Suppose the DGP is

	D	$P(Z)$	T_1	$g_1(T)$	U_1	Y_1	T_0	$g_0(T)$	U_0	Y_0	Y	T
	1	0.6	2	2	8	10	2	-2	-8	-10	10	2
	1	0.6	4	4	14	18	4	-4	-14	-18	18	4
	1	0.6	4	4	14	18	4	-4	-14	-18	18	4
	1	0.4	2	2	0	2	2	-2	0	-2	2	2
	1	0.4	4	4	0	4	4	-4	0	-4	4	4
	0	0.6	2	2	8	10	2	-2	-8	-10	-10	2
	0	0.6	4	4	14	18	4	-4	-14	-18	-18	4
	0	0.4	2	2	0	2	2	-2	0	-2	-2	2
	0	0.4	2	2	0	2	2	-2	0	-2	-2	2
	0	0.4	4	4	0	4	4	-4	0	-4	-4	4
means	0.5	0.5	3	3	$5\frac{4}{5}$	$8\frac{4}{5}$	3	-3	$-5\frac{4}{5}$	$-8\frac{4}{5}$	$1\frac{3}{5}$	3

What conditional and unconditional average treatment effects among ATT, $ATUT$, and ATE are identified by the saturated linear regression design (2a) compared with other linear regression designs? Conditional and unconditional average treatment effects for this DGP are

conditioning	$ATT(\cdot)$	$ATUT(\cdot)$	$ATE(\cdot)$
$\Im(p)=1$	$15\frac{1}{3}-\left(-15\frac{1}{3}\right)=30\frac{2}{3}$	$14-(-14)=28$	$14\frac{4}{5}-\left(-14\frac{4}{5}\right)=29\frac{3}{5}$
$\Im(p)=0$	$3-(-3)=6$	$2\frac{2}{3}-\left(-2\frac{2}{3}\right)=5\frac{1}{3}$	$2\frac{4}{5}-\left(-2\frac{4}{5}\right)=5\frac{3}{5}$
$T=2$	$6-(-6)=12$	$4\frac{2}{3}-\left(-4\frac{2}{3}\right)=9\frac{1}{3}$	$5\frac{1}{5}-\left(-5\frac{1}{5}\right)=10\frac{2}{5}$
$T=4$	$13\frac{1}{3}-\left(-13\frac{1}{3}\right)=26\frac{2}{3}$	$11-(-11)=22$	$12\frac{2}{5}-\left(-12\frac{2}{5}\right)=24\frac{4}{5}$
$\Im(p)=1,T=2$	$10-(-10)=20$	$10-(-10)=20$	$10-(-10)=20$
$\Im(p)=1,T=4$	$18-(-18)=36$	$18-(-18)=36$	$18-(-18)=36$
$\Im(p)=0,T=2$	$2-(-2)=4$	$2-(-2)=4$	$2-(-2)=4$
$\Im(p)=0,T=4$	$4-(-4)=8$	$4-(-4)=8$	$4-(-4)=8$
none	$10\frac{2}{5}-\left(-10\frac{2}{5}\right)=20\frac{4}{5}$	$7\frac{1}{5}-\left(-7\frac{1}{5}\right)=14\frac{2}{5}$	$8\frac{4}{5}-\left(-8\frac{4}{5}\right)=17\frac{3}{5}$

Propensity-score matched samples for ATT are constructed with the following proportions.

$$18:12:18:12$$

for

$$\begin{array}{rcl}(D=1,P(Z)=0.6) & : & (D=1,P(Z)=0.4):\\ (D=0,P(Z)=0.6) & : & (D=0,P(Z)=0.4)\end{array}$$

and for $ATUT$ we have

$$12:18:12:18$$

for

$$\begin{array}{rcl}(D=1,P(Z)=0.6) & : & (D=1,P(Z)=0.4):\\ (D=0,P(Z)=0.6) & : & (D=0,P(Z)=0.4)\end{array}$$

