→ This is what the first rockets used!

→ The SRB's on the Space Shuttle used a mixture of aluminum powder, ammonium perchlorate, iron oxide, and polybutadiene acrylic acid acrylonitrile.

→ Apparently solid rocket fuel feels like an eraser

→ Larger surface areas burn faster & produce more thrust

↳ Changing internal bore changes thrust curve



Pros: → Simpler, Safer, more Reliable than liquid-fueled rockets

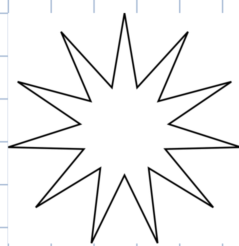
→ Easier to store, stable for a looonnnggg time

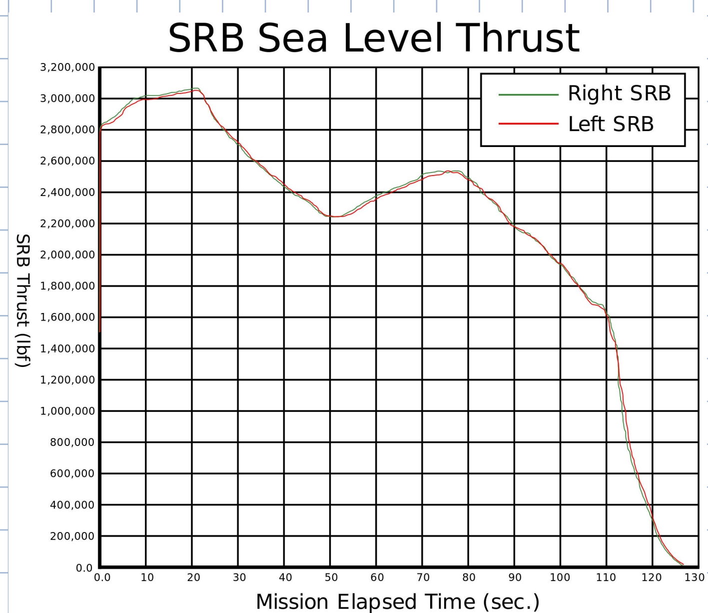
→ Denser, more bang for your buck

Cons: → Much lower I_{sp}, typically 200-300s (compare to 453s for Rocketdyne RS-25)

→ Once ignited, you can't "turn them off" or control the thrust

Fun fact: The space shuttle SRBs used an 11-point star →





← Space Shuttle SRB Thrust Curves

9.10 Rocket Equation

Summary: Using Newton's Second Law, the definition of specific impulse, and the mass of fuel burned, we can determine "burnout velocity" (V_b).

Derivation:

$$F = ma = m\frac{dV}{dt}$$

$$\downarrow$$

$$T = m\frac{dV}{dt}$$

$$I_{sp} \equiv \frac{T}{\dot{w}} \Rightarrow T = I_{sp}\,\dot{w} = I_{sp}\,g_0\,\dot{m}$$

$$\dot{m} = -\frac{dM_{propellant}}{dt} = -\frac{dM}{dt}$$

substitute for T and $\dot{m}$

$$T = -I_{sp}\,g_0\,\frac{dM}{dt} = M\frac{dV}{dt}$$

simplify

$$-\frac{dM}{dt} = \frac{dV}{g_0 I_{sp}}$$

integrate

$$\boxed{V_b = g_0 I_{sp} \ln\left(\frac{M_i}{M_f}\right)} \quad \text{OR} \quad \boxed{\frac{M_i}{M_f} = \exp\left(\frac{V_b}{g_0 I_{sp}}\right)}$$

Example Problems

9.3

9.3 Consider a turbojet mounted on a stationary test stand at sea level. The inlet and exit areas are the same, both equal to 0.45 m². The velocity, pressure, and temperature of the exhaust gas are 400 m/s, 1.0 atm, and 750 K, respectively. Calculate the static thrust of the engine. (*Note:* Static thrust of a jet engine is the thrust produced when the engine has no forward motion.)

