Ralph's quantum mutual information

This is an extension of Ralph's mutual information. Ralph is considering the following investment options expressed in returns form (this implies the investment cost is normalized to one for each Arrow-Debreu asset).

$$A = \begin{array}{ccc} & |0\rangle & |1\rangle \\ asset_1 & 2 & 0 \\ asset_2 & 0 & 2 \end{array}$$

State prices y solve $Ay = v$ where v is a vector of ones (the normalized investment cost).

In this frame, states and information are represented by qubits, or more precisely, density operators. The first qubit (labelled a) is the uncertain (future) state and the last two qubits (labelled bc) are Ralph's information.

$$\rho_{abc} = \frac{1}{4} \left(|000\rangle \langle 000| + |001\rangle \langle 001| + |110\rangle \langle 110| + |111\rangle \langle 111| \right)$$

Suggested:

1. Determine state prices y.

2. Trace out qubits bc (the second and third qubits), leaving ρ_a. Determine the eigenvalues for ρ_a and quantum entropy $S(\rho_a)$.

3. Determine expected logarithmic returns, $E[r]$, from a Kelly investment strategy without information. (hint: eigenvalues of density operators have probability interpretation.)

4. Trace out the first qubit leaving ρ_{bc}. Determine eigenvalues and quantum entropy for ρ_{abc} and ρ_{bc}. Determine quantum mutual information $S(\rho_a : \rho_{bc}) = S(\rho_a) + S(\rho_{bc}) - S(\rho_{abc}) = S(\rho_a) - S\left(\rho_{a|bc}\right)$ where $S\left(\rho_{a|bc}\right) = S(\rho_{abc}) - S(\rho_{bc})$.

5. Determine expected logarithmic returns given information Z, $E[r \mid Z] = p(Z = z_1) E[r \mid z_1] + p(Z = z_2) E[r \mid z_2]$ from a Kelly investment strategy.

6. Compare the expected gain from the information, $E[gain] = E[r \mid Z] - E[r]$, with mutual information $S(\rho_a : \rho_{bc})$.