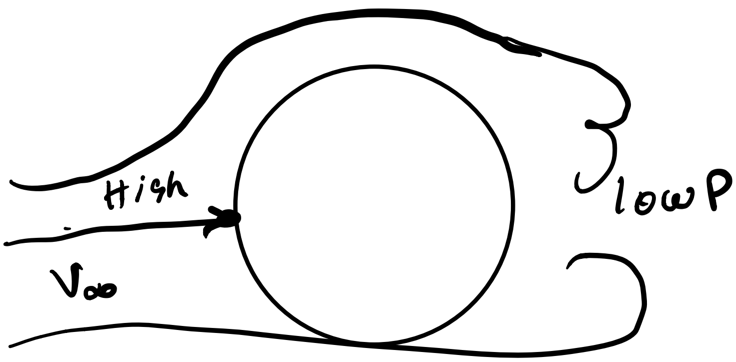
Midterm 3 review. Weeks 8-11

Viscous flow

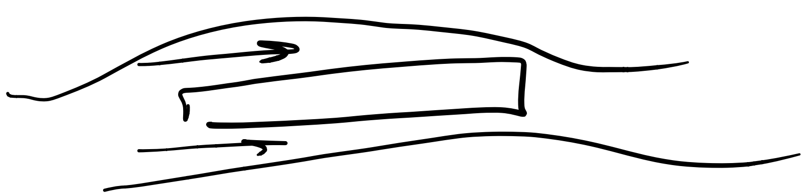
Types of drags:

Pressure drag: Mostly affect bluff bodies. It causes high pressure upstream and low pressure downstream, this is due to flow separation. Acts perpendicular to the surface

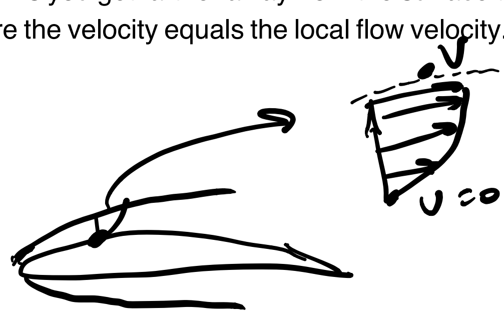
Flow separation: Where the streamlines can no longer follow the curvature of the of the body.



Skin friction drag: Is produced by the friction of the air molecules with the surface which creates a shear stress at the surface. This acts in a direction tangential to it.



Boundary layer: Is the region of fluid that is affected by viscosity. At this region, the fluid velocity is retarded, and right at the surface, the flow velocity is zero. The B.L has an impact on viscous forces, as well as pressure forces. As you get farther away from the surface the velocity increases, until you reach the edge of the B.L where the velocity equals the local flow velocity. The BL thickness grows as the flow moves over the body.



The shear stress at the wall is given by τ= μ(dV/dy)y=0

where μ is the viscosity of the gas, which varies with T

For air at standard sea-level temperature

μ = 1.7894*10^-5 kg/(m)(s) = 3.7373 × 10−7 slug/(ft)(s) → English

SI

Viscosity can be calculated

$$\mu = 1.458 \left(\frac{T^{3/2}}{T + 110.4} \right) \times 10^{-6} \text{ kg/m}\cdot\text{s}$$

$$\mu = 2.27 \left(\frac{T^{3/2}}{T + 199} \right) \times 10^{-8} \text{ slugs/ft}\cdot\text{s}$$

.

Reynolds number: Non dimensional parameter. Describes the behavior of viscosity. High Reynolds number indicates low viscosity, and low Reynolds number indicates high viscosity.

Re= $\dfrac{\rho V x}{\mu}$

Types of flow

Laminar flow: Streamlines are smooth and regular, and a fluid element moves smoothly along a streamline.

Boundary layer thickness $\delta = \dfrac{5.2\, x}{\sqrt{Re_x}}$

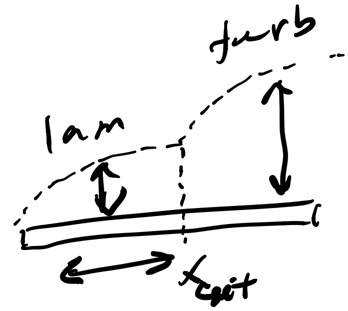
Total Skin friction coefficient $C_f = \dfrac{1.328}{\sqrt{Re_L}}$

Turbulent flow: The streamlines break up and a fluid element moves in a random, irregular fashion

Boundary layer thickness $\delta = \dfrac{0.37\, x}{Re^{0.2}}$

Total skin friction coefficient

$$C_f = \frac{0.074}{Re^{0.2}}$$



Laminar shear stress is less than the turbulent shear stress. Therefore, the skin friction is higher for turbulent flow. Turbulent boundary layer is thicker and grows faster.