$A_i = A_e = 0.45\,m^2$ $\quad V_e = 400\,m/s$ $\quad P_e = 1.0\,atm = 101325\,Pa$ $\quad T_e = 750\,K$

$$T = \dot{m}_{air}(V_e - V_\infty) + (P_e - P_\infty)A_e$$

$$\dot{m}_e = \rho_e V_e A_e = \frac{P_e}{RT_e} V_e A_e \Rightarrow \dot{m}_e = \frac{101325\,[Pa]}{287\left[\frac{J}{kg\cdot K}\right]\cdot 750[K]} \cdot 400\,[m/s]\cdot 0.45\,[m^2] = 84.732\,kg/s$$

Even though the engine is "static", it is still sucking in air!

$$\dot{m}_i = \dot{m}_e = \rho_i V_i A_i \Rightarrow V_i = \frac{\dot{m}_e}{\rho_i A_i} = \frac{84.732\,[kg/s]}{1.225\,[kg/m^3]\cdot 0.45\,[m^2]} = 153.708\,m/s$$

use S.L.

Now, plug & chug...

$$T = \dot{m}_{air}(V_e - V_\infty) + (P_e - P_\infty)A_e$$

$$T = 84.732\,[kg/s]\cdot(400\,[m/s] - 153.708\,[m/s]) + \cancel{(101325\,[Pa] - 101325\,[Pa])}^{\,0}\,0.45\,[m^2]$$

$$\boxed{T = 20868.7\,N = 20.9\,kN}$$

9.8]

9.8 Consider a rocket engine in which the combustion chamber pressure and temperature are 30 atm and 3756 K, respectively. The area of the rocket nozzle exit is 15 m² and is designed so that the exit pressure exactly equals ambient pressure at a standard altitude of 25 km. For the gas mixture, assume that $\gamma = 1.18$ and the molecular weight is 20. At a standard altitude of 25 km, calculate the (*a*) specific impulse, (*b*) exit velocity, (*c*) mass flow, (*d*) thrust, and (*e*) throat area.

$P_o = 30\ \text{atm} = 3039750\ \text{Pa}$ $\quad A_e = 15\ m^2$ $\quad P_\infty = 2.5273\cdot 10^3\ \text{Pa}$ $\quad \gamma = 1.18$

$T_o = 3756\ K$ $\quad h = 25\ km$ $\quad T_\infty = 216.66\ K$ $\quad \bar{M} = 20$

$\rho_\infty = 4.0639\cdot 10^{-2}\ kg/m^3$

$$R = \frac{\bar{R}}{\bar{M}} = \frac{8314.46\left[\frac{J}{kmol\cdot K}\right]}{20\ [g/mol]} = 415.72\ \frac{J}{kg\cdot K}$$

a) $I_{sp} = ?$

$$I_{sp} = \frac{1}{g_o}\left\{\frac{2\gamma R T_o}{\gamma-1}\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{(\gamma-1)}{\gamma}}\right]\right\}^{1/2} \Rightarrow I_{sp} = \frac{1}{9.81}\left\{\frac{2\cdot 1.18\cdot 415.72\cdot 3756}{1.18-1}\left[1-\left(\frac{2.5273\cdot 10^3}{3039750}\right)^{\frac{1.18-1}{1.18}}\right]\right\}^{1/2}$$

$$\boxed{I_{sp} = 375.0\, s}$$

b) $V_e = ?$

$$V_e = \left\{\frac{2\gamma R T_o}{\gamma-1}\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{(\gamma-1)}{\gamma}}\right]\right\}^{1/2} = I_{sp}\cdot g_o \Rightarrow V_e = 375.0\,[s]\cdot 9.81\,[m/s^2] = 3678.75\ m/s$$

$$\boxed{V_e = 3678.75\ m/s}$$

c) $\dot{m} = \rho V A = ?$

→ at the exit we know P_e, V_e, A_e ... need to get T_e to find ρ_e

Use isentropic relations: $\frac{T_e}{T_o} = \left(\frac{P_e}{P_o}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_e = T_o\left(\frac{P_e}{P_o}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_e = 3756\cdot\left(\frac{2.5273\cdot 10^3}{3039750}\right)^{\frac{1.18-1}{1.18}}$

