Ralph's quantum search

Quantum search algorithms exploit superposition to produce a quadratic increase in the rate a string is (probabilistically) identified from an unordered data base compared to classical search algorithms. Grover's algorithm proceeds as follows.

1. The quantity (search string) is represented in binary as n bits (e.g., 011 has $n = 3$ bits).

2. Begin with a quantum state $|0\rangle^{\otimes n}$ (for $n = 3$, this is $|000\rangle$).

3. Apply Hadamard operators to each qubit; call this state $|\psi\rangle$. This produces a uniform quantum state/superposition of all possible n bit strings.

$$|\psi\rangle = H^{\otimes n} |0\rangle^{\otimes n}$$

(for n = 3, $|\psi\rangle = \frac{1}{\sqrt{8}} (|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$)

4. Apply a unitary function (black box Oracle) f that assigns $+1$ to all components except it assigns -1 to the component that matches the desired string, a phase shift. A diagonal matrix of ones except the position of the string searched is - 1 produces the desired unitary operator.

$$|x\rangle = f |\psi\rangle$$

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5. Apply another unitary operator $D = 2|\psi\rangle\langle\psi| - I$ to $|x\rangle$.

$$|y\rangle = D |x\rangle$$

6. Repeat steps 4 and 5 $R = \frac{\pi}{4}\sqrt{2^n}$ (rounded to the nearest integer) times where $|\psi\rangle$ in step 4 is replaced by $|y\rangle$ from the most recent step 5 (but leave D unchanged).

7. Measure $|y\rangle$ in the computational basis. With high probability the quantity identified is the desired string.

Suggested:

Suppose Ralph is interested in finding 011. Apply Grover's algorithm. What is the probability the string is correctly found after 2 iterations? (hint: the probability is $\langle y_R | 011\rangle \langle 011 | y_R\rangle$ where $|y_R\rangle$ is the state reached in the Rth iteration) How would the success probability change if measurement occurred at an earlier stage in the algorithm?