Again, the ATE propensity-score matched sample has equal parts of the above samples. Design one yields

$$\begin{array}{cc}Y=-9.26+19.89D-0.07T+\varepsilon_{1a} & ATT \text{ p-score matched sample}\\ 19.89 & \text{suggested } ATT\\ Y=-7.16+15.14D-0.02T+\varepsilon_{1a} & ATUT \text{ p-score matched sample}\\ 15.14 & \text{suggested } ATUT\\ Y=-8.21+17.51D-0.43T+\varepsilon_{1a} & ATE \text{ p-score matched sample}\\ 17.51 & \text{suggested } ATE\end{array} \tag{1a}$$

and

$$\begin{array}{cc}Y=-2\frac{2}{3}+5\frac{2}{3}D-11\frac{1}{3}\Im(p) & \\ +23\frac{2}{3}\Im(p)\times D+\varepsilon_{1b} & ATT \text{ p-score matched sample}\\ 5\frac{2}{3} & \text{suggested } ATT\,(p=0.4)\\ 5\frac{2}{3}+23\frac{2}{3}=29\frac{1}{3} & \text{suggested } ATT\,(p=0.6)\\ 5\frac{2}{3}+23\frac{2}{3}\left(\frac{3}{5}\right)=19\frac{13}{15} & \text{suggested } ATT\\ Y=-2\frac{2}{3}+5\frac{2}{3}D-11\frac{1}{3}\Im(p) & \\ +23\frac{2}{3}\Im(p)\times D+\varepsilon_{1b} & ATUT \text{ p-score matched sample}\\ 5\frac{2}{3} & \text{suggested } ATUT\,(p=0.4)\\ 5\frac{2}{3}+23\frac{2}{3}=29\frac{1}{3} & \text{suggested } ATUT\,(p=0.6)\\ 5\frac{2}{3}+23\frac{2}{3}\left(\frac{2}{5}\right)=15\frac{2}{15} & \text{suggested } ATUT\\ Y=-2\frac{2}{3}+5\frac{2}{3}D-11\frac{1}{3}\Im(p) & \\ +23\frac{2}{3}\Im(p)\times D+\varepsilon_{1b} & ATE \text{ p-score matched sample}\\ 5\frac{2}{3} & \text{suggested } ATE\,(p=0.4)\\ 5\frac{2}{3}+23\frac{2}{3}=29\frac{1}{3} & \text{suggested } ATE\,(p=0.6)\\ 5\frac{2}{3}+23\frac{2}{3}\left(\frac{1}{2}\right)=17\frac{1}{2} & \text{suggested } ATE\end{array} \tag{1b}$$

where p-score refers to propensity-score $P(Z)$. Design two yields

$$\begin{array}{c|l}
Y = 0 + 0D - 1T + 2T \times D & \\
-2\Im(p) + 4\Im(p) \times D & ATT \text{ p-score matched sample} \\
-3T \times \Im(p) & \\
+6T \times \Im(p) \times D + \varepsilon_{2a} & \\
0 + 2(3) = 6 & \text{suggested } ATT\,(p = 0.4) \\
0 + 2\left(3\tfrac{1}{3}\right) + 4 + 6\left(3\tfrac{1}{3}\right) = 30\tfrac{2}{3} & \text{suggested } ATT\,(p = 0.6) \\
0 + 2(2) + 4\left(\tfrac{1}{2}\right) + 6\left(2 \times \tfrac{1}{2}\right) = 12 & \text{suggested } ATT\,(T = 2) \\
0 + 2(4) + 4\left(\tfrac{2}{3}\right) + 6\left(4 \times \tfrac{2}{3}\right) = 26\tfrac{2}{3} & \text{suggested } ATT\,(T = 4) \\
0 + 2(2) = 4 & \text{suggested } ATT\,(p = 0.4, T = 2) \\
0 + 2(2) + 4 + 6(2) = 20 & \text{suggested } ATT\,(p = 0.6, T = 2) \\
0 + 2(4) = 8 & \text{suggested } ATT\,(p = 0.4, T = 4) \\
0 + 2(4) + 4 + 6(4) = 36 & \text{suggested } ATT\,(p = 0.6, T = 4) \\
0 + 2\left(3\tfrac{1}{5}\right) + 4\left(\tfrac{3}{5}\right) + 6(2) = 20\tfrac{4}{5} & \text{suggested } ATT
\end{array} \tag{2a}$$