$T_e = 1273.11\ K$

$$\dot{m}_e = \rho_e V_e A_e = \frac{P_e}{R T_e} V_e A_e \Rightarrow \dot{m}_e = \frac{2.5273\cdot 10^3\ [Pa]}{(415.72\,[\frac{J}{kg\,K}])(1273\,[K])}(3678.75\,[m/s])(15\,[m^2])$$

$$\boxed{\dot{m} = 263.5\ kg/s}$$

d) $T = ?$

$$T = \dot{m}V_e + (P_e - P_\infty)A_e \Rightarrow T = (263.5\,[kg/s])(3679\,[m/s]) + (\cancel{P_e - P_\infty})A_e$$ 0

$$\boxed{T = 969341.5\ N}$$

e) $A^* = ?$

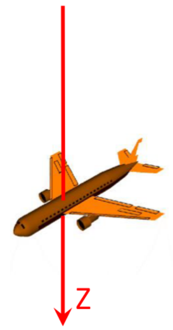
Use $\dot{m} = \frac{P_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \Longrightarrow A^* = \frac{\dot{m}\sqrt{T_0}}{P_0}\left[\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{-1/2}$

$$A^* = \frac{(263.5\,[\mathrm{kg/s}])(3756\,[\mathrm{K}])^{1/2}}{(3039750\,[\mathrm{Pa}])}\left[\frac{1.18}{415.72\,[\mathrm{J/kg\cdot K}]}\left(\frac{2}{1.18+1}\right)^{\frac{1.18+1}{1.18-1}}\right]^{-1/2} = 0.168\,\mathrm{m}^2$$

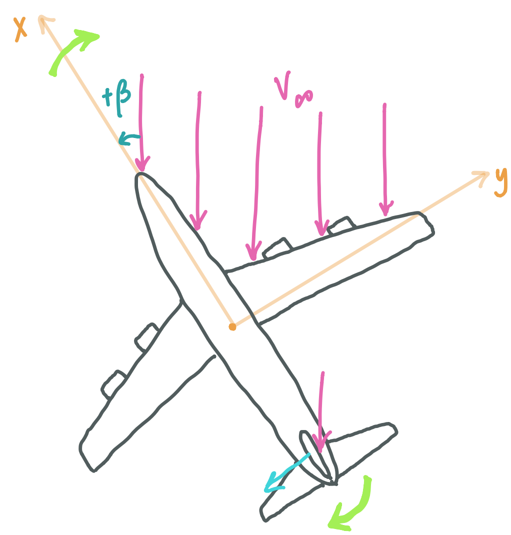
$$\boxed{A^* = 0.168\,\mathrm{m}^2}$$

Directional Static Stability

- Stability in yaw
- stability about the vertical / z-axis



- Coefficient of Yaw $C_{n_\beta} > 0$ for directional static stability



(shoutout to my AERDENG 3520 Instructor for the original illustration ◡̈)

