

Final review

Midterm 1

Forces acting on an aircraft

Steady
level
flight

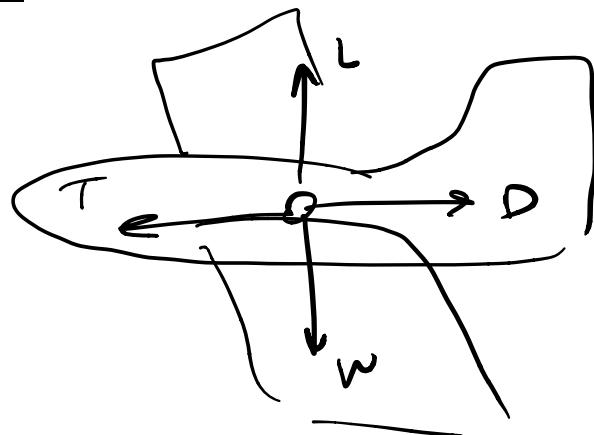
$$V_\infty \rightarrow$$

$$V \perp L$$

$$V \parallel D$$

$$L = \frac{1}{2} \rho V_\infty^2 C_L \cdot S$$

$$D = \frac{1}{2} \rho V_\infty^2 C_D \cdot S$$



$$T = D$$

$$L = W$$

Wing loading : $\frac{W}{S}$

Flight regimes: $M = \frac{V_\infty}{a_\infty}$ $\frac{V_\infty}{\sqrt{\gamma R T}}$

| | | |
|-----------------|-----------------------|---|
| $M < 0.3$ | incompressible | Assumes constant density. Air behaves as a perfect gas |
| $0.3 < M < 0.8$ | compressible subsonic | Density changes are now substantial and should be accounted for. Assumes air is a perfect gas |
| $0.8 < M < 1.2$ | Transonic | Mixed flow types. Including subsonic and supersonic in certain regions |
| $1.2 < M < 5$ | Supersonic | Shock waves formation. Large temperature increase |

| | | |
|-----|------------|--|
| M>5 | Hypersonic | Air is no longer behaving as a perfect gas. Shocks. Very high temperatures |
|-----|------------|--|

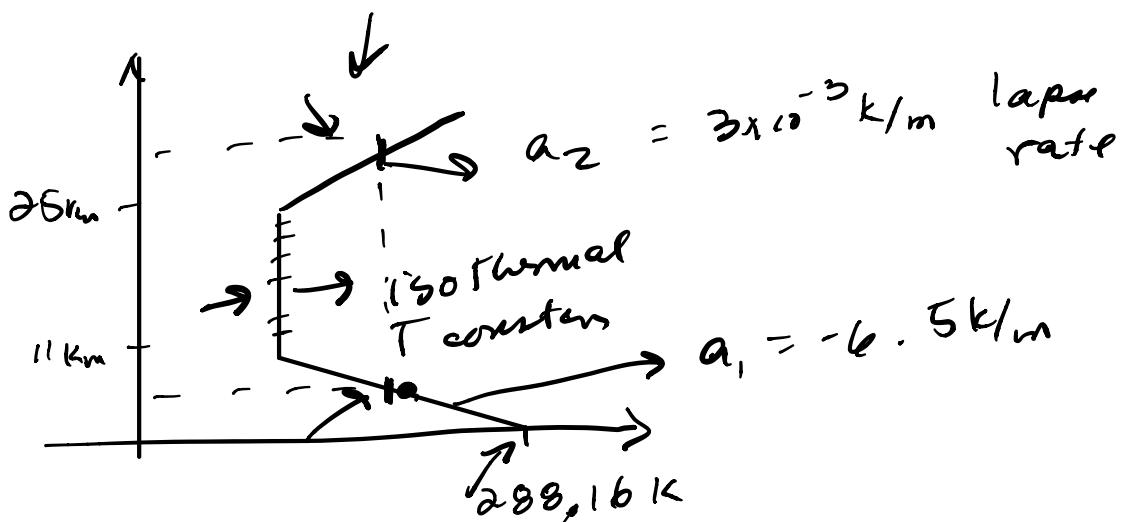
Standard atmosphere: Agreed upon standard for what pressure, temperature and density will likely be at a given altitude

Pressure= F/A Normal force per unit area. It decreases as we go up in altitude

Density=m/V Mass per unit volume

Temperature: Average kinetic energy of a collection of gas molecules.

The variation of temperature with altitude



You cannot find altitude based only on temperature because a value of temperature can correspond to more than one altitude

The pressure and density variations with altitude are obtained from this empirical temperature variation by using the laws of physics.

Temperature gradient regions

$$T_2 = T_1 + a \left(\frac{h_2 - h_1}{R} \right)$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{-\frac{g_0}{aR}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1} \right)^{-\left[\frac{g_0}{aR} + 1 \right]}$$

Isothermal regions

T is constant

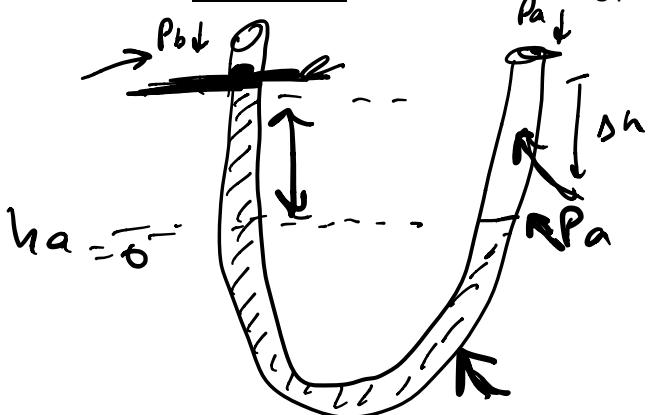
$$\frac{P_2}{P_1} = e^{\left[-\frac{g}{RT} (h_2 - h_1) \right]}$$
$$\frac{P_2}{P_1} = e^{-\frac{g}{RT} (h_2 - h_1)}$$

Equation of state

Assumes a perfect gas

$$P = \rho R T$$

Manometers: Useful for measuring pressure



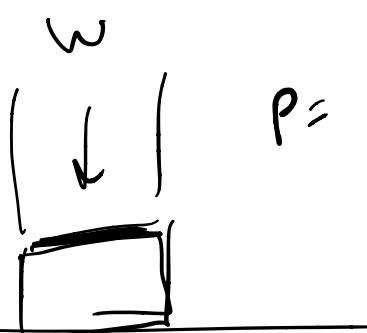
$$\Delta P = -\rho g \Delta h$$

$$P_b < P_a$$

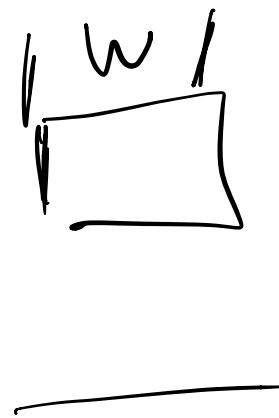
$$P_b - P_a = -\rho g (h_b - h_a)$$

$$P_b = -\rho g (h_b - h_a) + P_a$$

$$P_b = -\rho g \underbrace{h_b}_{h_b + P_a} + P_a$$



$$P = F/A$$



Continuity equation:

Based on conservation of mass: Mass can be neither created nor destroyed

Mass flow rate

$$\dot{m} = \rho v A$$

Looking at two points $\dot{m}_1 = \dot{m}_2$
 $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$

If incompressible

$$v_1 A_1 = v_2 A_2$$

Bernoulli

Derived from the conservation of momentum

Assumptions:

1. Inviscid
2. Neglect gravity (no body forces)
3. Steady
4. Incompressible
5. Same streamline

$$P_2 + \frac{1}{2} \rho v_2^2 = P_1 + \frac{1}{2} \rho v_1^2$$
$$M < 0.3$$
$$M = \frac{V}{a} = \sqrt{\frac{V^2 + RT}{\gamma RT}} \quad T = 288K$$

Midterm 2

For any process

-Energy equation (relates temperature and velocity)

$$C_p T_1 + \frac{1}{2} v_1^2 = C_p T_2 + \frac{1}{2} v_2^2$$

-Isentropic flow

Assumptions:

Adiabatic: No heat transfer

Reversible: No friction

Isentropic relations

$$\frac{P_2}{P_1} = \left(\frac{P_2}{P_1} \right)^\gamma \quad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\gamma/\gamma-1} \quad \left(\frac{P_2}{P_1} \right)^\gamma = \left(\frac{T_2}{T_1} \right)^{\gamma/\gamma-1}$$

We cannot assume incompressible; density must be allowed to change (Can't use Bernoulli)

Subsonic wind tunnels

Most of the times can assume incompressible because we are dealing with low speed.

