Ralph's Employee Stock Options

Ralph, chairman of the board, is contemplating offering employee stock options (ESOs)[1] as part of his firm's compensation arrangements. The firm is growth oriented and as it currently (and for the foreseeable near future) has no free cash flows, pays no dividends on its stock. Ralph believes ESOs may help to align the self-interest of employees with the welfare of the firm's owners by encouraging employees to work to increase the firm's stock price, increase employee retention, gain favorable income tax treatment, help identify gifted managers, and moderate risk aversion.[2]

Ralph also knows that after years of debate over so-called "intrinsic value" and "fair value" accounting for ESOs the FASB (along with the SEC) is gravitating towards "fair value" accounting. On the surface, these accounting methods differ primarily by the geography of their disclosures rather than by their information content. That is, compensation expense under "intrinsic value" accounting is the amount by which the options are "in the money" at the grant date. Since ESOs are almost always granted "at the money", compensation expense is rarely recognized under "intrinsic value" accounting. On the other hand, "fair value" accounting recognizes the "model-based value" of the ESOs at the grant date as compensation expense.

The trick is determining "value" of ESOs at the grant date. The FASB proposes employing either of the standard option valuation approaches: the continuous time Black-Scholes-Merton (BSM) model, or the discrete, more flexible binomial (also referred to as the lattice) model originally proposed by Cox, Ross, and Rubinstein (CRR).[3] There are a number of concerns with this approach. Rubinstein[4] identifies the following difficulties due to differences between traded options (the original object of the valuation methods) and ESOs: (1) ESOs typically have much longer *maturities*, (2) ESOs typically involve *delayed vesting*, (3) *forfeiture* associated with ESOs changes the timing and/or incidence of exercise, (4) ESOs are

[1] ESOs are really warrants as they are obligations of the issuing firm whereas traded options are obligations of other parties.
[2] See Arya and Mittendorf, 2005, "Offering stock options to gauge managerial talent," *Journal of Accounting & Economics*.
[3] Of course, in the limit as the number of discrete steps increases the binomial model yields the same valuation as the continuous time model, at least in the simplest cases where their inputs can be aligned.
[4] Rubinstein, 1994, "On the Accounting Valuation of Employee Stock Options," printed in *Journal of Derivatives*, Fall, 1995.

typically *non-transferable* which complicates employees' ability to diversify their holdings and potentially alters their perceptions of optimal exercise, (5) *income tax* implications for both the employee/investor and the firm differ for ESOs, (6) as ESOs are warrants they have *capital structure or dilution* implications not shared by traded options, and (7) ESOs may have *incentive, employee retention, and risk-taking* effects which change the firm's operating income (Rubinstein refers to this as *operating income effects*).

Ralph believes that first he should try to understand valuation of traded options, then try to extend this to warrants, and finally try to understand the implications for ESOs. The key idea to contingent claims analysis originally proposed by Black and Scholes,[5] is to equate a traded call option on a stock (option to buy the stock) with levered investment in the firm's stock. The discrete binomial approach conveys the intuition.

Suppose Ralph's stock is currently trading at \$100 per share. To keep things simple, there is one movement in the stock over the next period: either the stock increases to \$135 or decreases to \$74.[6] Further call options on the stock with an exercise date one year from now (this is a European option) and an exercise price (X) equal to \$100 (issued "at the money") are offered. What would investors be willing to pay for the option?

Equilibrium pricing satisfies a no arbitrage condition. That is, if the stock increases then the options are valuable ($\max[0, S^+ - X] = \max[0, 135 - 100] = 35$) and exercised, while if the stock price falls then the options are worthless ($\max[0, S^- - X]$ $= \max[0, 74 - 100] = 0$), and expire unexercised. These payoffs can be mimicked by levered investment in the stock as follows. Suppose investors can borrow at the riskless interest rate, say $r = 5\%$, then if an investor borrows the present value of the low future stock price, 74/(1.05) = \$70.55 and purchases the stock, what are the investor's future payoffs?

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[5] Black and Scholes, 1973, "The pricing of options and corporate liabilities," *Journal of Political Economy*. See also Merton, 1973, "Theory of rational option pricing," *Bell Journal of Economics and Management Science*.

[6] A simple relation between discrete movements and continuous-time volatility is derived from equating the limit of the mean and variance for a binomial random variable as the number of steps or movements (h) increases with the mean and variance of lognormal stock returns. In this setting, suppose the volatility (standard deviation σ of continuously compounded returns) is 0.3 then percentage up movements are $u = exp(\sigma\sqrt{t/h}) = exp(0.3\sqrt{1/1}) \approx 1.35$, and percentage down movements are $d = 1/u = \exp(-\sigma\sqrt{t/h}) = exp(-0.3\sqrt{1/1}) \approx 0.74$ where t is the time to maturity and h is the number of steps or movements; see Cox, Ross, and Rubinstein, 1979, "Option Pricing: A Simplified Approach," *Journal of Financial Economics*.

