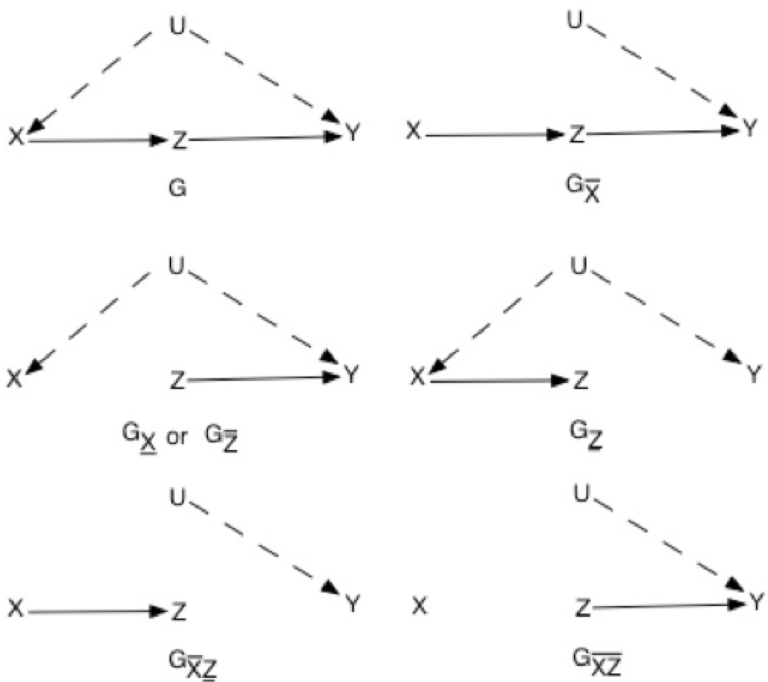
Ralph's front-door adjustment

Ralph is attempting to assess the causal effect of X on Y where the effect is mediated by Z as in the DAG G and its subgraphs below.



While conventional statistics lore suggests Z hopelessly confounds the causal effect of interest, Ralph has learned that a front-door adjustment may overcome this challenge.

$$\Pr\left(y \mid do\left(x\right)\right) = \sum_{z} \Pr\left(z \mid x\right) \sum_{x'} \Pr\left(y \mid x', z\right) \Pr\left(x'\right) \qquad \text{(front-door adj)}$$

Definition 1 (front-door) *A set of variables Z is defined a front-door for the ordered pair (X, Y) if*
(i) Z intercepts all directed paths from X to Y,
(ii) there is no unblocked back-door path from X to Z, and
(iii) all back-door paths from Z to Y are blocked by X.

The front-door adjustment can be considered a two-step adjustment: $X \to Z$ and $Z \to Y$

$$\Pr\left(z \mid do\left(x\right)\right)$$

and

$$\Pr(y \mid do(z))$$

where the latter employs a back-door adjustment utlizing covariate X.

Suggested:

1. Utilize rules from the do-calculus theorem (see Ralph's back-door adjustment) to determine if the causal effect of X on Y is identified.

Suppose U, X, Y, and Z are binary with DGP (data generating process

U	X	Z	$\Pr(Y = 1 \mid u, x, z)$	$\Pr(u, x, z)$
0	0	0	1/8	5/64
0	0	1	1/4	5/64
0	1	0	1/8	9/64
0	1	1	1/4	3/64
1	0	0	3/8	3/64
1	0	1	3/4	3/64
1	1	0	3/8	27/64
1	1	1	3/4	9/64

2. Determine the causal effect of X on Y, $\Pr(Y = 1 \mid do(x))$ for $x = 0, 1$. How does this compare with $\Pr(y \mid x)$ for $x = 0, 1$?

3. Suppose U is observable. Use the back-door adjustment with confounder U to identify the causal effect of X on Y. Compare with the results in 2.