In reality, the flow always starts out from the leading edge as laminar, and then at the transition point the boundary layer becomes completely turbulent where the boundary layer grows at a faster rate.

This point where transition occurs is called the critical point, which corresponds to a critical Reynolds number

Re_crit= $\frac{\rho V x_{crit}}{\mu}$

200,000<Re transition>1*10^6 .

$\uparrow$ laminar $\quad\quad$ $\downarrow$ turbulent

Airfoil nomenclature

NACA Airfoils

-4-digit series:

1 digit: max camber

2nd :location of max camber

3rd and 4th thickness of airfoil

-5-digit series:

1st digit: design lift coefficient, multiplied by 3/20

2nd digit: max camber, divided by 20

3rd digit: 0 refers to normal camber, 1 refers to reflex camber

4th and 5th : thickness

There is also a 6th digit series

NACA charts

-Angle of attack

-Cl (does not depend on Re unless we want to know Cl max)

-Cl with flaps

-a0: lift curve slope for Cl vs alpha. It's value is 2pi or 0.11

-AlphaL=0. This is the angle where the lift equals zero. It equals zero degrees for a symmetric airfoil. $\alpha_{l=0}$

-Stall: corresponds to the point where we have a max Cl and you get a dramatic loss of lift.

-Pitching moment coefficient about the quarter cord Cmc/4

$L = \frac{1}{2} \rho v^2 S C_{l\,max}$

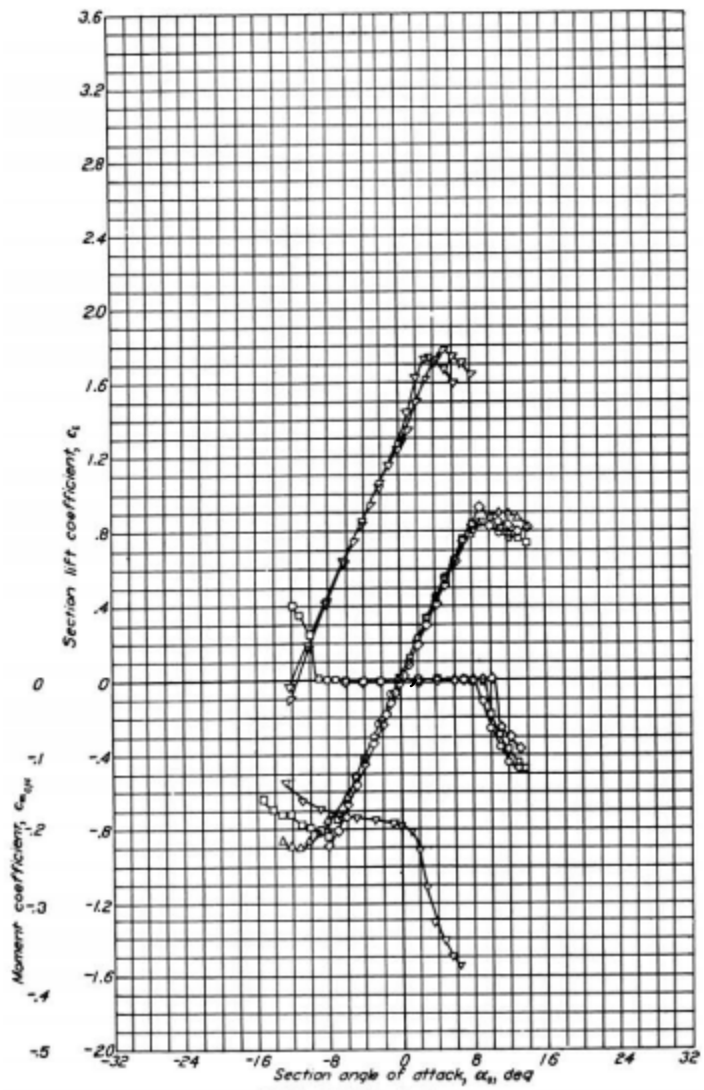
stall

-Drag polar (Cl vs Cd)