$$\begin{array}{c|l}
Y = 0 + 0D - 1T + 2T \times D & \\
-2\Im(p) + 4\Im(p) \times D & ATUT \text{ p-score matched sample} \\
-3T \times \Im(p) & \\
+6T \times \Im(p) \times D + \varepsilon_{2a} & \\
0 + 2\left(2\tfrac{2}{3}\right) = 5\tfrac{1}{3} & \text{suggested } ATUT\,(p = 0.4) \\
0 + 2(3) + 4 + 6(3) = 28 & \text{suggested } ATUT\,(p = 0.6) \\
0 + 2(2) + 4\left(\tfrac{1}{3}\right) + 6\left(2 \times \tfrac{1}{3}\right) = 9\tfrac{1}{3} & \text{suggested } ATUT\,(T = 2) \\
0 + 2(4) + 4\left(\tfrac{1}{2}\right) + 6\left(4 \times \tfrac{1}{2}\right) = 22 & \text{suggested } ATUT\,(T = 4) \\
0 + 2(2) = 4 & \text{suggested } ATUT \left(\begin{array}{c} p = 0.4, \\ T = 2 \end{array}\right) \\
0 + 2(2) + 4 + 6(2) = 20 & \text{suggested } ATUT \left(\begin{array}{c} p = 0.6, \\ T = 2 \end{array}\right) \\
0 + 2(4) = 8 & \text{suggested } ATUT \left(\begin{array}{c} p = 0.4, \\ T = 4 \end{array}\right) \\
0 + 2(4) + 4 + 6(4) = 36 & \text{suggested } ATUT \left(\begin{array}{c} p = 0.6, \\ T = 4 \end{array}\right) \\
0 + 2\left(2\tfrac{4}{5}\right) + 4\left(\tfrac{2}{5}\right) + 6\left(1\tfrac{1}{5}\right) = 14\tfrac{2}{5} & \text{suggested } ATUT
\end{array} \tag{2a}$$

$$
\begin{array}{cc}
Y = 0 + 0D - 1T + 2T \times D & \\
-2\Im(p) + 4\Im(p) \times D & ATE \text{ p-score matched sample} \\
-3T \times \Im(p) & \\
+6T \times \Im(p) \times D + \varepsilon_{2a} & \\
0 + 2\left(2\frac{4}{5}\right) = 5\frac{3}{5} & \text{suggested } ATE\,(p = 0.4) \\
0 + 2\left(3\frac{1}{5}\right) + 4 + 6\left(3\frac{1}{5}\right) = 29\frac{3}{5} & \text{suggested } ATE\,(p = 0.6) \\
0 + 2(2) + 4\left(\frac{2}{5}\right) + 6\left(2 \times \frac{2}{5}\right) = 10\frac{2}{5} & \text{suggested } ATE\,(T = 2) \\
0 + 2(4) + 4\left(\frac{3}{5}\right) + 6\left(4 \times \frac{3}{5}\right) = 24\frac{4}{5} & \text{suggested } ATE\,(T = 4) \\
0 + 2(2) = 4 & \text{suggested } ATE \left(\begin{array}{c} p = 0.4, \\ T = 2 \end{array} \right) \\
0 + 2(2) + 4 + 6(2) = 20 & \text{suggested } ATE \left(\begin{array}{c} p = 0.6, \\ T = 2 \end{array} \right) \\
0 + 2(4) = 8 & \text{suggested } ATE \left(\begin{array}{c} p = 0.4, \\ T = 4 \end{array} \right) \\
0 + 2(4) + 4 + 6(4) = 36 & \text{suggested } ATE \left(\begin{array}{c} p = 0.6, \\ T = 4 \end{array} \right) \\
0 + 2(3) + 4\left(\frac{1}{2}\right) + 6\left(1\frac{3}{5}\right) = 17\frac{3}{5} & \text{suggested } ATE
\end{array}
\tag{2a}
$$

