Ralph's quantum parallelism

Ralph knows superposition allows a kind of quantum parallel computing such that a classical operation that requires n executions can be executed in a single quantum measurement. The Deutsch algorithm offers a glimpse into this fascinating world.

Suppose we have a binary function $f(x) : \{0,1\} \to \{0,1\}$ that is either constant or balanced. A constant function produces the same result for any input, $f(0) = f(1) = 0$ or $f(0) = f(1) = 1$. On the other hand, a balanced function balances outcomes 0 and 1, $f(0) = 0, f(1) = 1$ or $f(0) = 1, f(1) = 0$. Classically, distinguishing whether the function is constant or balanced requires two operations while a quantum operation determines the nature of the function in one measurement by exploiting superposition.

The procedure is as follows.

1. Create the state $|01\rangle$.

2. Apply the Hadamard operator, H, to each qubit creating $|xy\rangle = |+-\rangle$ where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

3. Apply a unitary operator U_f that takes the first register $x \to x$ and the second register $y \to y + f(x) \bmod 2$. If the function is constant the resultant state is $\pm|+-\rangle$. However, if the function is balanced the transformed state is $\pm|--\rangle$.

4. Apply H to the first register or qubit. If the function is constant the result is $\pm|0-\rangle$, but if the function is balanced the result is $\pm|1-\rangle$.

5. Finally, measure the first qubit with $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. If the measurement result is 1 the function is constant and if the measurement result is -1 the function is balanced.

Suggested:

Verify the above steps.