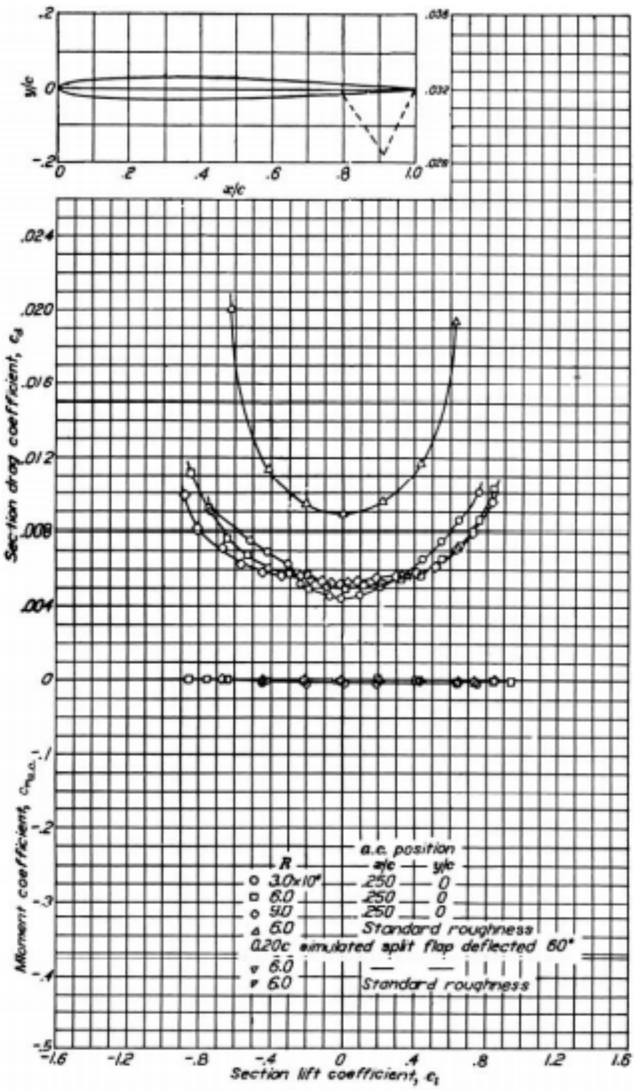
-Cd

-Pitching moment coefficient about the aerodynamic center Cmac

-Cdmin



NACA 0006 Wing Section



NACA 0006 Wing Section (continued)

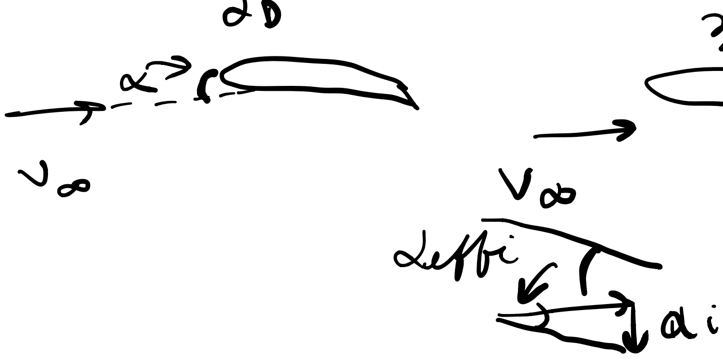
For a 2D infinite airfoil (Cl,Cd,Cm) and a 3D finite wing (CL,CD,CM), the lift and drag coefficient are different.

This is because for an airfoil section, the end effects are removed when testing in a wing tunnel.

For a 3 D wing these end effects produce a downward component called downwash. This causes an induced drag, which increases the total drag and reduces the lift.

Downwash causes the relative wind in the proximity of the airfoil section to be inclined slightly downward through a small angle called the induced angle of attack . This in turn reduces the angle of attack felt by the local airfoil section to a value smaller than the geometric angle of attack. This smaller angle of attack is called the effective angle of attack. The effective angle of attack for a 3D wing is equivalent to the geometric angle of attack for a 2D airfoil.

$C_l \rightarrow C_L$



$$\alpha = \alpha_i + \alpha_{eff}$$

3D

α_{2D}

Induced AoA= $\frac{C_L}{\pi A R e}$

(in radians)

Induced AoA $= 57.3 \frac{C_L}{\pi A R e}$

(in degrees)

CDi= $\frac{C_L^2}{\pi A R e}$

AR= b^2/S

If we have an elliptical wing(ideal case)then e=1

Where e is the span efficiency factor

CDi= $C_L^2/\pi A R$

Induced Drag= $q_\infty S \frac{C_L^2}{\pi A R e}$

To find total drag:

$\alpha = \alpha_{eff} + \alpha_{in}$

3D

CD=Cd+CDi

Dtot= $q_\infty S (C_d + C_{Di})$

Where Cd is the profile drag. Cd for a 2D case

For lift coefficient 3D

CL= $a(\alpha - \alpha_{L=0})$ → chart

For the 3D lift curve slope

a= $\frac{a_0}{1 + \frac{57.3 a_0}{\pi A R e}}$ > degrees $= \frac{dC_L}{d\alpha} \rightarrow$ 3D