- Velocity increases as the area decreases through the convergent nozzle, and the opposite occurs for the divergent part

Different types of pressures

- Static pressure is the pressure we would feel if we were moving along with the flow (Standard atmosphere table) $\rightarrow p_s \rightarrow T_s \rightarrow \rho_s$

- Total pressure or stagnation pressure: The pressure obtained at a point where the flow velocity has been decreased to zero. p_0

- It is a property of the flow.

- Constant throughout. (We can use it to find the pressure at other points)

- It is measured by a pitot tube

- If we assume incompressible the Bernoulli's equation can be used, which relates dynamic pressure, total pressure, and static pressure.

$$p_0 + \frac{1}{2} \rho v^2 = p_s + \frac{1}{2} \rho v^2$$

$$p_0 - p_s = \frac{1}{2} \rho v^2 \leftarrow \text{dynamic}$$

- Solving for true airspeed which deals with the actual density.

$$\sqrt{\frac{(p_0 - p_s)}{\rho}}^{1/2} = V_{true}$$

Equivalent airspeed is the airspeed measured by an airspeed indicator and deals with sea level density. If assuming incompressible flow, then

$$V_{eq} = \sqrt{\frac{(p_0 - p_s)}{\rho_{SL}}}^{1/2}$$

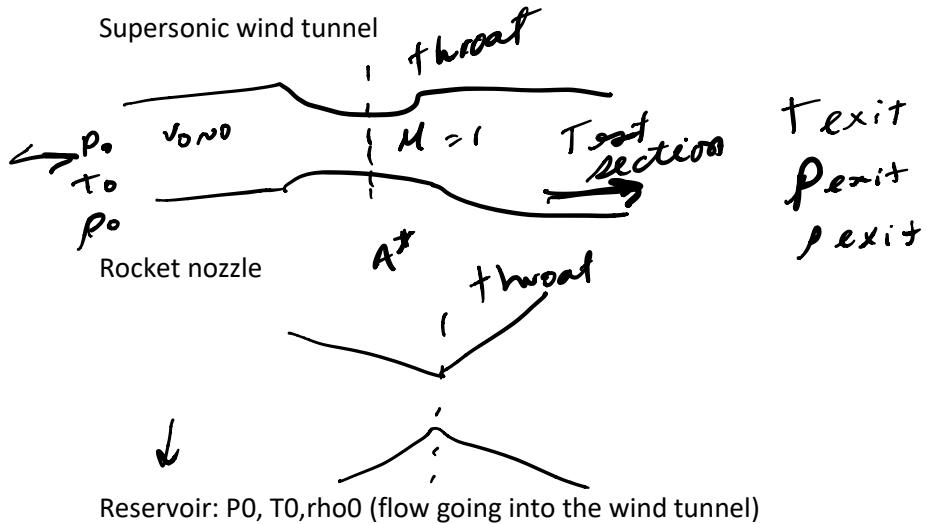
An equation that relates true airspeed and equivalent airspeed:

$$V_{true} = V_{eq} \sqrt{\frac{\rho}{\rho_{SL}}} \quad \leftarrow \begin{array}{l} \text{incompressible} \\ \text{Bernoulli;} \\ M=0.45 \end{array}$$

We can find Mach number using the isentropic Mach relations

Supersonic wind tunnels

-For the velocity to increase the area must increase



Test section: $P_{exit}, T_{exit}, \rho_{exit}$ (flow going out).

$$\left(\frac{A_{exit}}{A^*}\right)^2 = \frac{1}{M_{exit}^2} \left[\frac{\gamma}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{\gamma-1}}$$

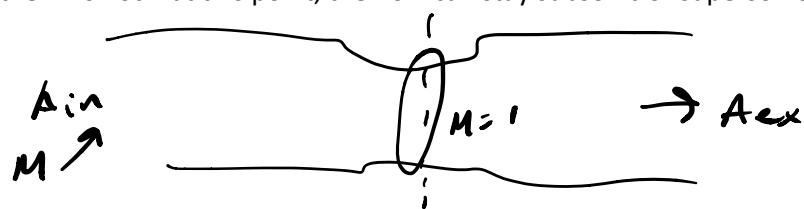
Area/Mach relations

We are given a Mach number or an area ratio, we can get either of those from the table

- Be careful when using the table, since your result will depend on if the flow is subsonic or supersonic
- It's important to know that a throat is the point where the smallest area of a wind tunnel or rocket nozzle can be found, but having a throat does not necessarily mean that you have a choke point where $M=1$.
- A^* which is the area where $M=1$ can be thought as a property of the flow like P_0, T_0 and ρ_0 . Even if we do not physically have it, we can still solve for it, and use this value to find other variables.
- You will see indications that wind tunnel has a physical choke point (e.g. if you are told that the flow goes from subsonic to supersonic)
- Even if there is throat, if the M is not 1 at this point, the flow can stay subsonic or supersonic.

Midterm 3 review. Weeks 8-11

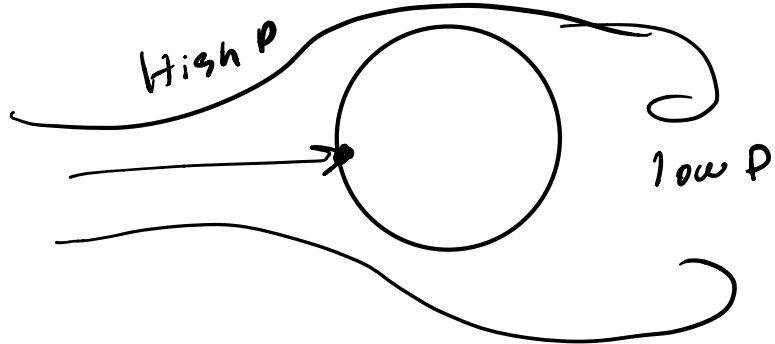
Viscous flow



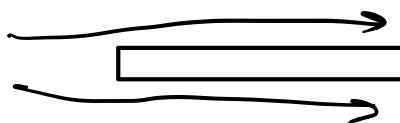
Types of drags:

Pressure drag: Mostly affect bluff bodies. It causes high pressure upstream and low pressure downstream, this is due to flow separation. Acts perpendicular to the surface

Flow separation: Where the streamlines can no longer follow the curvature of the body.



Skin friction drag: Is produced by the friction of the air molecules with the surface which creates a shear stress at the surface. This acts in a direction tangential to it.



Viscosity for air at standard sea-level temperature

$$\mu = 1.7894 \times 10^{-5} \text{ kg/(m)(s)} = 3.7373 \times 10^{-7} \text{ slug/(ft)(s)} \rightarrow \text{English}$$

SI Viscosity can be calculated

$$\mu = 1.458 \left(\frac{T^{3/2}}{T + 110.4} \right) \times 10^{-6} \text{ kg/m.s}$$

$$\mu = 2.27 \left(\frac{T^{3/2}}{T + 199} \right) \times 10^{-8} \text{ slug/ft.s}$$

Reynolds number: Non dimensional parameter. Describes the behavior of viscosity. High Reynolds number indicates low viscosity, and low Reynolds number indicates high viscosity.

$$Re = \frac{\rho v' x}{\mu}$$



Types of flow

Laminar flow: Streamlines are smooth and regular, and a fluid element moves smoothly along a streamline.

Boundary layer thickness

$$\delta = \frac{5.2 x}{\sqrt{Re_x}}$$

Total Skin friction coefficient

$$C_f = \frac{1.328}{\sqrt{Re_x}}$$

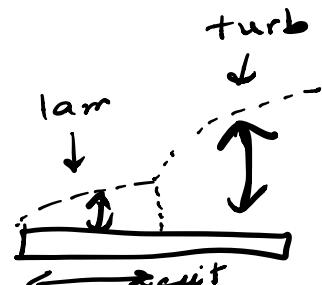
Turbulent flow: The streamlines break up and a fluid element moves in a random, irregular fashion

Boundary layer thickness

$$\delta = \frac{0.37 x}{R_c^{0.2}}$$

Total skin friction coefficient

$$C_f = \frac{0.074}{Re^{0.2}}$$



Laminar shear stress is less than the turbulent shear stress. Therefore, the skin friction is higher for turbulent flow. Turbulent boundary layer is thicker and grows faster.

