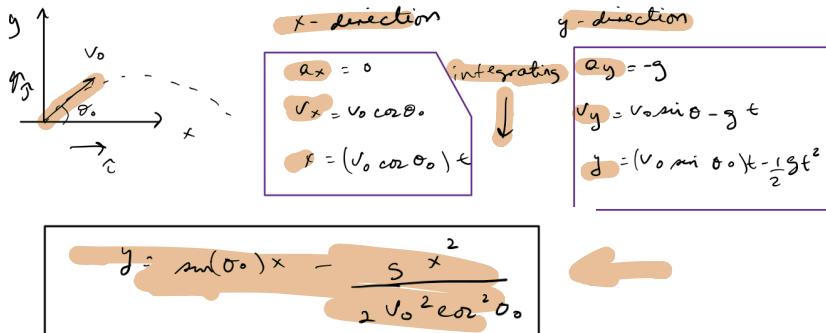
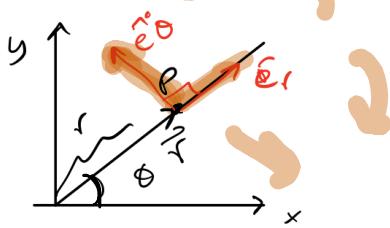


Projectile problem



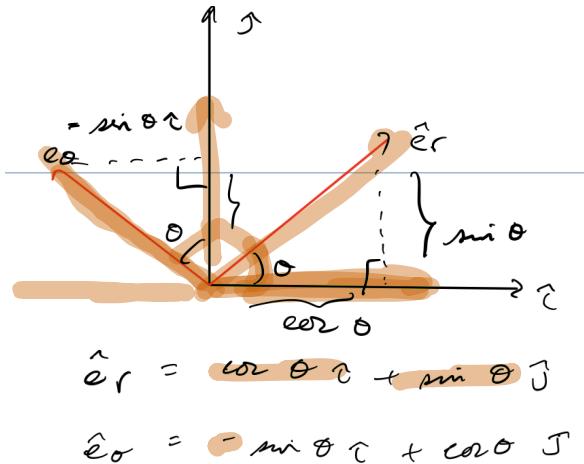
Polar coordinates



$$\vec{v} = r \hat{e}_r + r\dot{\theta} \hat{e}_\theta$$

$$v_r = r \quad v_\theta = r\dot{\theta}$$

Polar to cartesian

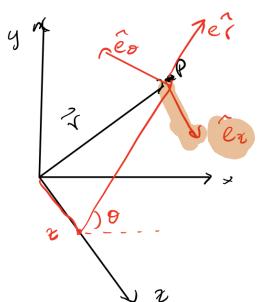


$$a = a_r \hat{e}_r + a_\theta \hat{e}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

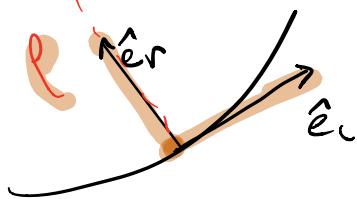
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Cylindrical coordinates



$$\begin{aligned} \vec{r} &= r \hat{e}_r + z \hat{e}_z \\ \vec{r} &= r \hat{e}_r + r\dot{\theta} \hat{e}_\theta + i \hat{e}_x \\ \vec{a} &= [\ddot{r} - r\dot{\theta}^2] \hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}] \hat{e}_\theta + \ddot{z} \hat{e}_z \end{aligned}$$

Normal to tangential

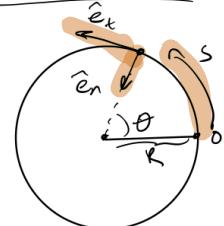


v is always defined in the tangential direction

$$\vec{a} = \frac{dv}{dt} \hat{e}_t + \frac{v^2}{r} \hat{e}_n$$

$$\vec{a} = \frac{dv}{dt} \hat{e}_t + mv\omega \hat{e}_n$$

Circular motion



Distance velocity acceleration :
 $s = R\theta$
 $\vec{v} = R\omega \hat{e}_t$
 $\vec{a} = R\alpha \hat{e}_t + R\omega^2 \hat{e}_n$