$a_0 = \frac{dC_l}{d\alpha} \rightarrow$ 2D → 0.11

$Cl = ?$ $\alpha_{3D} \rightarrow 1^{0}$ ~~cbf~~ need α_{3D}

$CD = Cd + CD_i$

$CD = Cd + CDi$

Cp_1 P_1

P_∞

Pressure coefficient

q_∞

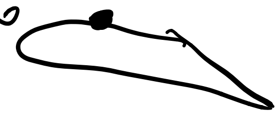
$Cp = \dfrac{P - P_\infty}{q_\infty}$

$Cp = 1 - \left(\dfrac{V}{V_\infty}\right)^2 \rightarrow$ incompressible

Compressible flow

Prandtl Glauert Compressibility Correction

M_∞ Cp



$$Cp = \frac{Cp_{M=0}}{\sqrt{1 - M_\infty^2}}$$ ← incompressible

$M = 0.4 \rightarrow Cl$ $\alpha \rightarrow Cl$

Compressibility correction for lift coefficient

$$Cl = \frac{Cl_{M=0}}{\sqrt{1 - M_\infty^2}}$$ ← incompressible

Critical Mach number and Critical Pressure Coefficient

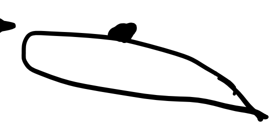
Critical Mach number: The freestream Mach number at which the flow around the airfoil first reaches sonic conditions (M=1)

Critical Pressure Coefficient: Pressure coefficient on the airfoil that correspond to M=1. This is the most negative value, corresponds to the highest velocity on the airfoil.

$$Cpcrit = \frac{Cp_{M=0}}{\sqrt{1 - M_\infty^2}}$$ $\rightarrow M_{\infty crit}$

freestream

$M_\infty \rightarrow$ Cp M



$$Cp = \frac{2}{\gamma M_\infty^2}\left\{\left[\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} \cancel{M}^2}\right]^{\gamma/\gamma-1} - 1\right\}$$

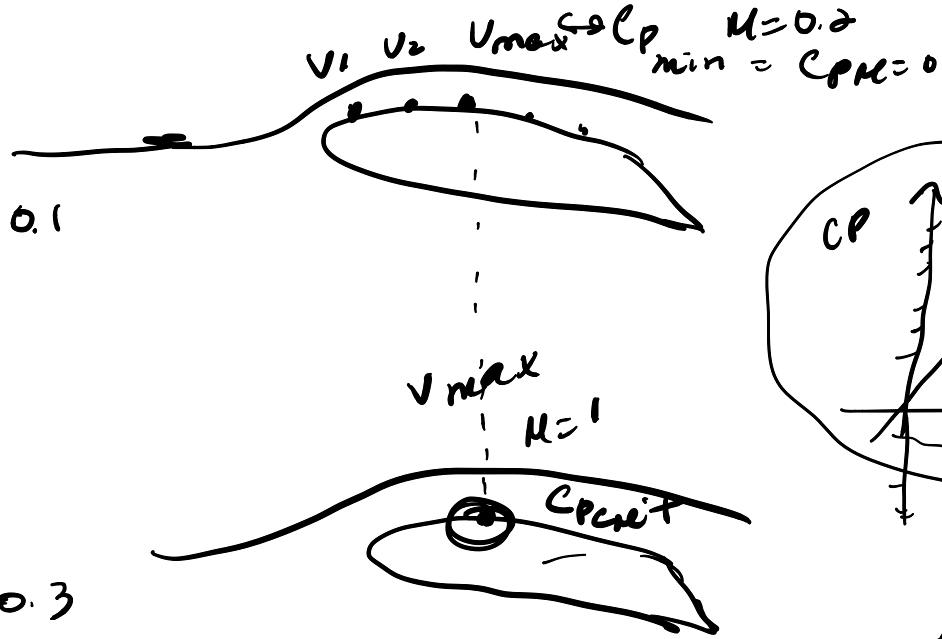
local

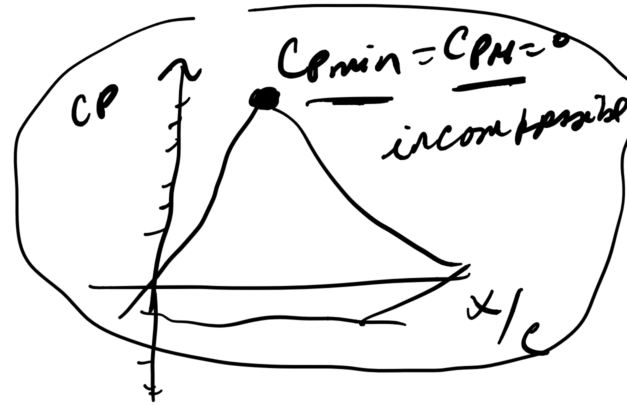
$$Cpcrit = \frac{2}{\gamma M_\infty^2}\left\{\left[\frac{2 + (\gamma-1) M_\infty^2}{\gamma+1}\right]^{\gamma/\gamma-1} - 1\right\}$$