In reality, the flow always starts out from the leading edge as laminar, and then at the transition point the boundary layer becomes completely turbulent where the boundary layer grows at a faster rate.

This point where transition occurs is called the critical point, which corresponds to a critical Reynolds number

$$Re_{crit} = \frac{\rho v x_{crit}}{\mu}$$

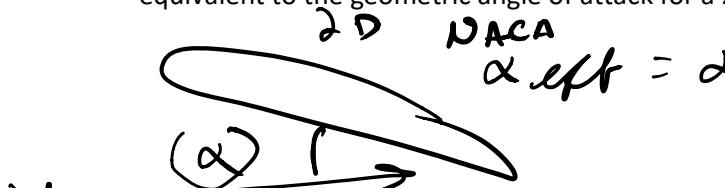
For a 2D infinite airfoil (Cl,Cd,Cm) and a 3D finite wing (CL,CD,CM), the lift and drag coefficient are different.

This is because for an airfoil section, the end effects are removed when testing in a wing tunnel.

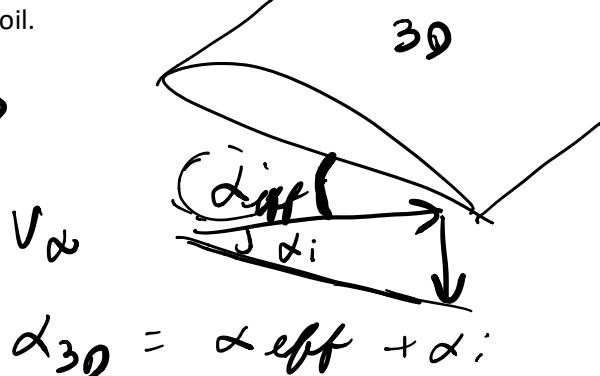
For a 3 D wing these end effects produce a downward component called downwash. This causes an induced drag, which increases the total drag and reduces the lift.

Downwash causes the relative wind in the proximity of the airfoil section to be inclined slightly downward through a small angle called the induced angle of attack. This in turn reduces the angle of attack felt by the local airfoil section to a value smaller than the geometric angle of attack. This smaller

angle of attack is called the effective angle of attack. The effective angle of attack for a 3D wing is equivalent to the geometric angle of attack for a 2D airfoil.



$$\alpha \rightarrow C_L \rightarrow C_D$$



Induced AoA=

(in radians)

Induced AoA

(in degrees)

$$CD_i = \frac{C_L^2}{\pi A R e} \leftarrow$$

AR=

If we have an elliptical wing(ideal case)then $e=1$

Where e is the span efficiency factor

$$CD_i = \frac{C_L^2}{\pi A R}$$

Induced Drag=

$$D_i = \frac{1}{2} \rho v^2 S C_D i$$

To find total drag:

$$CD = CD_d + CD_i$$

Dtot=

$$D = \frac{1}{2} \rho v^2 (C_L + CD_i)$$

Where C_d is the profile drag. C_d for a 2D case



For lift coefficient

$$C_L = a(\alpha - \alpha_{L=0})$$

\uparrow \uparrow
 a α

For the 3D lift curve slope

$$a = \frac{ab}{1 + 57.3 \frac{\alpha_0}{\pi A Re}} \rightarrow \text{per degree}$$

$$\frac{\partial C_L}{\partial \alpha} \rightarrow \beta_D$$

$$\alpha_0 \rightarrow \frac{\partial C_L}{\partial \alpha} \rightarrow \beta_D = 0.11$$

Pressure coefficient

$$C_p = \frac{P - P_\infty}{\rho_\infty}$$

$$C_p = 1 - \left(\frac{V}{V_\infty} \right)^2 \rightarrow \text{incompressible}$$

\uparrow

Compressible flow

Prandtl Glauert Compressibility Correction

$$C_p = \frac{C_p M_\infty = 0}{\sqrt{1 - M_\infty^2}}$$

$V_\infty = 60 \text{ m/s}$ ↗
 $V_\infty = 80 \text{ m/s}$ ↗
 $V_\infty = 240 \text{ m/s}$ ↗
 $C_p \text{ crit} = ?$

~~$1 - \left(\frac{80}{240} \right)^2$~~

$C_p M_\infty = 0 = 1 - \left(\frac{80}{60} \right)^2$

Compressibility correction for lift coefficient

$$C_L = C_{L M_\infty = 0} \sqrt{1 - M_\infty^2}$$

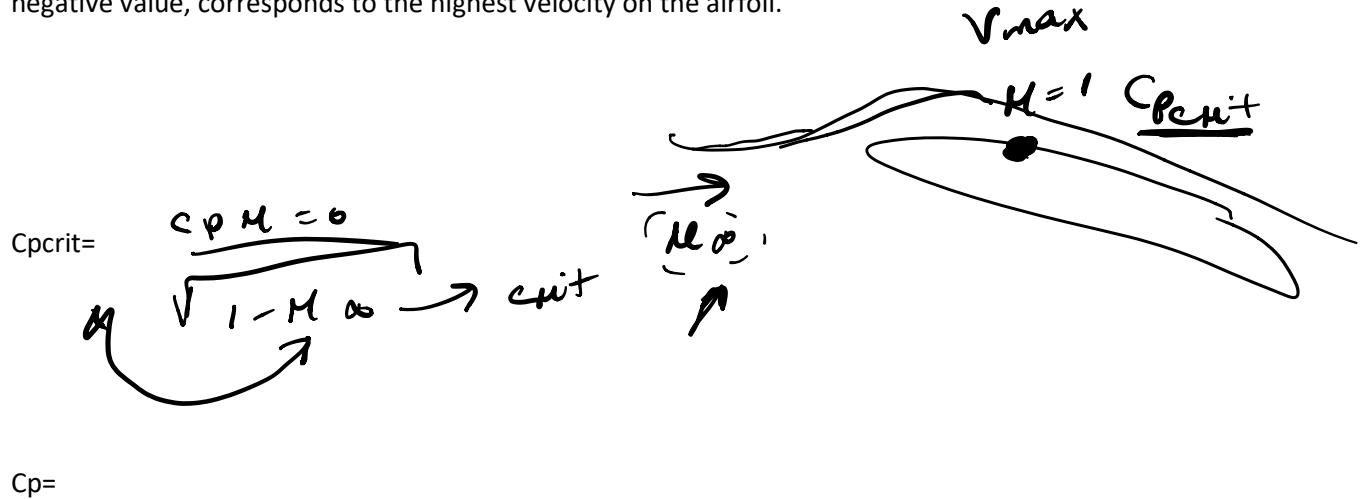
\downarrow \downarrow
 $M_\infty = 0.3$ 60 m/s

$$C_L = 0.3$$

Critical Mach number and Critical Pressure Coefficient

Critical Mach number: The freestream Mach number at which the flow around the airfoil first reaches sonic conditions ($M=1$)

Critical Pressure Coefficient: Pressure coefficient on the airfoil that correspond to $M=1$. This is the most negative value, corresponds to the highest velocity on the airfoil.



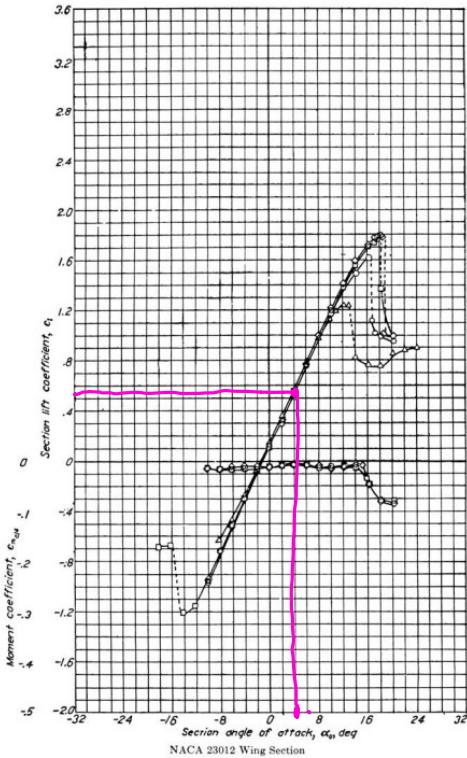
$$C_p =$$

$$C_{p\text{crit}} =$$

Consider a wing with an aspect ratio of 10 and an NACA 23012 airfoil section. Assume that $Re = 5 \times 10^6$. The span efficiency factor is $e = e_1 = 0.95$. If the wing is at a 4° angle of attack, calculate C_L and C_D .