Newton 2nd law

$$\sum \vec{F} = m\vec{a}$$

Normal + tangential

$$\sum \vec{F}_t = m\vec{a}_t = m\frac{d\vec{v}}{dt}$$

$$\sum \vec{F}_n = m\vec{a}_n = m\frac{\vec{v}^2}{r} = mv\omega$$

Polar :

$$\sum \vec{F}_r = m\vec{a}_r = m(r\ddot{\theta} - r\omega^2)$$

$$\sum \vec{F}_\theta = m\vec{a}_\theta = m(r\dot{\theta} + r\omega)$$

$$\alpha = \ddot{\theta} \quad \omega = \dot{\theta}$$

Work and Energy

$$W_{1,2} = \int_{r_1}^{r_2} \sum \vec{F} \cdot d\vec{r}$$

Work done = Change in KE

$$W_{1,2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$PE_1 + KE_1 = \text{constant}$$

$$dPE = - \sum \vec{F} \cdot d\vec{r} \rightarrow \text{work done}$$

$$PE_1 + KE_1 = PE_2 + KE_2$$

$$PE_{\text{spring}} = \frac{1}{2} k s^2$$

$$PE_{\text{weight}} = mgh$$

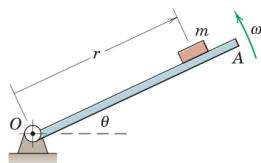
Conservation of linear momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Coefficient of restitution (relates the velocities before and after impact)

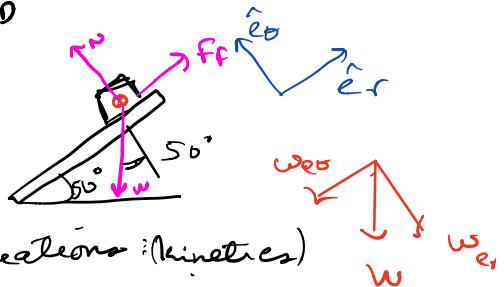
$$e = \frac{v_A - v_B}{v_B' - v_A'}$$

- 3/54** The member OA rotates about a horizontal axis through O with a constant counterclockwise angular velocity $\omega = 3 \text{ rad/sec}$. As it passes the position $\theta = 0$, a small block of mass m is placed on it at a radial distance $r = 18 \text{ in}$. If the block is observed to slip at $\theta = 50^\circ$, determine the coefficient of static friction μ_s between the block and the member.



1) Reference system: Polar

2) PBD



3) Equations (kinetics)

$$\sum \vec{F}_r = m \vec{a}_r$$

$$f_f - w \sin 50^\circ = m(r\dot{\theta}^2)$$

$$\mu_s N - w \cos 50^\circ = m(-r\ddot{\theta})$$

$$\mu_s = \frac{(m(-r\ddot{\theta}) + w \cos 50^\circ)}{w \cos 50^\circ}$$

$$\mu_s = \frac{-r\ddot{\theta} + g \sin 50^\circ}{g \cos 50^\circ}$$

$$\mu_s = \frac{\frac{18}{12} \cdot 3^2 + 32.12 \sin 50^\circ}{32.12 \cos 50^\circ}$$

$$\sum \vec{F}_\theta = m \vec{a}_\theta$$

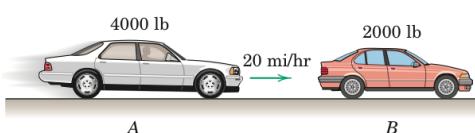
$$N - w \cos 50^\circ = m(r\dot{\theta}^2 + r\ddot{\theta})$$

$$N = mg \cos 50^\circ$$

$$\mu_s = 0.54$$

Problem 3/199

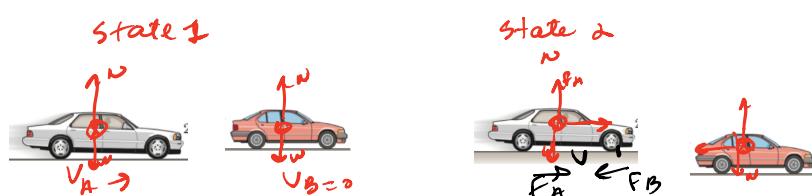
- 3/200** Car B is initially stationary and is struck by car A moving with initial speed $v_1 = 20 \text{ mi/hr}$. The cars become entangled and move together with speed v' after the collision. If the time duration of the collision is 0.1 sec, determine (a) the common final speed v' , (b) the average acceleration of each car during the collision, and (c) the magnitude R of the average force exerted by each car on the other car during the impact. All brakes are released during the collision.