$\uparrow M_{\infty crit}$

V_∞

$M_\infty = 0.1$

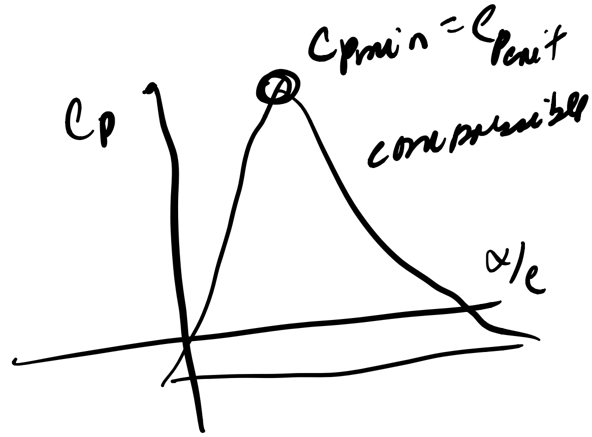




V_∞

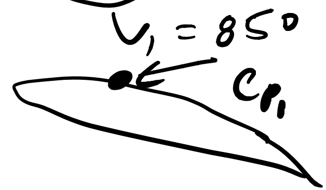
$M_\infty = 0.3$

$M_{\infty\,crit}$



$M_{crit} = ?$ $C_{p\,crit} = ?$

3.11 Consider a wing in a high-speed wind tunnel. At a point on the wing, the velocity is 850 ft/s. If the test-section fl ow is at a velocity of 780 ft/s, with a pressure and temperature of 1 atm and 505°R, respectively, calculate the pressure coeffi cient at the point



Given

$V_1 = 850$ ft/s

$V_\infty = 780$ ft/s

$P_\infty = 1$ atm $= 2116.8$ lb·ft^2

$T = 505°R$

$C_{p_1} = ?$

P_∞, T_∞, V_∞

$$C_{p_1} = \frac{P_1 - P_\infty}{q_\infty}$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$$

$$\cancel{C_p = 1 - \left(\frac{V}{V_\infty}\right)^2}$$

incomp

$$M_\infty = \frac{V_\infty}{\sqrt{\gamma R T_\infty}}$$

$$M_\infty = \frac{780}{\sqrt{1.4 \cdot 1716 \cdot 505}}$$

$$\underline{M_\infty = 0.708}$$

compressible (can't use Bernoulli)

$$\frac{P_1}{P_\infty} = \left(\frac{T_1}{T_\infty}\right)^{\gamma/\gamma-1}$$

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_\infty + \frac{V_\infty^2}{2}$$

$$T_1 = \frac{c_p T_\infty + \frac{V_\infty^2}{2} - \frac{V_1^2}{2}}{c_p}$$

$c_p \rightarrow 6000$

$$T_1 = \frac{6000 \cdot 505 + \frac{780^2}{2} - \frac{850^2}{2}}{6000}$$

$$T_1 = 495.492°R$$

$$P_1 = \left(\frac{T_1}{T_\infty}\right)^{\gamma/\gamma-1} \cdot P_\infty$$

$$P_1 = \left(\frac{495.492}{505}\right)^{1.4/1.4-1} \cdot 2116.8$$

$$P_1 = 1980 \text{ lb/ft}^2$$

$$\rho_\infty = \frac{P_\infty}{T_\infty R}$$

$$\rho_\infty = \frac{2116.8}{505 \cdot 1716}$$

$$\rho_\infty = 0.00244$$

$$C_{p_1} = \frac{P_1 - P_\infty}{q_\infty}$$

$$C_{p_1} = \frac{1980 - 2116.8}{\frac{1}{2} \cdot 0.00244 \cdot 780^2}$$

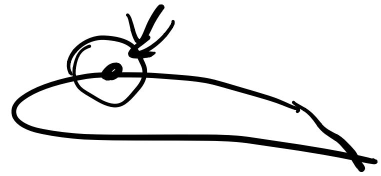
$$\boxed{C_{p_1} = -0.184}$$

5.12 If the test-section flow velocity in Prob. 5.11 is reduced to 100 ft/s, what will the pressure coefficient become at the same point on the wing?