and

$$
\begin{array}{cc}
Y = 1.22 - 2.56D - 3.73T & \\
+7.40T \times D + \varepsilon_{2b} & ATT \text{ p-score matched sample} \\
-2.56 + 7.40(2) = 12.24 & \text{suggested } ATT\,(T = 2) \\
-2.56 + 7.40(4) = 27.03 & \text{suggested } ATT\,(T = 4) \\
-2.56 + 7.40\left(3\frac{1}{5}\right) = 21.11 & \text{suggested } ATT \\
Y = 1.67 - 3.33D - 3.17T & \\
+6.23T \times D + \varepsilon_{2b} & ATUT \text{ p-score matched sample} \\
-3.33 + 6.23(2) = 9.13 & \text{suggested } ATUT\,(T = 2) \\
-3.33 + 6.23(4) = 21.59 & \text{suggested } ATUT\,(T = 4) \\
-3.33 + 6.23\left(2\frac{4}{5}\right) = 14.11 & \text{suggested } ATUT \\
Y = 1.54 - 3.14D - 3.49T & \\
+6.89T \times D + \varepsilon_{2b} & ATE \text{ p-score matched sample} \\
-3.14 + 6.89(2) = 10.63 & \text{suggested } ATE\,(T = 2) \\
-3.14 + 6.89(4) = 24.4 & \text{suggested } ATE\,(T = 4) \\
-3.14 + 6.89(3) = 17.51 & \text{suggested } ATE
\end{array}
\tag{2b}
$$

Contrast this with the following DGP.

	D	$P(Z)$	T_1	$g_1(T)$	U_1	Y_1	T_0	$g_0(T)$	U_0	Y_0	Y	T
	1	0.6	2	2	8	10	2	-2	-8	-10	10	2
	1	0.6	4	4	14	18	4	-4	-14	-18	18	4
	1	0.4	2	2	0	2	2	-2	0	-2	2	2
	1	0.4	4	4	0	4	4	-4	0	-4	4	4
	0	0.6	2	2	8	10	2	-2	-8	-10	-10	2
	0	0.6	4	4	14	18	4	-4	-14	-18	-18	4
	0	0.4	2	2	0	2	2	-2	0	-2	-2	2
	0	0.4	4	4	0	4	4	-4	0	-4	-4	4
means	0.5	0.5	3	3	$5\frac{1}{2}$	$8\frac{1}{2}$	3	-3	$-5\frac{1}{2}$	$-8\frac{1}{2}$	0	3

Conditional and unconditional average treatment effects for this DGP are

conditioning	$ATT(\cdot)$	$ATUT(\cdot)$	$ATE(\cdot)$
$\Im(p)=1$	$14-(-14)=28$	$14-(-14)=28$	$14-(-14)=28$
$\Im(p)=0$	$3-(-3)=6$	$3-(-3)=6$	$3-(-3)=6$
$T=2$	$6-(-6)=12$	$6-(-6)=12$	$6-(-6)=12$
$T=4$	$11-(-11)=22$	$11-(-11)=22$	$11-(-11)=22$
$\Im(p)=1, T=2$	$10-(-10)=20$	$10-(-10)=20$	$10-(-10)=20$
$\Im(p)=1, T=4$	$18-(-18)=36$	$18-(-18)=36$	$18-(-18)=36$
$\Im(p)=0, T=2$	$2-(-2)=4$	$2-(-2)=4$	$2-(-2)=4$
$\Im(p)=0, T=4$	$4-(-4)=8$	$4-(-4)=8$	$4-(-4)=8$
none	$8\frac{1}{2}-\left(-8\frac{1}{2}\right)=17$	$8\frac{1}{2}-\left(-8\frac{1}{2}\right)=17$	$8\frac{1}{2}-\left(-8\frac{1}{2}\right)=17$