1) Reference system: Cartesian

Type of problem: Impulse + momentum, kinematics

2) PBD + state,



kinematics

$$a_A = \frac{dv}{dt}$$

$$\int_a^t a_A dt = \int v dv$$

$$v_0$$

$$a_A t = v - v_0$$

$$a_A = \frac{v - v_0}{t}$$

$$a_A = \frac{19.56 - 29.33}{0.1} = -97.7 \text{ ft/s}^2$$

$$a_B = \frac{dv}{dt}$$

$$\int_a^t a_B dt = \int v dv$$

$$v_0$$

$$a_B t = v - v_0$$

$$a_B = \frac{v}{t}$$

$$a_B = \frac{19.56}{0.1} = 195.6 \text{ ft/s}^2$$

$$\Sigma F_x = m a_x$$

$$F_A = m_A a_A$$

$$F_A = 4000 \cdot 97.7$$

$$F_A = 121452.38 \text{ lb}$$

3) Momentum Equations

$$m_A v_A + m_B \cancel{v_B} = (m_A + m_B) v'$$

$$v' = \frac{m_A v_A}{m_A + m_B}$$

$$v' = \frac{4000 \cdot 20}{4000 + 2000} = \frac{40}{3} \text{ mi/h} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ sec}}{60 \text{ sec}} \frac{5280 \text{ ft}}{1 \text{ mi}}$$

$$19.56 \text{ ft/s}$$

Kinematics

$$a_A = \frac{dv}{dt}$$

$$\int_0^t a_A dt = \int_0^t dv$$

$$\frac{20 \text{ mi}}{h} = 29.33 \text{ ft/s}$$

$$a_B t = v - v_0$$

$$a_A = 19.56 - 29.33$$

$$a_A = -97.77 \text{ ft/s}^2$$

$$a_B = \frac{dv}{dt}$$

$$\int_0^t a_B dt = \int_0^t dv$$

$$a_B t = v$$

$$a_B = \frac{19.56}{0.1} = 195.6 \text{ ft/s}^2$$

Newton 2nd Law

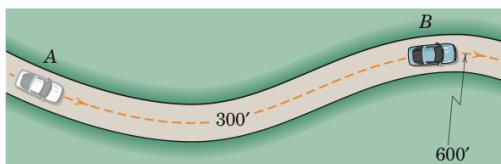
$$\sum F_x = m a_x$$

$$F_A = m a_A$$

$$F_A = \frac{4000 \cdot 97.77}{22.2}$$

$$F_A = 121452.38 \text{ lb}$$

- 3/64** A 3220-lb car enters an S-curve at *A* with a speed of 60 mi/hr with brakes applied to reduce the speed to 45 mi/hr at a uniform rate in a distance of 300 ft measured along the curve from *A* to *B*. The radius of curvature of the path of the car at *B* is 600 ft. Calculate the total friction force exerted by the road on the tires at *B*. The road at *B* lies in a horizontal plane.

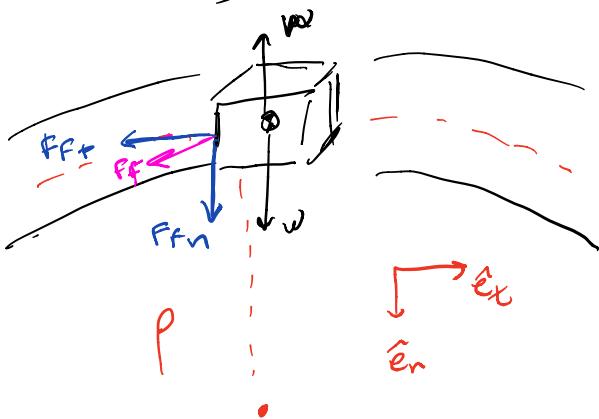


D Reference system: normal tangential

$$45 \text{ mi/hr} = 66 \text{ ft/s}$$

$$60 \text{ mi/hr} = 88 \text{ ft/s}$$

2) F_{BD}



3) Equations kinematics

$$F_p = m \vec{a}_t \quad a_t = \frac{dv}{dt} \quad \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \frac{dv}{ds}$$