$$C_p = \frac{C_{p_{M=0}}}{\sqrt{1-M_\infty^2}}$$

← need incompressible

M_∞



$$C_p\sqrt{1-M_\infty^2} = C_{p_{M=0}}$$

$$-0.184\sqrt{1-0.708^2} = C_{p_{M=0}}$$

$$\boxed{= -0.129}$$

$M=1$ $C_{p_{crit}}$

$M_{\infty_{crit}}$

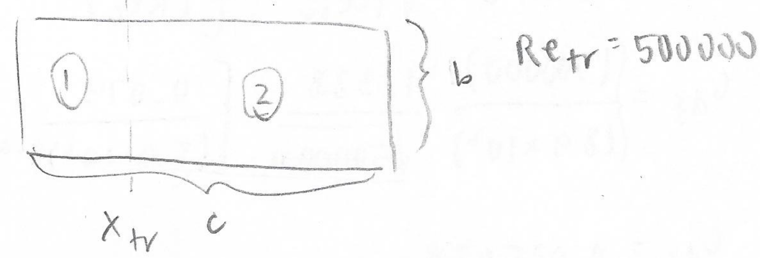
Anderson 5-36

From 5-34:
NACA 2412 In low speed flow

$\alpha = 0 \quad Re = 8.9 \times 10^6$

Flat plate approximation

Laminar Turbulent



For ① $S = x_{tr} \cdot b$

For ② $S = c \cdot b$

At $\alpha = 0$, $c_l = 0.2$ $c_d = 0.006$

$$D_{Total} = D_F + D_p \rightarrow \cancel{q_\infty} S c_d = \cancel{q_\infty} S c_{d_f} + \cancel{q_\infty} S c_{d_p} \longrightarrow c_d = c_{d_f} + c_{d_p}$$

$$D_F = D_L + D_T$$

$$D_T = \left[q_\infty bc\, c_{d_{f,T_{1+2}}} - q_\infty x_{tr} b\, c_{d_{f,T_1}}\right] 2 \rightarrow$$ Account for top + bottom of flat plate

$$D_L = \left[q_\infty b x_{tr} c_{d_{f,L}}\right] 2$$

$$D_F = q_\infty bc\, c_{d_f}$$

$$\frac{q_\infty bc\, c_{d_f}}{q_\infty bc} = \frac{\left[(q_\infty x_{tr} b\, c_{d_{f,L}}) + (q_\infty bc\, c_{d_{f,T_{1+2}}} - q_\infty x_{tr} b\, c_{d_{f,T_1}})\right] 2}{q_\infty bc}$$

$$c_{d_f} = \left[\frac{x_{tr}}{c} \cdot c_{d_{f,L}} + \left(c_{d_{f,T_{1+2}}} - \frac{x_{tr}}{c} \cdot c_{d_{f,T_1}}\right)\right] 2$$

Emperical correlations for a flat plate - Total skin friction coefficient

$$c_{d_{f,L}} = \frac{1.328}{\sqrt{Re_{tr}}} \qquad c_{d_{f,T_{1+2}}} = \frac{0.074}{(Re_c)^{0.2}} \qquad c_{d_{f,T_1}} = \frac{0.074}{(Re_{tr})^{0.2}}$$

$$\frac{x_{tr}}{c} = \frac{\rho V x_{tr} / \mu}{\rho V c / \mu} = \frac{Re_{tr}}{Re_c}$$

$$C_{df} = \left\{ \frac{Re_{tr}}{Re_c} \cdot \frac{1.328}{\sqrt{Re_{tr}}} + \left[\frac{0.074}{(Re_c)^{0.2}} - \frac{Re_{tr}}{Re_c} \cdot \frac{0.074}{(Re_{tr})^{0.2}} \right] \right\} 2$$

$$C_{df} = \left\{ \frac{(500000)}{(8.9 \times 10^6)} \cdot \frac{1.328}{\sqrt{500000}} + \left[\frac{0.074}{(8.9 \times 10^6)^{0.2}} - \frac{(500000)}{(8.9 \times 10^6)} \cdot \frac{0.074}{(500000)^{0.2}} \right] \right\} 2$$

$$C_{df} = 0.005639$$

$$C_d = C_{df} + C_{dp}$$

$$C_d - C_{df} = C_{dp}$$

$$0.006 - 0.005639 = C_{dp}$$

$$3.607 \times 10^{-4} = C_{dp}$$

$$\frac{C_{dp}}{C_d} \times 100 = \frac{3.607 \times 10^{-4}}{0.006} \times 100 \approx \boxed{6\%}$$

5.2 Consider an infinite wing with a NACA 1412 airfoil section and a chord length of 3 ft. The wing is at an angle of attack of 5° in an airflow velocity of 100 ft/s at standard sea-level conditions. Calculate the lift, drag, and moment about the quarter-chord per unit span.

