Ralph's accounting code

Ralph recognizes accounting is a simple linear code that at least for the auditor has error detection and correction capabilities. The input y is an n-element vector of transactions amount and output x is an m-element vector containing the changes in the accounts reported in the financial statements.

Linear codes can be described by a generator matrix G for encoding and a parity check matrix H for error detection and correction utlizing built-in redundancy. The accounting code involves the accounting journal entry, T-account $m \times n$ matrix A augmented by the identity matrix.

$$G = \begin{bmatrix} I_n \\ A \end{bmatrix}$$

so G is $(m+n) \times n$. The accounting parity check matrix is $m \times (m+n)$

$$H = \begin{bmatrix} -A & I_m \end{bmatrix}$$

and their product is $m \times n$

$$HG = -A + A = 0$$

In other words, the parity check matrix resides in the left-nullspace of the augmented A matrix G.

Encoding is $Gy = \begin{bmatrix} y \\ x \end{bmatrix}$ and the syndrome for error detection and correction is

$$s = H \begin{bmatrix} y \\ x \end{bmatrix}$$

If there are no errors (or possibly, more than two errors) the syndrome will be zero. Otherwise, it will indicate both the position and magnitude of error in the changes in accounts. In other words, the accounts can be corrected by

$$x = (x+e) - s$$

where $(x+e)$ refers to the accounts with error.

Suggested:

1. Suppose

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

audited transactions and reported changes in account balances for the period are

$$y = \begin{bmatrix} 60 & collections \\ 30 & p\&e - invest \\ 0 & payments \\ 0 & bad - debts \\ 70 & sales \\ 0 & period - depr \\ 30 & cgs \\ 30 & accrued - \exp \\ 0 & purch - invent \\ 20 & product - depr \end{bmatrix}, x = \begin{bmatrix} 30 & cash \\ 10 & receivables \\ -10 & inventory \\ 10 & p\&e \\ -30 & payables \\ -70 & sales \\ 30 & cgs \\ 30 & sg\&a \end{bmatrix}$$

Does the balancing vector b, a vector of m ones, detect any errors? That is, is $b^T x = 0$? Does the syndrome s detect any errors? If so, correct the changes in account balances.

2. Suppose everything is as in 1 except changes in account balances are reported as

$$x = \begin{bmatrix} 30 & cash \\ 0 & receivables \\ -10 & inventory \\ 10 & p\&e \\ -20 & payables \\ -70 & sales \\ 30 & cgs \\ 30 & sg\&a \end{bmatrix}$$

Does the balancing vector b detect any errors? Does the syndrome s detect any errors? If so, correct the changes in account balances.