Ralph's superdense coding

Alice and Ralph are separated by great distance. Alice possesses two bits of information she would like to transmit to Ralph but only has channel capacity to transmit one qubit. Suppose Alice can access the first qubit of an entangled pair and Ralph possesses the second qubit of the pair where the Bell state is

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Further, suppose Alice and Ralph agree on the following code:

$$\begin{aligned}
|\beta_{00}\rangle &= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \to 00 \\
|\beta_{01}\rangle &= \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right) \to 01 \\
|\beta_{10}\rangle &= \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right) \to 10 \\
|\beta_{11}\rangle &= \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right) \to 11
\end{aligned}$$

That is, each Bell state uniquely corresponds to the two bits Alice wishes to communicate to Ralph.

Suggested:

1. Alice can only interact with her first qubit. Suppose Alice does nothing to her first qubit and transmits it to Ralph. What Bell state does Ralph possess?

2. Suppose Alice applies the bit flip operator X to her first qubit and transmits it to Ralph. What Bell state does Ralph possess?

3. Suppose Alice applies the phase flip operator Z to her first qubit and transmits it to Ralph. What Bell state does Ralph possess?

4. Suppose Alice applies both operators XZ to her first qubit and transmits it to Ralph. What Bell state does Ralph possess?

5. After receiving Alice's transmitted qubit, Ralph measures the resultant two qubits with the following (Hermitian) device.

$$A = (1)\,|\beta_{00}\rangle\langle\beta_{00}| + (2)\,|\beta_{01}\rangle\langle\beta_{01}| + (3)\,|\beta_{10}\rangle\langle\beta_{10}| + (4)\,|\beta_{11}\rangle\langle\beta_{11}|$$

For each potential measurement result (1,2,3,4) indicate the two bits Ralph has received from Alice. (Note: measurement occcurs in the Bell basis rather than the more familiar computational basis for this device.)