$\underbrace{\text{Infinite}}_{\text{i.e. 2D}}$ NACA 1412, $c = 3\,\text{ft}$, $\alpha = 5°$, $V_\infty = 100\,\text{ft/s}$

max camber is 0.01c

max camber location is at 0.4c

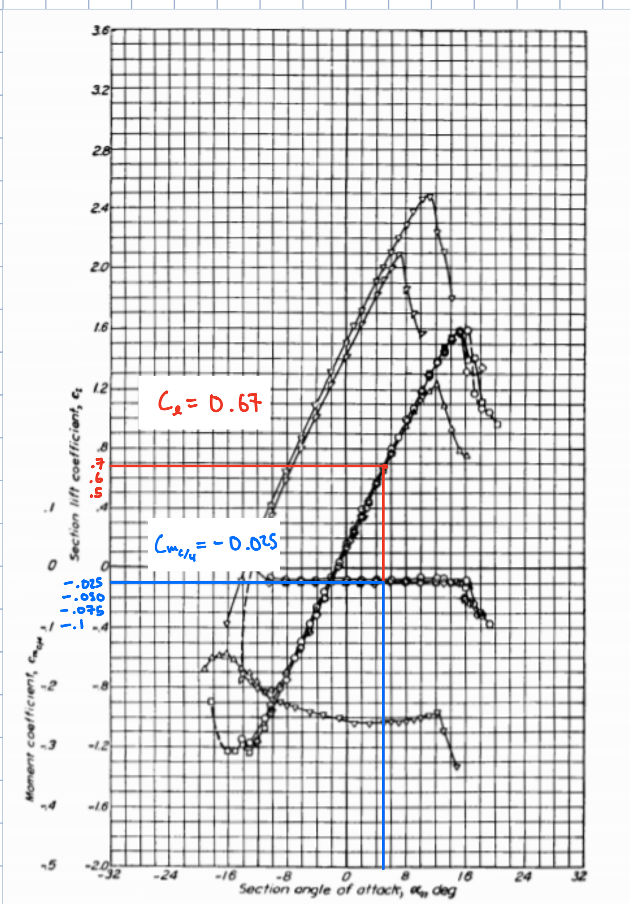
0.12·c at max thickness

' denotes "per unit length"

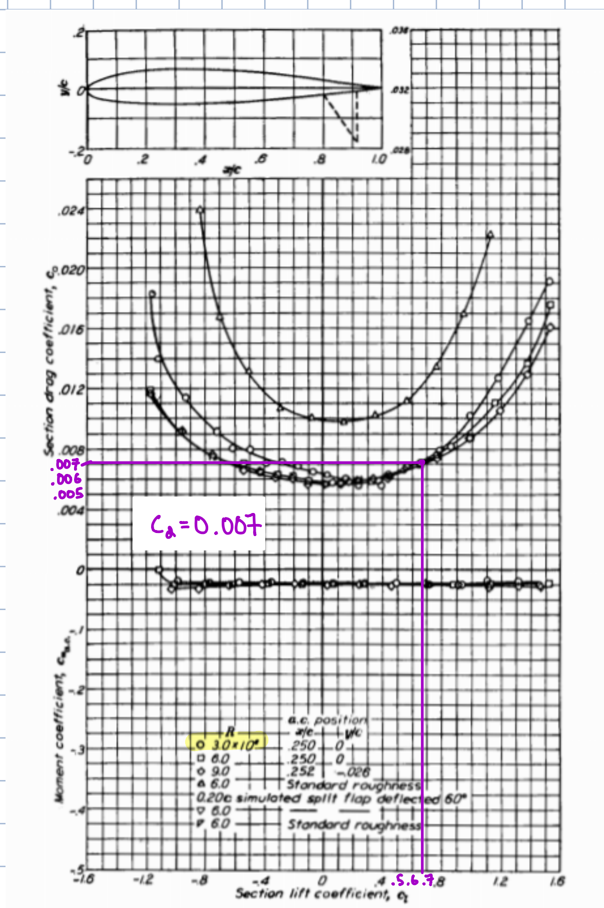
Find L', D', $M'_{c/4}$: NEED TO CHECK AIRFOIL DATA!

