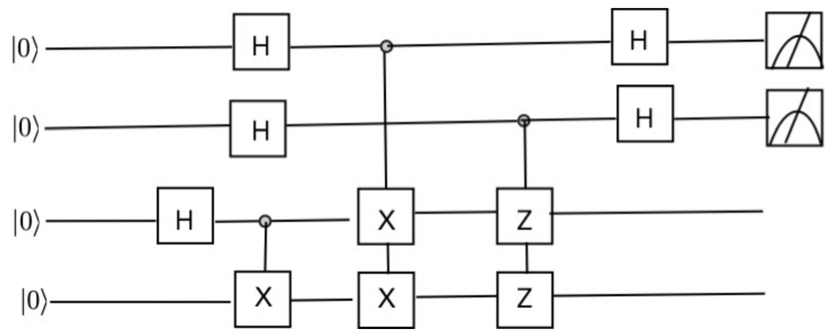
Stabilizer quantum circuit for Bell state

The Bell or EPR state $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ has stabilizers X_1X_2 and Z_1Z_2. The former serves to detect any single phase flip while the latter detects a single bit flip. If the syndrome is $+1, +1$ (with measurement in the above indicated order) then there is no phase or bit flip, $-1, +1$ indicates a phase flip or $|\beta_{10}\rangle$, $+1, -1$ indicates a bit flip or $|\beta 01\rangle$, and $-1, -1$ indicates both a phase flip and a bit flip or $|\beta_{11}\rangle$ is the received state.

A quantum circuit that employs an ancilla for implicit measurement is pictured below.



The first two qubits are the ancilla, tensoring with the constructed Bell state (qubits three and four) creates a four qubit system. Hadamard operators are applied to each of the first two qubits followed by controlled-not conditional on the first applied to the received Bell state (the third and fourth qubits). This is followed by controlled-Z conditional on the second qubit and again applied to the received Bell state. Then, Hadamard operators are applied to the first two qubits. Finally, successive measurements of the first two qubits using Z-basis (computational or $|0\rangle / |1\rangle$-basis) produce the syndrome. Quantum error correction can then be applied to the received (but unperturbed by the circuit) Bell state after dismissing the ancilla. Error correction for syndrome $+1, +1$ is I (the identity matrix), $-1, +1$ calls for Z_1, $+1, -1$ calls for X_1 and $-1, -1$ calls for X_1Z_1.

An equivalent quantum circuit is pictured below.