$$\frac{\partial n}{\partial \beta} = C_{n_\beta}$$

Positive sideslip angle β → Positive yawing moment

- Contributions by plane components

Vertical Tail : Positive Directional Stability
*Largest Contribution

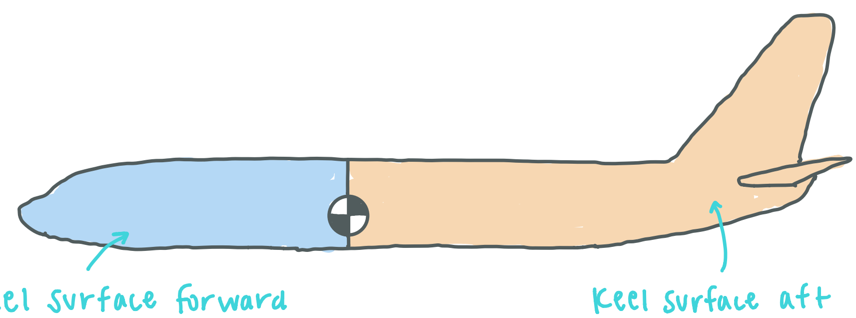
Wings: Positive Directional Stability

Fuselage: Negative Directional Stability

* Also very influential on stability

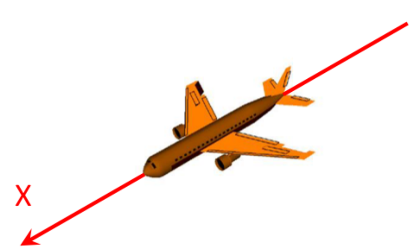
For directional static stability,

Keel surface forward of the CG < Keel surface aft of the CG



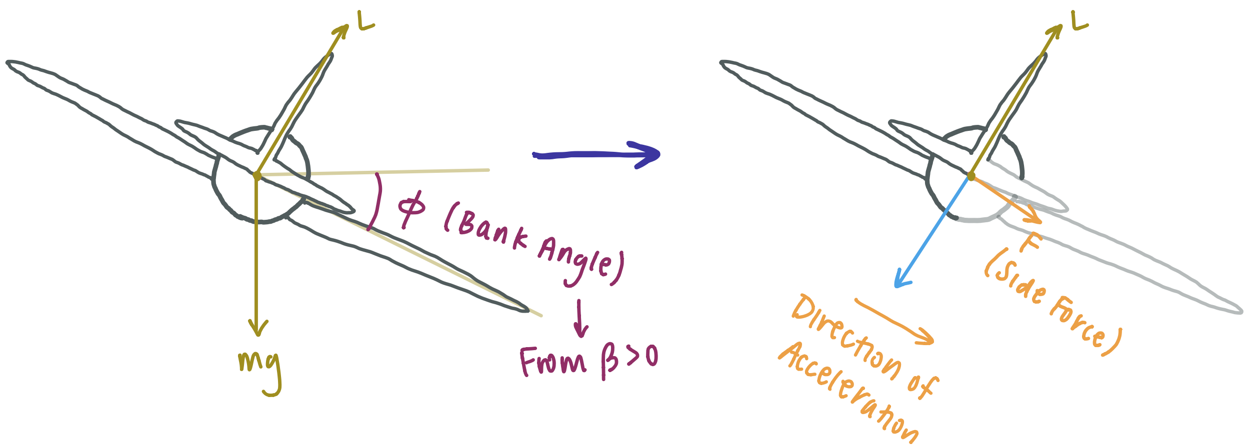
Lateral Static Stability

- Stability in roll
- Stability about the longitudinal/x-axis



- Coefficient of Roll $C_{l_\beta} < 0$ for lateral static stability

* Yawing and Rolling motions are coupled: Notice how sideslip causes the plane to experience rolling and yawing moments at the same time



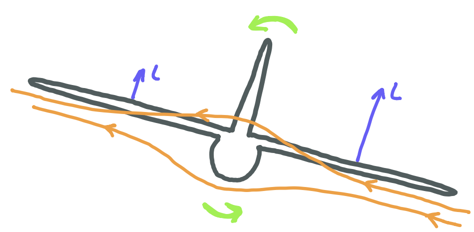
$$\frac{\partial l}{\partial \beta} = C_{l_\beta}$$

Aircraft must roll left wing down to stabilize:

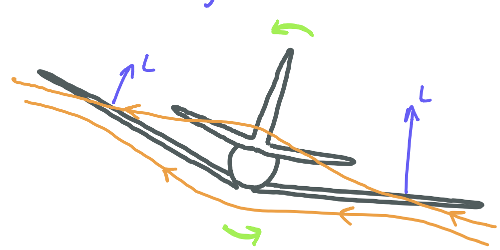
Positive sideslip angle $\beta \longrightarrow$ Negative rolling moment

- Contributions by plane components

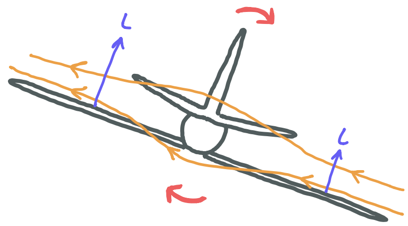
Straight, top mounted wing: Positive lateral static stability



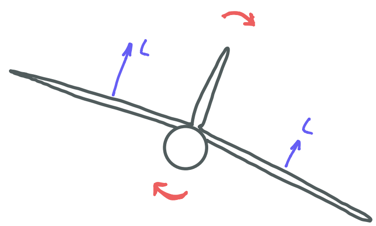
Dihedral wing: Positive lateral static stability



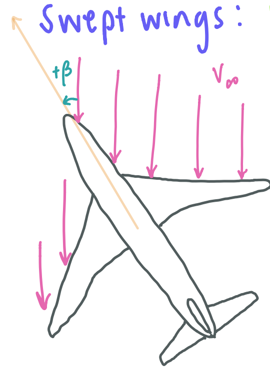
Straight, bottom mounted wing: Negative lateral static stability



Anhedral wing: Negative lateral static stability



Swept wings: Positive lateral static stability



Right wing will have more lift than left wing → plane rolls left

Rudder: Positive lateral static stability