First, Get Reynolds Number: $Re = \dfrac{\rho_\infty V_\infty c}{\mu_\infty}$ (Using S.I. Values $\left\{ \begin{array}{l} \rho_\infty = 2.3769 \cdot 10^{-3}\ \text{slug/ft}^3 \\ \mu_\infty = 3.7373 \cdot 10^{-7}\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2} \end{array} \right.$)

$$Re = \frac{2.3769 \cdot 10^{-3} \cdot 100 \cdot 3}{3.7373 \cdot 10^{-7}} \approx 1.9 \cdot 10^6 \approx 3 \cdot 10^6$$



NACA 1412 Wing Section



NACA 1412 Wing Section (Continued)

q_∞

$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 \Rightarrow q_\infty = \frac{1}{2} \cdot 2.3769 \cdot 10^{-3} \cdot 100^2 = 11.8845 \ \frac{lb}{ft^2}$

L'

$L' = q_\infty c_l c \Rightarrow L' = 11.8845 \cdot 0.67 \cdot 3 = 23.89 \ lb/ft$

$M'_{c/4}$

$M'_{c/4} = q_\infty c_{m,c/4} c^2 \Rightarrow M'_{c/4} = 11.8845 \cdot (-0.025) \cdot 3^2 = -2.67 \ \frac{lb \cdot ft}{ft}$

D'

$D' = q_\infty c_d c \Rightarrow D' = 11.8845 \cdot 0.007 \cdot 3 = 0.250 \ lb/ft$

Ex 5.14 From Intro to Flight 8th Edition, Anderson

NACA 4415 in a high speed, subsonic wind tunnel

c_l measured as 0.85

Mach number is 0.7

Find α.

1. Mach number is $0.7 > 0.3$, consider compressibility.

Eq. 5.40

$$c_l = \frac{c_{l,0}}{\sqrt{1-M_\infty^2}}$$

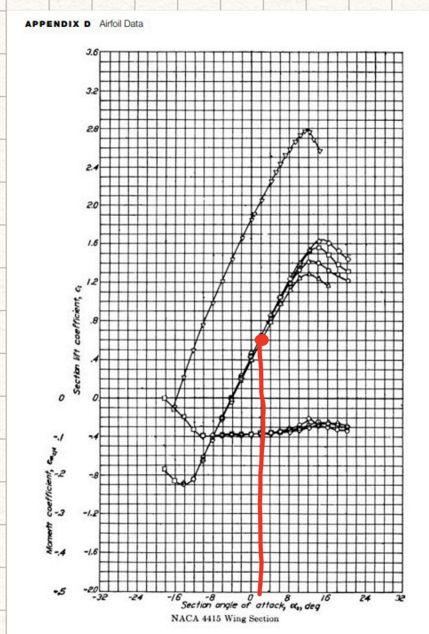
$$\Rightarrow c_{l,0} = c_l\sqrt{1-M_\infty^2}$$

$$= 0.85\sqrt{1-0.7^2}$$

$$c_{l,0} = 0.6070$$

2. Look at airfoil data.

$$\boxed{\alpha = 2^\circ}$$



Ex 5.21 (From the same book)

Cessna Cardinal, with Wing area = 16.2 $[m^2]$
Aspect ratio = 7.31
Span efficiency factor = 0.62
Weight = 9800 [N]

Flying at standard sea level conditions, $V_\infty = 251\ [\frac{km}{hr}]$

Find induced drag.

$$D_i = \underbrace{\tfrac{1}{2}\rho_\infty}_{\text{St. Atm.}} \underbrace{V_\infty^2 S}_{\text{given}} C_{D,i}$$

$$C_{D,i} = \frac{\overbrace{C_L^2}^{\text{find.}}}{\pi \underbrace{e AR}_{\text{given}}}$$

Find C_L.

$$L = W = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$\Rightarrow \quad C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{9800\ [N]}{\frac{1}{2}\left(1.225 \frac{kg}{m^3}\right)\left(251 \frac{km}{hr}\cdot\frac{1000}{3600}\right)^2 16.2\ m^2}$$

$$C_L = 0.2032$$

Plug into D_i equation:

$$D_i = \tfrac{1}{2}\rho_\infty V_\infty^2 S \frac{C_L^2}{\pi e AR} = \tfrac{1}{2}\left(1.225 \tfrac{kg}{m^3}\right)\left(251 \cdot \tfrac{1000}{3600} \tfrac{m}{s}\right)^2 \left(16.2\ m^2\right)\left(\frac{0.2032^2}{\pi (0.62)(7.31)}\right)$$

$$\boxed{D_i = 139.8\ [N]}$$