Design one yields

$$\begin{array}{cc} Y=-8\frac{1}{2}+17D+0T+\varepsilon_{1a} & ATT \text{ p-score matched sample} \\ 17 & \text{suggested } ATT \\ Y=-8\frac{1}{2}+17D+0T+\varepsilon_{1a} & ATUT \text{ p-score matched sample} \\ 17 & \text{suggested } ATUT \\ Y=-8\frac{1}{2}+17D+0T+\varepsilon_{1a} & ATE \text{ p-score matched sample} \\ 17 & \text{suggested } ATE \end{array} \tag{1a}$$

and

$$
\begin{array}{cc}
Y = -3 + 6D - 11\Im(p) & \\
+22\Im(p) \times D + \varepsilon_{1b} & ATT \text{ p-score matched sample} \\
6 & \text{suggested } ATT\,(p = 0.4) \\
6 + 22 = 28 & \text{suggested } ATT\,(p = 0.6) \\
6 + 22\left(\frac{1}{2}\right) = 17 & \text{suggested } ATT \\
Y = -3 + 6D - 11\Im(p) & \\
+22\Im(p) \times D + \varepsilon_{1b} & ATUT \text{ p-score matched sample} \\
6 & \text{suggested } ATUT\,(p = 0.4) \\
6 + 22 = 28 & \text{suggested } ATUT\,(p = 0.6) \\
6 + 22\left(\frac{1}{2}\right) = 17 & \text{suggested } ATUT \\
Y = -3 + 6D - 11\Im(p) & \\
+22\Im(p) \times D + \varepsilon_{1b} & ATE \text{ p-score matched sample} \\
6 & \text{suggested } ATE\,(p = 0.4) \\
6 + 22 = 28 & \text{suggested } ATE\,(p = 0.6) \\
6 + 22\left(\frac{1}{2}\right) = 17 & \text{suggested } ATE
\end{array}
\tag{1b}
$$

where p-score refers to propensity-score $P(Z)$. Design two yields

$$
\begin{array}{cc}
Y = 0 + 0D - 1T + 2T \times D & \\
-2\Im(p) + 4\Im(p) \times D & \\
-3T \times \Im(p) & ATT \text{ p-score matched sample} \\
+6T \times \Im(p) \times D + \varepsilon_{2a} & \\
0 + 2(3) = 6 & \text{suggested } ATT\,(p = 0.4) \\
0 + 2(3) + 4 + 6(3) = 28 & \text{suggested } ATT\,(p = 0.6) \\
0 + 2(2) + 4\left(\frac{1}{2}\right) + 6\left(2 \times \frac{1}{2}\right) = 12 & \text{suggested } ATT\,(T = 2) \\
0 + 2(4) + 4\left(\frac{1}{2}\right) + 6\left(4 \times \frac{1}{2}\right) = 22 & \text{suggested } ATT\,(T = 4) \\
0 + 2(2) = 4 & \text{suggested } ATT\,(p = 0.4, T = 2) \\
0 + 2(2) + 4 + 6(2) = 20 & \text{suggested } ATT\,(p = 0.6, T = 2) \\
0 + 2(4) = 8 & \text{suggested } ATT\,(p = 0.4, T = 4) \\
0 + 2(4) + 4 + 6(4) = 36 & \text{suggested } ATT\,(p = 0.6, T = 4) \\
0 + 2(3) + 4\left(\frac{1}{2}\right) + 6\left(1\frac{1}{2}\right) = 17 & \text{suggested } ATT
\end{array}
\tag{2a}
$$

$$
\begin{array}{c}
Y = 0 + 0D - 1T + 2T \times D \\
-2\Im(p) + 4\Im(p) \times D \\
-3T \times \Im(p) \\
+6T \times \Im(p) \times D + \varepsilon_{2a}
\end{array}
\qquad ATUT \text{ p-score matched sample}
$$

