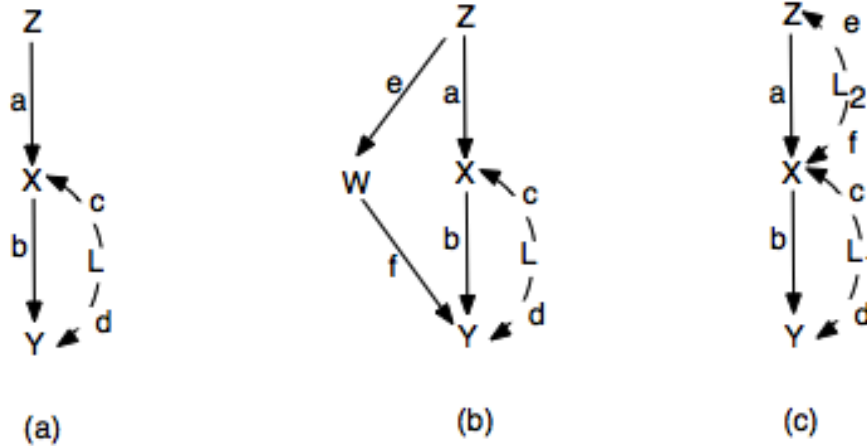# Ralph's instrumental variables

Ralph is considering the following causal graphs (DAGs — directed acyclic graphs) in which only W, X, Y, Z are observable and the objective is identification of the causal effect of X on Y, $X \rightarrow Y$.

(a) (b) (c)

Ralph knows a graphical definition of an instrument is as follows. A variable Z is an instrument with respect to the causal effect $X \rightarrow Y$ if Z is related to X but independent of Y in the graph with the direct path from X to Y deleted. The IV strategy is regress X and Y on Z (and possibly some set of covariates S) to produce estimands $r_{xz \cdot s}$ and $r_{yz \cdot s}$, then the causal effect $X \rightarrow Y$ can be retrieved from the ratio of estimands $\frac{r_{yz \cdot s}}{r_{xz \cdot s}}$ times whatever value $X = x$ is set.

$$
\begin{aligned}
E\left[Y \mid do\left(X = x\right)\right] &= \frac{r_{yz \cdot s}}{r_{xz \cdot s}} x \\
&= bx
\end{aligned}
$$

Suggested:

1. Can $X \rightarrow Y$ be identified by a back-door adjustment in any of the three settings? Explain.

2. Is conditioning on W in (b) helpful for satisfying the instrumental variable criteria? Explain.

3. For each case, demonstrate how instruments Z can be employed to identify the causal effect of interest, $X \rightarrow Y$. That is, derive the estimand for each step in the linear-IV strategy and explain how confounding due to the omitted, correlated (unobserved) variables is addressed.