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APPENDIX D Airfoil Data



$$\alpha_L = -1.5^\circ$$

$$d = 40 \Rightarrow \text{geometric}$$

$$a = \frac{a_0}{1 + \frac{\zeta 7.3 a_0}{\pi e A R}} \rightarrow 20$$

(3D)

$$a_0 = \frac{\Delta C_L}{\Delta \alpha} = \frac{1.2 - 0.14}{10^\circ - 0^\circ} = \frac{1.06}{10^\circ}$$

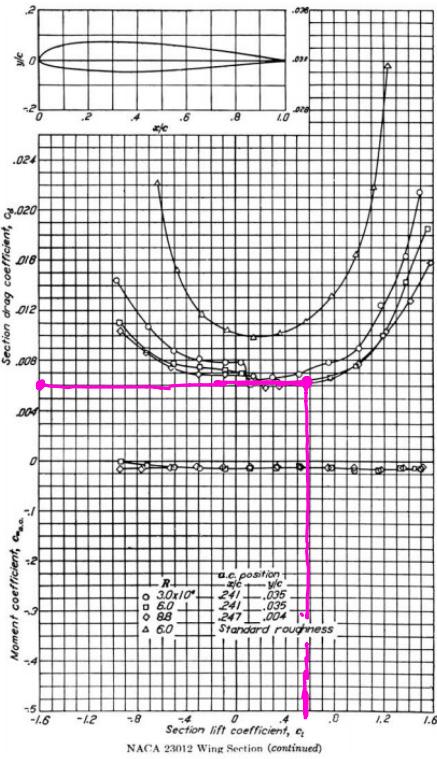
$$a_0 = 0.106 \left[\frac{1}{\text{deg}} \right]$$

20 Lift slope

$$a = \frac{0.106 \left[\frac{1}{\text{deg}} \right]}{1 + \frac{\zeta 7.3 (0.106 \left[\frac{1}{\text{deg}} \right])}{\pi e A R}} = 0.088 \left[\frac{1}{\text{deg}} \right]$$

APPENDIX D Airfoil Data

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$$\frac{C_L}{\pi (0.95)(10)}$$

\downarrow
 e_1

\downarrow
 R

● $a = \frac{\Delta C_L}{\Delta \alpha} = \frac{C_L - 0}{4^\circ - \alpha_{L=0}} = \frac{C_L}{4^\circ - (-1.5^\circ)}$

3D Lift slope

$$\frac{\alpha_{L=0}|_{2D}}{\alpha_{L=0}|_{3D}} =$$

● $\frac{C_L}{4^\circ + 1.5^\circ} = 0.088 \left[\frac{1}{\alpha_a} \right]$

$$C_L = 0.484$$

● $C_D = C_d + \frac{C_L^2}{\pi e R}$

$\hookrightarrow \alpha_{3D} \rightarrow C_L \rightarrow C_d$

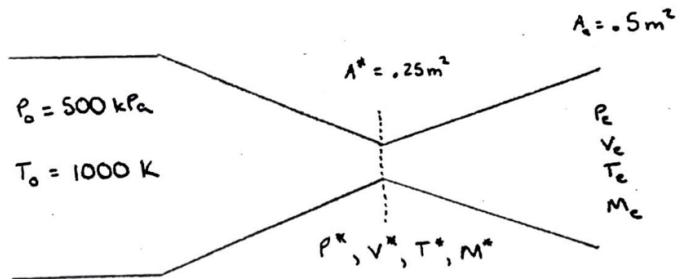
Geometrisch

$$C_D = 0.006 + \frac{(0.484)^2}{\pi (0.95)(10)}$$

$$C_D = 0.0138$$

Question: Compressed air flows through a convergent-divergent nozzle and exits supersonically. Given the throat and exit areas, as well as the stagnation pressure and temperature, determine the pressure, temperature, Mach number, and flow velocity at both the throat and exit of the nozzle. Assume the flow is isentropic throughout.

Assume $\gamma = 1.4$ (constant C_p, C_v)



$M^* = 1$ since flow exits supersonically

$$\frac{P_0}{P^*} = \left[1 + \frac{\gamma - 1}{2} M^*{}^2 \right]^{\frac{\gamma}{\gamma - 1}}$$

$$P^* = \frac{P_0}{\left[1 + \frac{\gamma - 1}{2} M^*{}^2 \right]^{\frac{\gamma}{\gamma - 1}}}$$

$$P^* = \frac{500}{\left[1 + \frac{1.4 - 1}{2} (1)^2 \right]^{\frac{1.4}{1.4 - 1}}}$$

$$P^* = 264.15 \text{ kPa}$$

$$\frac{T_0}{T^*} = 1 + \frac{\gamma - 1}{2} M^{*2}$$

$$T^* = \frac{T_0}{1 + \frac{\gamma - 1}{2} M^{*2}}$$

$$T^* = \frac{1000}{1 + \frac{1.4 - 1}{2} (1)^2}$$

$$T^* = 833.33 \text{ K}$$

$$a^* = \sqrt{\gamma R T^*} \quad R = 287 \text{ J/kg} \cdot \text{K}$$

$$a^* = \sqrt{(1.4)(287)(833.33)}$$

$$a^* = 578.65 \text{ m/s}$$

$$M^* = \frac{V^*}{a^*}$$

$$1 = \frac{V^*}{a^*}$$

$$V^* = a^* \rightarrow V^* = 578.65 \text{ m/s}$$

$$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M_e^2 \right) \right]^{\frac{\gamma + 1}{\gamma - 1}}$$

$$\frac{A_e}{A^*} = \frac{0.5}{0.25} = 2 \rightarrow \text{use Isentropic-Mach Tables}$$

Linearly interpolate from table

$$\underbrace{\frac{2.2 - M_e}{2.2 - 2.15}}_M = \frac{2.005 - 2}{2.005 - 1.919} \underbrace{\frac{A_e}{A^*}}$$