$$-F_{f_t} = m \vec{a}_t$$

$$-F_{f_t} = m \frac{dv}{dt}$$

$$-F_{f_t} = m v \frac{dv}{ds}$$

$$\int -F_{f_t} ds = \int m v dv$$

$$-F_{f_t} s = \left[\frac{mv^2}{2} \right]_0^v$$

$$F_{f_t} = -m \left(\frac{v^2 - v_0^2}{2s} \right)$$

$$F_{f_t} = -\frac{3220}{32.2} \left(\frac{(66-45)^2}{2 \cdot 300} \right) = 564.7 \text{ lb}$$

$$e \vec{F}_n = m \vec{a}_n$$

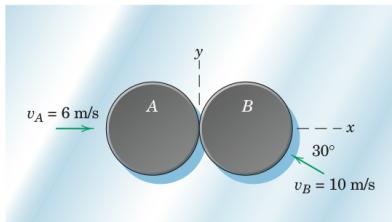
$$\vec{F}_F = m \left(\frac{\vec{v}^2}{r} \right)$$

$$F_{Fn} = \frac{3220}{32 \cdot 2} \cdot \frac{66^2}{600} = 726$$

$$F_F = \sqrt{F_{Fn}^2 + F_{Fr}^2}$$

$$F_F = \sqrt{726^2 + 564.7^2} = 920 \text{ N}$$

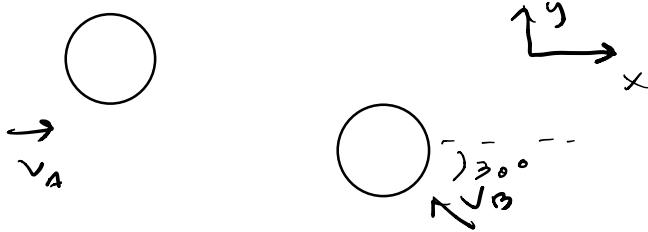
- 3/261 Two identical hockey pucks moving with initial velocities v_A and v_B collide as shown. If the coefficient of restitution is $e = 0.75$, determine the velocity (magnitude and direction θ with respect to the positive x -axis) of each puck just after impact. Also calculate the percentage loss n of system kinetic energy.



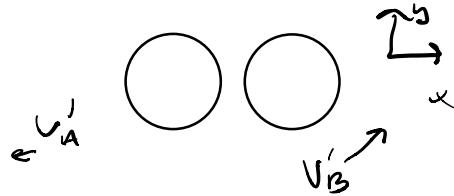
Problem 3/261

2) $F_{BO} + \text{state}$

State 1 Before impact



State 2 Right after impact



3) Coefficient of restitution equation (x axis)

$$e = \frac{v_{Bx}' - v_{Ax}'}{v_{Ax} - v_{Bx}}$$

$$0.75 (v_{Ax} - v_{Bx}) = v_{Bx}' - v_{Ax}'$$

$$0.75(6 - (-10 \cos 30)) = v_{Bx'} - v_{Ax'}$$

$$10.995 = v_{Bx'} - v_{Ax'} \quad (1)$$

$$10.995 + v_{Ax'} = v_{Bx'}$$

4) Conservation of momentum (x axis)

$$m_A v_{Ax} - m_B v_B \cos 30^\circ = m_A v_{Ax'} + m_B v_{Bx'}$$

$$v_A - v_B \cos 30^\circ = v_{Ax'} + v_{Bx'}, \quad (2)$$

(1) and (2)

$$6 - 10 \cos 30^\circ = v_{Ax'} + 10.995 + v_{Ax'}$$

$$6 - 10 \cos 30^\circ - 10.995 = 2v_{Ax'}$$

$$v_{Ax'} = -6.828 \text{ m/s}$$

$$v_{Bx'} = 10.995 - 6.828$$

$$v_{Bx'} = 4.167$$

Conservation of momentum (y direction)

