

issue

private key

public key

1. base theorem

Fermat: $a^{p-1} \equiv 1 \pmod{p}$;

Euler: $a^{\varphi(m)} \equiv 1 \pmod{m}$

$a < p$; p (prime)

$a \in \text{residues}(m) \stackrel{\circ}{=} r(m)$

$GCD(a, m) = 1$

$\varphi(m) = m \prod_{p|m} \left(1 - \frac{1}{p}\right)$

where $p|m$ are the distinct prime factors of m

2. proof sketch

$1a, 2a, \dots, (p-1)a$

$r_1(m)a, r_2(m)a, \dots, r_{\varphi(m)}(m)a$

$\equiv 1, 2, \dots, (p-1) \pmod{p}$

$\equiv r_1(m), r_2(m), \dots, r_{\varphi(m)}(m) \pmod{m}$

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$(p-1)!a^{p-1} \equiv (p-1)! \pmod{p}$

$\prod_{i=1}^{\varphi(m)} ar_i(m) \equiv \prod_{i=1}^{\varphi(m)} r_i(m) \pmod{m}$

$a^{p-1} \equiv 1 \pmod{p}$

$a^{\varphi(m)} \prod_{i=1}^{\varphi(m)} r_i(m) \equiv \prod_{i=1}^{\varphi(m)} r_i(m) \pmod{m}$

$a^{\varphi(m)} \equiv 1 \pmod{m}$

issue	private key	public key
3. encryption		
a. public disclosure	nothing (e and p privately communicated)	sender announces e receiver announces m $m = r * s * t$ $\varphi(m) = rst \left(1 - \frac{1}{r}\right) \left(1 - \frac{1}{s}\right) \left(1 - \frac{1}{t}\right)$ $= rst \left(\frac{r-1}{r}\right) \left(\frac{s-1}{s}\right) \left(\frac{t-1}{t}\right)$ $= (r-1)(s-1)(t-1)$
b. decryptor	$a^{(p-1)n+1} \equiv a \pmod{p}$ $(a^e)^d = a^{ed} \equiv a \pmod{p}$ $ed = (p-1)n + 1$ $ed \equiv 1 \pmod{p-1}$ $e^{-1} \equiv d \pmod{p-1}$	$a^{\varphi(m)n+1} \equiv a \pmod{m}$ $a^{ed} \equiv a \pmod{m}$ $ed = \varphi(m)n + 1$ $ed \equiv 1 \pmod{\varphi(m)}$ $e^{-1} \equiv d \pmod{\varphi(m)}$
c. encoding	text $\rightarrow a$	text $\rightarrow a$
encrypting	$a^e \equiv \text{cyphertext} \pmod{p}$	$a^e \equiv \text{cyphertext} \pmod{m}$
decrypting	$(a^e)^d \equiv a \pmod{p}$	$(a^e)^d \equiv a \pmod{m}$
decoding	$a \rightarrow \text{text}$	$a \rightarrow \text{text}$