$$M_e = 2.197$$

Subsonic

Table A.1 Isentropic flow properties

| M | $\frac{p_e}{p}$ | $\frac{\rho_e}{\rho}$ | $\frac{T_e}{T}$ | $\frac{A}{A^*}$ |
|-------------|-----------------|-----------------------|-----------------|-----------------|
| 0.2000 + 01 | 0.1000 + 01 | 0.1000 + 01 | 0.1000 + 01 | 0.2894 + 02 |
| 0.4000 + 01 | 0.1001 + 01 | 0.1001 + 01 | 0.1000 + 01 | 0.1448 + 02 |
| 0.6000 + 01 | 0.1003 + 01 | 0.1002 + 01 | 0.1001 + 01 | 0.9666 + 01 |
| 0.8000 + 01 | 0.1004 + 01 | 0.1003 + 01 | 0.1001 + 01 | 0.7262 + 01 |
| 0.1000 + 00 | 0.1007 + 01 | 0.1005 + 01 | 0.1002 + 01 | 0.5822 + 01 |
| 0.1200 + 00 | 0.1010 + 01 | 0.1007 + 01 | 0.1003 + 01 | 0.4864 + 01 |
| 0.1400 + 00 | 0.1014 + 01 | 0.1010 + 01 | 0.1004 + 01 | 0.4182 + 01 |
| 0.1600 + 00 | 0.1018 + 01 | 0.1013 + 01 | 0.1005 + 01 | 0.3673 + 01 |
| 0.1800 + 00 | 0.1023 + 01 | 0.1016 + 01 | 0.1006 + 01 | 0.3278 + 01 |
| 0.2000 + 00 | 0.1028 + 01 | 0.1020 + 01 | 0.1008 + 01 | 0.2964 + 01 |
| 0.2200 + 00 | 0.1034 + 01 | 0.1024 + 01 | 0.1010 + 01 | 0.2708 + 01 |
| 0.2400 + 00 | 0.1041 + 01 | 0.1029 + 01 | 0.1012 + 01 | 0.2496 + 01 |
| 0.2600 + 00 | 0.1048 + 01 | 0.1034 + 01 | 0.1014 + 01 | 0.2317 + 01 |
| 0.2800 + 00 | 0.1056 + 01 | 0.1040 + 01 | 0.1016 + 01 | 0.2166 + 01 |
| 0.3000 + 00 | 0.1064 + 01 | 0.1046 + 01 | 0.1018 + 01 | 0.2035 + 01 |
| 0.3200 + 00 | 0.1074 + 01 | 0.1052 + 01 | 0.1020 + 01 | 0.1922 + 01 |
| 0.3400 + 00 | 0.1083 + 01 | 0.1059 + 01 | 0.1023 + 01 | 0.1823 + 01 |
| 0.3600 + 00 | 0.1094 + 01 | 0.1066 + 01 | 0.1026 + 01 | 0.1736 + 01 |
| 0.3800 + 00 | 0.1105 + 01 | 0.1074 + 01 | 0.1029 + 01 | 0.1659 + 01 |
| 0.4000 + 00 | 0.1117 + 01 | 0.1082 + 01 | 0.1032 + 01 | 0.1590 + 01 |
| 0.4200 + 00 | 0.1129 + 01 | 0.1091 + 01 | 0.1035 + 01 | 0.1529 + 01 |
| 0.4400 + 00 | 0.1142 + 01 | 0.1100 + 01 | 0.1039 + 01 | 0.1474 + 01 |
| 0.4600 + 00 | 0.1156 + 01 | 0.1109 + 01 | 0.1042 + 01 | 0.1425 + 01 |
| 0.4800 + 00 | 0.1171 + 01 | 0.1119 + 01 | 0.1046 + 01 | 0.1380 + 01 |
| 0.5000 + 00 | 0.1186 + 01 | 0.1130 + 01 | 0.1050 + 01 | 0.1340 + 01 |
| 0.5200 + 00 | 0.1202 + 01 | 0.1141 + 01 | 0.1054 + 01 | 0.1303 + 01 |
| 0.5400 + 00 | 0.1219 + 01 | 0.1152 + 01 | 0.1058 + 01 | 0.1270 + 01 |
| 0.5600 + 00 | 0.1237 + 01 | 0.1164 + 01 | 0.1063 + 01 | 0.1240 + 01 |
| 0.5800 + 00 | 0.1256 + 01 | 0.1177 + 01 | 0.1067 + 01 | 0.1213 + 01 |
| 0.6000 + 00 | 0.1276 + 01 | 0.1190 + 01 | 0.1072 + 01 | 0.1188 + 01 |
| 0.6200 + 00 | 0.1296 + 01 | 0.1203 + 01 | 0.1077 + 01 | 0.1166 + 01 |
| 0.6400 + 00 | 0.1317 + 01 | 0.1218 + 01 | 0.1082 + 01 | 0.1143 + 01 |
| 0.6600 + 00 | 0.1340 + 01 | 0.1232 + 01 | 0.1087 + 01 | 0.1127 + 01 |
| 0.6800 + 00 | 0.1363 + 01 | 0.1247 + 01 | 0.1092 + 01 | 0.1110 + 01 |
| 0.7000 + 00 | 0.1387 + 01 | 0.1263 + 01 | 0.1098 + 01 | 0.1094 + 01 |
| 0.7200 + 00 | 0.1412 + 01 | 0.1280 + 01 | 0.1104 + 01 | 0.1081 + 01 |
| 0.7400 + 00 | 0.1439 + 01 | 0.1297 + 01 | 0.1110 + 01 | 0.1068 + 01 |
| 0.7600 + 00 | 0.1466 + 01 | 0.1314 + 01 | 0.1116 + 01 | 0.1057 + 01 |
| 0.7800 + 00 | 0.1493 + 01 | 0.1333 + 01 | 0.1122 + 01 | 0.1047 + 01 |
| 0.8000 + 00 | 0.1524 + 01 | 0.1351 + 01 | 0.1128 + 01 | 0.1038 + 01 |
| 0.8200 + 00 | 0.1555 + 01 | 0.1371 + 01 | 0.1134 + 01 | 0.1030 + 01 |
| 0.8400 + 00 | 0.1587 + 01 | 0.1391 + 01 | 0.1141 + 01 | 0.1024 + 01 |
| 0.8600 + 00 | 0.1621 + 01 | 0.1412 + 01 | 0.1148 + 01 | 0.1018 + 01 |
| 0.8800 + 00 | 0.1655 + 01 | 0.1433 + 01 | 0.1155 + 01 | 0.1013 + 01 |
| 0.9000 + 00 | 0.1691 + 01 | 0.1456 + 01 | 0.1162 + 01 | 0.1009 + 01 |
| 0.9200 + 00 | 0.1729 + 01 | 0.1478 + 01 | 0.1169 + 01 | 0.1006 + 01 |
| 0.9400 + 00 | 0.1767 + 01 | 0.1502 + 01 | 0.1177 + 01 | 0.1003 + 01 |
| 0.9600 + 00 | 0.1808 + 01 | 0.1526 + 01 | 0.1184 + 01 | 0.1001 + 01 |
| 0.9800 + 00 | 0.1850 + 01 | 0.1552 + 01 | 0.1192 + 01 | 0.1000 + 01 |
| 0.1000 + 01 | 0.1893 + 01 | 0.1577 + 01 | 0.1200 + 01 | 0.1000 + 01 |