$$
\begin{array}{cc}
0 + 2(3) = 6 & \text{suggested } ATUT(p = 0.4) \\
0 + 2(3) + 4 + 6(3) = 28 & \text{suggested } ATUT(p = 0.6) \\
0 + 2(2) + 4\left(\tfrac{1}{2}\right) + 6\left(2 \times \tfrac{1}{2}\right) = 12 & \text{suggested } ATUT(T = 2) \\
0 + 2(4) + 4\left(\tfrac{1}{2}\right) + 6\left(4 \times \tfrac{1}{2}\right) = 22 & \text{suggested } ATUT(T = 4) \\
0 + 2(2) = 4 & \text{suggested } ATUT \left(\begin{array}{c} p = 0.4, \\ T = 2 \end{array} \right) \\
0 + 2(2) + 4 + 6(2) = 20 & \text{suggested } ATUT \left(\begin{array}{c} p = 0.6, \\ T = 2 \end{array} \right) \\
0 + 2(4) = 8 & \text{suggested } ATUT \left(\begin{array}{c} p = 0.4, \\ T = 4 \end{array} \right) \\
0 + 2(4) + 4 + 6(4) = 36 & \text{suggested } ATUT \left(\begin{array}{c} p = 0.6, \\ T = 4 \end{array} \right) \\
0 + 2(3) + 4\left(\tfrac{1}{2}\right) + 6\left(1\tfrac{1}{2}\right) = 17 & \text{suggested } ATUT
\end{array}
\tag{2a}
$$

$$
\begin{array}{c}
Y = 0 + 0D - 1T + 2T \times D \\
-2\Im(p) + 4\Im(p) \times D \\
-3T \times \Im(p) \\
+6T \times \Im(p) \times D + \varepsilon_{2a}
\end{array}
\qquad ATE \text{ p-score matched sample}
$$

$$
\begin{array}{cc}
0 + 2(3) = 6 & \text{suggested } ATE(p = 0.4) \\
0 + 2(3) + 4 + 6(3) = 28 & \text{suggested } ATE(p = 0.6) \\
0 + 2(2) + 4\left(\tfrac{1}{2}\right) + 6\left(2 \times \tfrac{1}{2}\right) = 12 & \text{suggested } ATE(T = 2) \\
0 + 2(4) + 4\left(\tfrac{1}{2}\right) + 6\left(4 \times \tfrac{1}{2}\right) = 22 & \text{suggested } ATE(T = 4) \\
0 + 2(2) = 4 & \text{suggested } ATE \left(\begin{array}{c} p = 0.4, \\ T = 2 \end{array} \right) \\
0 + 2(2) + 4 + 6(2) = 20 & \text{suggested } ATE \left(\begin{array}{c} p = 0.6, \\ T = 2 \end{array} \right) \\
0 + 2(4) = 8 & \text{suggested } ATE \left(\begin{array}{c} p = 0.4, \\ T = 4 \end{array} \right) \\
0 + 2(4) + 4 + 6(4) = 36 & \text{suggested } ATE \left(\begin{array}{c} p = 0.6, \\ T = 4 \end{array} \right) \\
0 + 2(3) + 4\left(\tfrac{1}{2}\right) + 6\left(1\tfrac{1}{2}\right) = 17 & \text{suggested } ATE
\end{array}
\tag{2a}
$$

and

$$
\begin{array}{ll}
Y = -1 + 2D - 2\frac{1}{2}T & \\
\quad +5T \times D + \varepsilon_{2b} & ATT \text{ p-score matched sample} \\
\quad 2 + 5\,(2) = 12 & \text{suggested } ATT\,(T = 2) \\
\quad 2 + 5\,(4) = 22 & \text{suggested } ATT\,(T = 4) \\
\quad 2 + 5\,(3) = 17 & \text{suggested } ATT \\
Y = -1 + 2D - 2\frac{1}{2}T & ATUT \text{ p-score matched sample} \\
\quad +5T \times D + \varepsilon_{2b} & \\
\quad 2 + 5\,(2) = 12 & \text{suggested } ATUT\,(T = 2) \\
\quad 2 + 5\,(4) = 22 & \text{suggested } ATUT\,(T = 4) \\
\quad 2 + 5\,(3) = 17 & \text{suggested } ATUT \\
Y = -1 + 2D - 2\frac{1}{2}T & ATE \text{ p-score matched sample} \\
\quad +5T \times D + \varepsilon_{2b} & \\
\quad 2 + 5\,(2) = 12 & \text{suggested } ATE\,(T = 2) \\
\quad 2 + 5\,(4) = 22 & \text{suggested } ATE\,(T = 4) \\
\quad 2 + 5\,(3) = 17 & \text{suggested } ATE
\end{array}
\tag{2b}
$$