Because there is no force in the y -direction

$$m_A v_{Ay'} = m_A v_{Ay}$$

$$v_{Ay'} = 0$$

$$m_B v_{By'} = m_B v_{By}$$

$$v_{By'} = v_{By}$$

$$v_{By'} = 10 \sin 30^\circ$$

$$v_{By'} = 5 \text{ m/s}$$

$$v_A' = \sqrt{v_{Ax'}^2 + v_{Ay'}^2} = \sqrt{(-6.828)^2 + 0^2}$$

$$V_A' = 6.83 \text{ m/s}$$

$$\theta_A = 180^\circ$$

(opposite x direction)

$$V_B' = \sqrt{(V_{Bx}')^2 + (V_{By}')^2} = \sqrt{4.17^2 + 5^2}$$

$$V_B' = 6.51 \text{ m/s}$$

$$\theta_B = \tan^{-1} \left(\frac{V_{By}'}{V_{Bx}'} \right) = \tan^{-1} \left(\frac{5}{4.17} \right) \quad \theta_B = 50.2^\circ$$

$$n = \frac{\Delta KE}{KE_1} \times 100\% = \frac{KE_1 - KE_2}{KE_1}$$

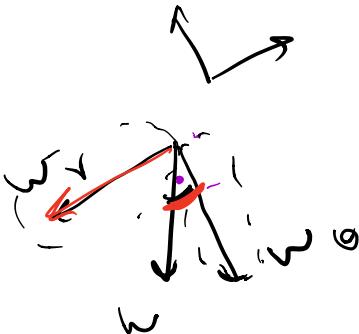
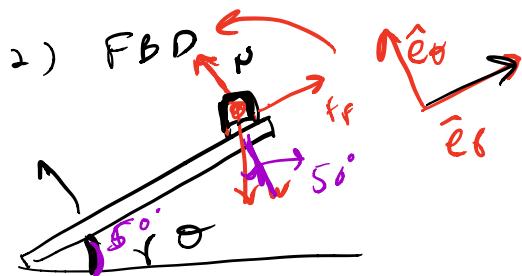
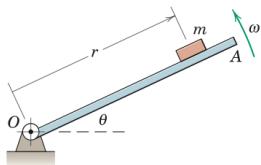
$$= \frac{\frac{1}{2}m(V_A'^2 + V_B'^2) - \frac{1}{2}m(V_A^2 + V_B^2)}{\frac{1}{2}m(V_A^2 + V_B^2)} \times 100\%$$

$$n = \frac{(6^2 + 10^2) - (6.83^2 + 6.51^2)}{(6^2 + 10^2)} \times 100\%$$

$$n = 34.53\%$$

- 3/54 The member OA rotates about a horizontal axis through O with a constant counterclockwise angular velocity $\omega = 3 \text{ rad/sec}$. As it passes the position $\theta = 0$, a small block of mass m is placed on it at a radial distance $r = 18 \text{ in}$. If the block is observed to slip at $\theta = 50^\circ$, determine the coefficient of static friction μ_s between the block and the member.

1) Ref system: Polar



3) Equations (2 minitcs)

$$\Sigma F_r = m a_r$$

$$f_f - w \sin \theta = m (r \dot{\theta}^2)$$

$$\mu_s N - mg \sin \theta = -m r \dot{\theta}^2$$

$$\mu_s = \frac{-\mu_s r \dot{\theta}^2 + mg \sin \theta}{mg \cos \theta}$$

$$\mu_s = \frac{-\frac{18}{12} \cdot 3^2 + 32.2 \sin 50^\circ}{32.2 \cos 50^\circ}$$

$$\Sigma F_\theta = m a_\theta$$

$$N - w \cos \theta = m (r \dot{\theta}^2 + r \ddot{\theta})$$

$$N = w \cos \theta$$

$$N = mg \cos \theta$$

$\mu_s = 0.54$