Supersonic

Table A.1—Continued

| M | $\frac{p_e}{p}$ | $\frac{\rho_e}{\rho}$ | $\frac{T_e}{T}$ | $\frac{A}{A^*}$ |
|-------------|-----------------|-----------------------|-----------------|-----------------|
| 0.1020 + 01 | 0.1938 + 01 | 0.1604 + 01 | 0.1208 + 01 | 0.1000 + 01 |
| 0.1040 + 01 | 0.1985 + 01 | 0.1612 + 01 | 0.1216 + 01 | 0.1001 + 01 |
| 0.1060 + 01 | 0.2033 + 01 | 0.1660 + 01 | 0.1225 + 01 | 0.1003 + 01 |
| 0.1080 + 01 | 0.2082 + 01 | 0.1689 + 01 | 0.1233 + 01 | 0.1005 + 01 |
| 0.1100 + 01 | 0.2135 + 01 | 0.1719 + 01 | 0.1242 + 01 | 0.1008 + 01 |
| 0.1120 + 01 | 0.2189 + 01 | 0.1750 + 01 | 0.1251 + 01 | 0.1011 + 01 |
| 0.1140 + 01 | 0.2245 + 01 | 0.1782 + 01 | 0.1260 + 01 | 0.1015 + 01 |
| 0.1160 + 01 | 0.2303 + 01 | 0.1814 + 01 | 0.1269 + 01 | 0.1020 + 01 |
| 0.1180 + 01 | 0.2363 + 01 | 0.1848 + 01 | 0.1278 + 01 | 0.1025 + 01 |
| 0.1200 + 01 | 0.2425 + 01 | 0.1883 + 01 | 0.1288 + 01 | 0.1030 + 01 |
| 0.1220 + 01 | 0.2489 + 01 | 0.1918 + 01 | 0.1298 + 01 | 0.1037 + 01 |
| 0.1240 + 01 | 0.2556 + 01 | 0.1955 + 01 | 0.1308 + 01 | 0.1043 + 01 |
| 0.1260 + 01 | 0.2623 + 01 | 0.1992 + 01 | 0.1318 + 01 | 0.1050 + 01 |
| 0.1280 + 01 | 0.2697 + 01 | 0.2031 + 01 | 0.1328 + 01 | 0.1058 + 01 |
| 0.1300 + 01 | 0.2771 + 01 | 0.2071 + 01 | 0.1338 + 01 | 0.1066 + 01 |
| 0.1320 + 01 | 0.2847 + 01 | 0.2112 + 01 | 0.1348 + 01 | 0.1075 + 01 |
| 0.1340 + 01 | 0.2927 + 01 | 0.2153 + 01 | 0.1359 + 01 | 0.1084 + 01 |
| 0.1360 + 01 | 0.3009 + 01 | 0.2197 + 01 | 0.1370 + 01 | 0.1094 + 01 |
| 0.1380 + 01 | 0.3094 + 01 | 0.2241 + 01 | 0.1381 + 01 | 0.1104 + 01 |
| 0.1400 + 01 | 0.3182 + 01 | 0.2286 + 01 | 0.1392 + 01 | 0.1115 + 01 |
| 0.1420 + 01 | 0.3273 + 01 | 0.2333 + 01 | 0.1403 + 01 | 0.1126 + 01 |
| 0.1440 + 01 | 0.3368 + 01 | 0.2381 + 01 | 0.1413 + 01 | 0.1138 + 01 |
| 0.1460 + 01 | 0.3465 + 01 | 0.2430 + 01 | 0.1426 + 01 | 0.1150 + 01 |
| 0.1480 + 01 | 0.3566 + 01 | 0.2480 + 01 | 0.1438 + 01 | 0.1161 + 01 |
| 0.1500 + 01 | 0.3671 + 01 | 0.2532 + 01 | 0.1450 + 01 | 0.1176 + 01 |
| 0.1520 + 01 | 0.3779 + 01 | 0.2585 + 01 | 0.1462 + 01 | 0.1190 + 01 |
| 0.1540 + 01 | 0.3891 + 01 | 0.2639 + 01 | 0.1474 + 01 | 0.1204 + 01 |
| 0.1560 + 01 | 0.4007 + 01 | 0.2695 + 01 | 0.1487 + 01 | 0.1219 + 01 |
| 0.1580 + 01 | 0.4127 + 01 | 0.2752 + 01 | 0.1499 + 01 | 0.1234 + 01 |
| 0.1600 + 01 | 0.4250 + 01 | 0.2811 + 01 | 0.1512 + 01 | 0.1250 + 01 |
| 0.1620 + 01 | 0.4378 + 01 | 0.2871 + 01 | 0.1525 + 01 | 0.1267 + 01 |
| 0.1640 + 01 | 0.4511 + 01 | 0.2933 + 01 | 0.1538 + 01 | 0.1284 + 01 |
| 0.1660 + 01 | 0.4648 + 01 | 0.2996 + 01 | 0.1551 + 01 | 0.1301 + 01 |
| 0.1680 + 01 | 0.4790 + 01 | 0.3061 + 01 | 0.1564 + 01 | 0.1319 + 01 |
| 0.1700 + 01 | 0.4936 + 01 | 0.3128 + 01 | 0.1578 + 01 | 0.1338 + 01 |
| 0.1720 + 01 | 0.5087 + 01 | 0.3196 + 01 | 0.1592 + 01 | 0.1357 + 01 |
| 0.1740 + 01 | 0.5244 + 01 | 0.3266 + 01 | 0.1606 + 01 | 0.1376 + 01 |
| 0.1760 + 01 | 0.5406 + 01 | 0.3338 + 01 | 0.1620 + 01 | 0.1397 + 01 |
| 0.1780 + 01 | 0.5573 + 01 | 0.3411 + 01 | 0.1634 + 01 | 0.1418 + 01 |
| 0.1800 + 01 | 0.5746 + 01 | 0.3487 + 01 | 0.1648 + 01 | 0.1439 + 01 |
| 0.1820 + 01 | 0.5924 + 01 | 0.3564 + 01 | 0.1662 + 01 | 0.1461 + 01 |
| 0.1840 + 01 | 0.6109 + 01 | 0.3643 + 01 | 0.1677 + 01 | 0.1484 + 01 |
| 0.1860 + 01 | 0.6300 + 01 | 0.3723 + 01 | 0.1692 + 01 | 0.1507 + 01 |
| 0.1880 + 01 | 0.6497 + 01 | 0.3806 + 01 | 0.1707 + 01 | 0.1531 + 01 |
| 0.1900 + 01 | 0.6701 + 01 | 0.3891 + 01 | 0.1722 + 01 | 0.1555 + 01 |
| 0.1920 + 01 | 0.6911 + 01 | 0.3978 + 01 | 0.1737 + 01 | 0.1580 + 01 |
| 0.1940 + 01 | 0.7128 + 01 | 0.4067 + 01 | 0.1753 + 01 | 0.1606 + 01 |
| 0.1960 + 01 | 0.7353 + 01 | 0.4158 + 01 | 0.1768 + 01 | 0.1633 + 01 |
| 0.1980 + 01 | 0.7585 + 01 | 0.4251 + 01 | 0.1784 + 01 | 0.1660 + 01 |
| 0.2000 + 01 | 0.7824 + 01 | 0.4347 + 01 | 0.1800 + 01 | 0.1687 + 01 |

| M | $\frac{p_e}{p}$ | $\frac{\rho_e}{\rho}$ | $\frac{T_e}{T}$ | $\frac{A}{A^*}$ |
|-------------|-----------------|-----------------------|-----------------|-----------------|
| 0.2050 + 01 | 0.8458 + 01 | 0.4596 + 01 | 0.1840 + 01 | 0.1760 + 01 |
| 0.2100 + 01 | 0.9145 + 01 | 0.4859 + 01 | 0.1882 + 01 | 0.1837 + 01 |
| 0.2150 + 01 | 0.9888 + 01 | 0.5138 + 01 | 0.1924 + 01 | 0.1919 + 01 |
| 0.2200 + 01 | 0.1069 + 02 | 0.5431 + 01 | 0.1968 + 01 | 0.2015 + 01 |
| 0.2250 + 01 | 0.1156 + 02 | 0.5746 + 01 | 0.2012 + 01 | 0.2096 + 01 |
| 0.2300 + 01 | 0.1250 + 02 | 0.6076 + 01 | 0.2058 + 01 | 0.2193 + 01 |
| 0.2350 + 01 | 0.1352 + 02 | 0.6425 + 01 | 0.2104 + 01 | 0.2295 + 01 |
| 0.2400 + 01 | 0.1462 + 02 | 0.6794 + 01 | 0.2152 + 01 | 0.2403 + 01 |
| 0.2450 + 01 | 0.1581 + 02 | 0.7183 + 01 | 0.2200 + 01 | 0.2517 + 01 |
| 0.2500 + 01 | 0.1709 + 02 | 0.7594 + 01 | 0.2250 + 01 | 0.2637 + 01 |
| 0.2550 + 01 | 0.1817 + 02 | 0.8027 + 01 | 0.2310 + 01 | 0.2761 + 01 |
| 0.2600 + 01 | 0.1995 + 02 | 0.8484 + 01 | 0.2352 + 01 | 0.2896 + 01 |
| 0.2650 + 01 | 0.2156 + 02 | 0.8965 + 01 | 0.2404 + 01 | 0.3036 + 01 |
| 0.2700 + 01 | 0.2328 + 02 | 0.9472 + 01 | 0.2458 + 01 | 0.3183 + 01 |
| 0.2750 + 01 | 0.2514 + 02 | 0.1001 + 02 | 0.2512 + 01 | 0.3338 + 01 |
| 0.2800 + 01 | 0.2714 + 02 | 0.1057 + 02 | 0.2568 + 01 | 0.3500 + 01 |
| 0.2850 + 01 | 0.2929 + 02 | 0.1116 + 02 | 0.2624 + 01 | 0.3671 + 01 |
| 0.2900 + 01 | 0.3159 + 02 | 0.1178 + 02 | 0.2682 + 01 | 0.3830 + 01 |
| 0.2950 + 01 | 0.3407 + 02 | 0.1243 + 02 | 0.2740 + 01 | 0.4038 + 01 |
| 0.3000 + 01 | 0.3673 + 02 | 0.1312 + 02 | 0.2800 + 01 | 0.4235 + 01 |
| 0.3050 + 01 | 0.3959 + 02 | 0.1384 + 02 | 0.2860 + 01 | 0.4441 + 01 |
| 0.3100 + 01 | 0.4265 + 02 | 0.1459 + 02 | 0.2922 + 01 | 0.4657 + 01 |
| 0.3150 + 01 | 0.4593 + 02 | 0.1539 + 02 | 0.2984 + 01 | 0.4884 + 01 |
| 0.3200 + 01 | 0.4944 + 02 | 0.1622 + 02 | 0.3048 + 01 | 0.5121 + 01 |
| 0.3250 + 01 | 0.5320 + 02 | 0.1709 + 02 | 0.3112 + 01 | 0.5369 + 01 |
| 0.3300 + 01 | 0.5722 + 02 | 0.1800 + 02 | 0.3178 + 01 | 0.5629 + 01 |
| 0.3350 + 01 | 0.6152 + 02 | 0.1896 + 02 | 0.3244 + 01 | 0.5900 + 01 |
| 0.3400 + 01 | 0.6612 + 02 | 0.1996 + 02 | 0.3312 + 01 | 0.6184 + 01 |
| 0.3450 + 01 | 0.7103 + 02 | 0.2101 + 02 | 0.3380 + 01 | 0.6480 + 01 |
| 0.3500 + 01 | 0.7627 + 02 | 0.2211 + 02 | 0.3450 + 01 | 0.6790 + 01 |

$$\frac{P_0}{P_e} = \left[1 + \frac{\gamma - 1}{2} M_e^2 \right]^{\frac{\gamma}{\gamma - 1}} \rightarrow 46.98 \text{ kPa}$$

$$\frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 \rightarrow 508.81 \text{ K}$$

$$a_e = \sqrt{\gamma R T_e} \rightarrow 452.15 \text{ m/s}$$

$$V_e = M_e a_e \rightarrow 993.37 \text{ m/s}$$

Good Luck :)



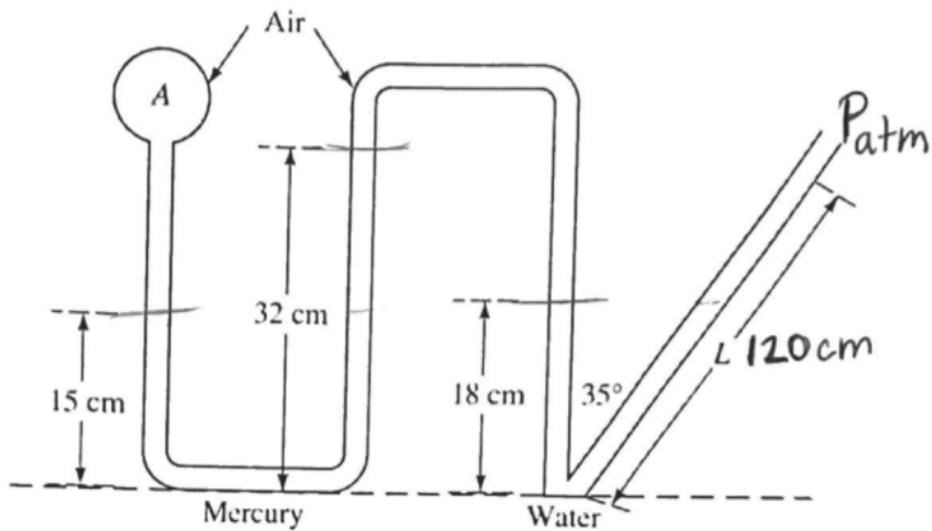


Figure 7: Inclined manometer.

Question 8

Consider the inclined manometer in Fig. 7 filled with Mercury ($\rho_{Hg} = 13,600 \text{ kg/m}^3$) and water ($\rho_{H_2O} = 1000 \text{ kg/m}^3$). The system is open to 1 atm (101,325 Pa) air ($\rho_{air} = 1.225 \text{ kg/m}^3$) on the right side. If $L = 120 \text{ cm}$, what is the air pressure in container A?

Solution:

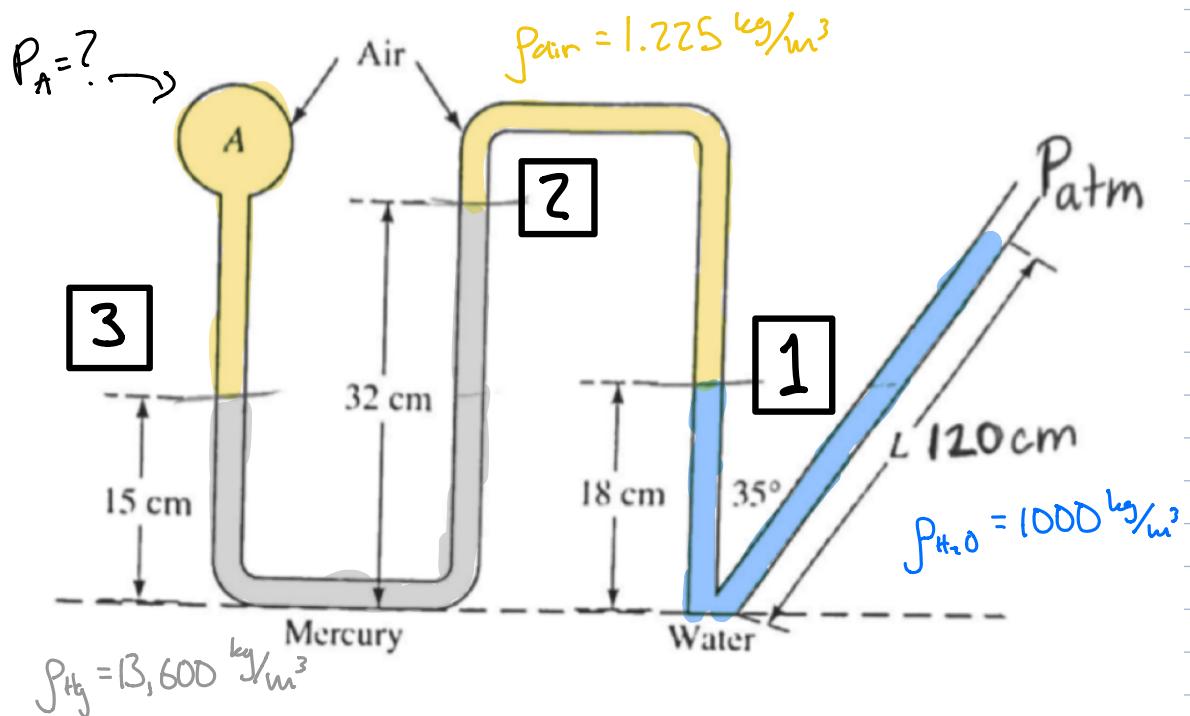
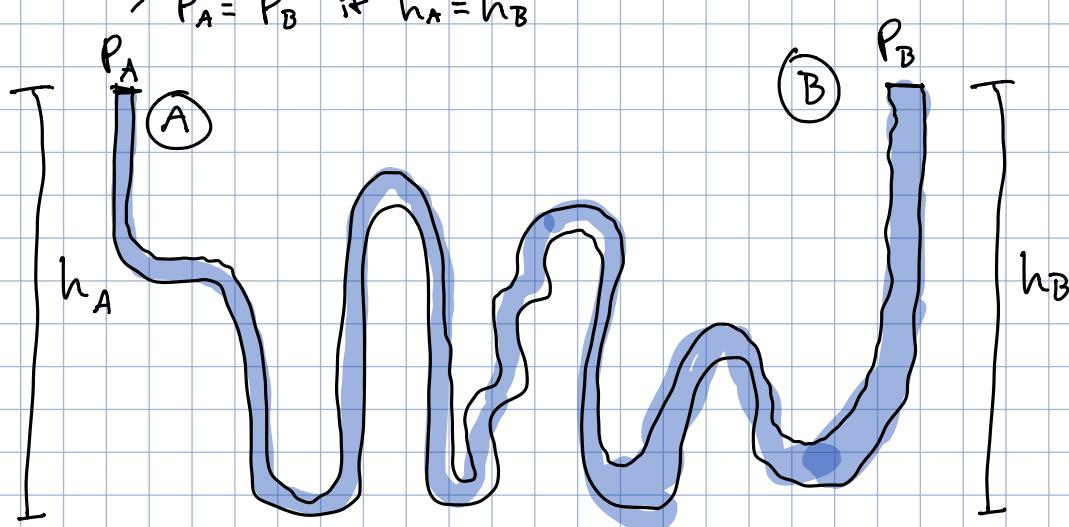


Figure 7: Inclined manometer.

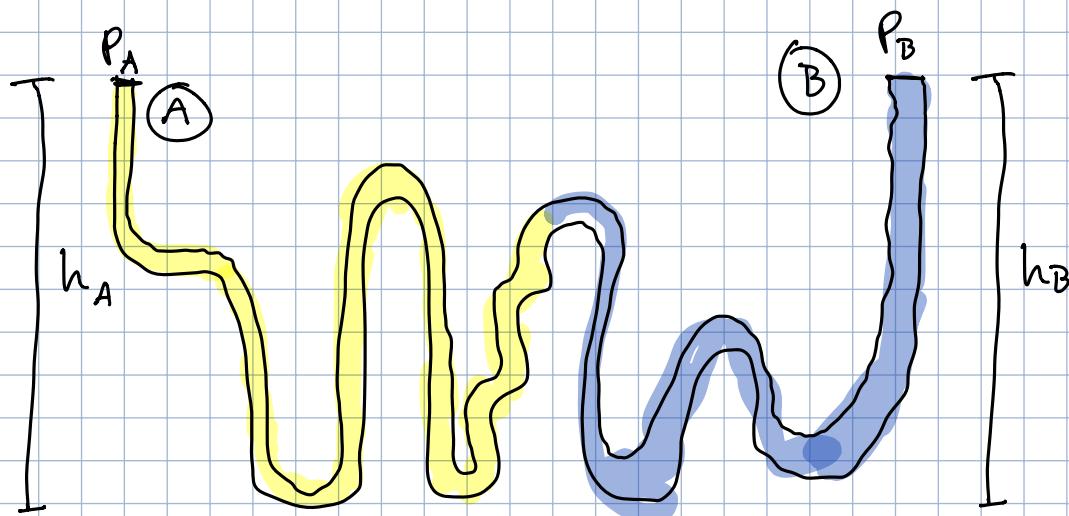
Theory to Remember w/ Manometers:

1. Pressures in continuous fluids are the SAME at the SAME height.

i.e. $\rightarrow P_A = P_B$ if $h_A = h_B$



However! $P_A \neq P_B$ if there is a different fluid (w/ different ρ)



HEIGHT MATTERS! Using hydrostatic equation:

$$\Delta P = -\rho g \Delta h$$

\hookrightarrow No matter how crazy the manometer looks, for a single fluid change in height is the only thing that will create ΔP

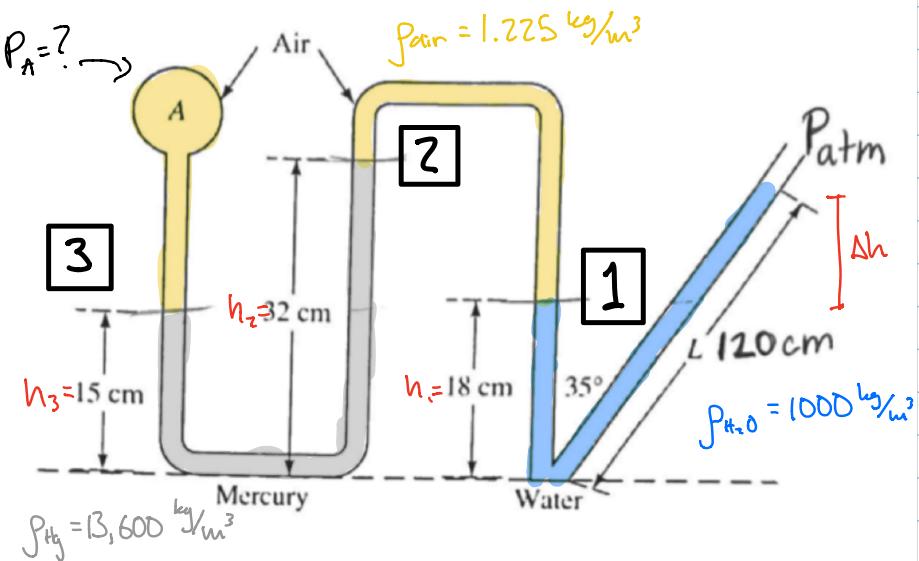


Figure 7: Inclined manometer.

P_1

$$\Delta P = -\rho g \Delta h \Rightarrow P_1 - P_{atm} = -\rho_{H2O} g \Delta h \Rightarrow \Delta h = (0.18 - 1.2 \cos 35^\circ)$$

$$P_1 = 101325 - 1000 \cdot 9.81 \cdot (0.18 - 1.2 \cos 35^\circ)$$

$$P_1 = 109,202.76 \text{ Pa}$$

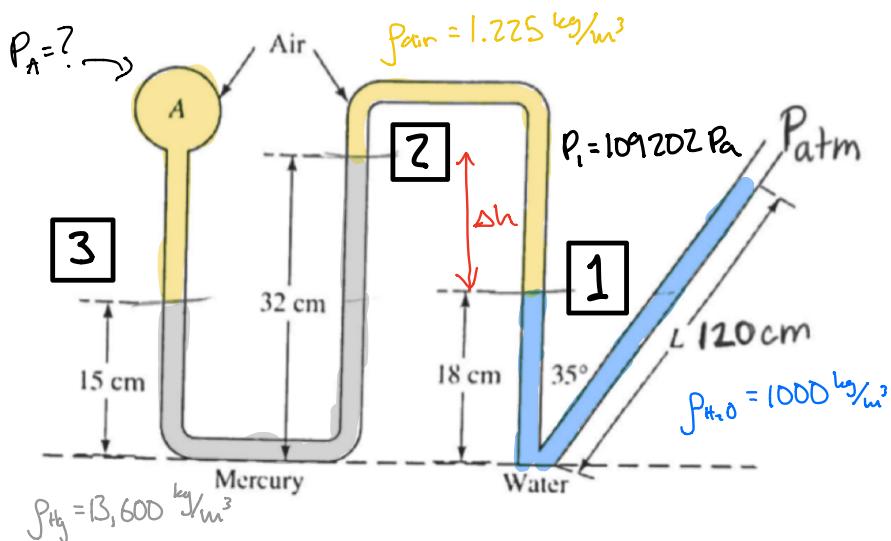


Figure 7: Inclined manometer.

$\cancel{P_2}$

$$P_2 - P_1 = -\rho_{air} g \Delta h \Rightarrow P_2 = 109202 - 1.225 \cdot 9.81 \cdot (-32 - 18)$$

$$P_2 = 109,200.58 \text{ Pa}$$

* Notice $P_1 \approx P_2$, air is much less dense than H₂O or Hg and over small Δh will have a negligible effect.

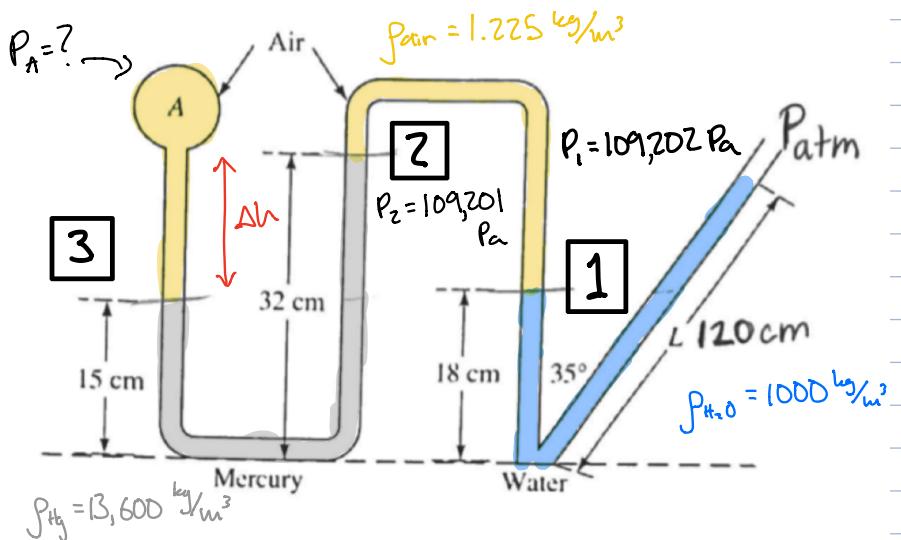


Figure 7: Inclined manometer.

P_3

$$P_3 - P_Z = -\rho_{Hg} g \Delta h \Rightarrow P_3 = 109,201 - 13,600 \cdot 9.81 \cdot (0.15 - 0.32)$$

$$P_3 = 131,881.30 \text{ Pa}$$

Because no height info was given for P_A , assume equilibrium...

Therefore, $P_A = P_3 = 131,881.30 \text{ Pa}$

* Note: Due to rounding errors your answers may not be exactly the same. As long as you're close you should be good.