	Levered investment	Option
If the stock price rises:		
value of stock	\$135	
less: repayment of debt (70.55 x 1.05)	74	
payoff	\$ 61	\$ 35
If the stock price falls:		
value of stock	\$ 74	
less: repayment of debt (70.55 x 1.05)	74	
payoff	\$ 0	\$ 0

These state-contingent payoffs are perfectly aligned if 35/61 shares of levered investment in stock per call option are purchased. This is referred to as the hedge ratio or option delta Δ = $\frac{\text{spread of possible option prices}}{\text{spread of possible share prices}} = \frac{35-0}{61-0}$. Since the payoffs are the same they must be priced equivalently to prevent arbitrage opportunities. In general, Δ and B are determined by matching state-for-state the payoffs from the call option with the payoffs from levered investment in the "hedge" portfolio $C = \Delta S + B$. This works as follows. $\Delta\, uS + (1+r)B = C^+$, and $\Delta\, dS + (1+r)B = C^-$. Solving for Δ and B yields $\Delta = \frac{C^+ - C^-}{(u-d)S}$ and $B = \frac{uC^- - dC^+}{(u-d)(1+r)}$. Hence,

$C = \Delta S + B = 35/61\ (100) - 40.53 = 16.91$ (rounding).

The binomial option valuation approach exploits this ability to replicate investment in the option by a levered investment in the stock. A quick method for calculating the no arbitrage equilibrium option value involves so-called "risk neutral probabilities" and discounting "expected" payoffs at the riskless interest rate r. A general method to impute the risk neutral probability p is to identify "risk neutral probabilities" is to recognize p and (1 – p) are the non-negative weights that satisfy

$$C = \Delta S + B = \frac{C^+ - C^-}{(u-d)S}\,\mathrm{S} + \frac{uC^- - dC^+}{(u-d)(1+r)} = \frac{C^+ - C^-}{(u-d)S}S + \frac{uC^- - dC^+}{(u-d)(1+r)} = \frac{pC^+ + (1-p)C^-}{1+r}$$

and, of course, sum to one. In other words, $p = \frac{1+r-d}{u-d}$.

Based on the risk neutral probability, the future payoff on the option, and the riskless interest rate the value of a one-step call option is simply the present value of the "expected" future payoff.

$C = [p\ C^{+} + (1 - p)\ C^{-}]/(1 + r)$

$= [0.508(35) + 0.492(0)]/1.05 = 16.91$ (rounding)

Then, say, for a two-step option held to maturity, the current value of the call is

$C = \dfrac{p^2C^{++} + p(1-p)C^{+-} + (1-p)pC^{-+} + (1-p)^2C^{--}}{(1+r2)^2}$, where $r2$ is the interest rate that compounds over two periods to 1.05.

Of course, this is the same as the value of levered stock investment. So, we have two (equivalent) ways to value traded options based on the discrete binomial approach. In addition, the BSM continuous-time option valuation model could be employed (the expression only appears to be unwieldy).

$C = S/(1 + \delta)^{t}\ \mathrm{N}(d_1) - X/(1 + r)^{t}\ \mathrm{N}(d_2)$

where $d_1 = \dfrac{Ln[S/(1+\delta)^t] - Ln[X/(1+r)^t]}{\sigma\sqrt{t}} + \tfrac{1}{2}\sigma\sqrt{t}$,

$d_2 = d_1 - \sigma\sqrt{t} = \dfrac{Ln[S/(1+\delta)^t] - Ln[X/(1+r)^t]}{\sigma\sqrt{t}} - \tfrac{1}{2}\sigma\sqrt{t}$,

N = cumulative standard normal distribution function,

S = current stock price,

X = exercise price,

σ = volatility (standard deviation of continuously compounded return) of stock,

t = time (in years) to maturity (expiration) of option,

r = annualized rate of interest,

δ = annualized dividend yield, and

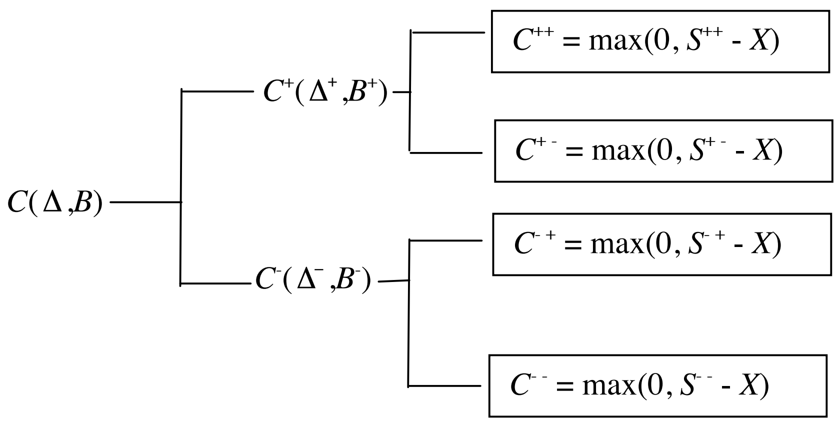
Ln = natural logarithm.

For the example, $C = 100/(1+0)^1\,\mathrm{N}\left[\dfrac{\mathrm{Ln}[100/(1+0)^1] - \mathrm{Ln}[100/(1.05)^1]}{0.3\sqrt{1}} + \tfrac{1}{2}0.3(1)\right]$

$- 100/(1.05)^1\,\mathrm{N}\left[\dfrac{\mathrm{Ln}[100/(1+0)^1] - \mathrm{Ln}[100/(1.05)^1]}{0.3\sqrt{1}} - \tfrac{1}{2}0.3(1)\right] = 14.17$

Of course, the latter differs from the first two valuations since an arbitrarily large number of stock price movements are incorporated into the BSM value. If more steps are added to the binomial approach it will approach the above result pretty quickly. For example, C(five-step binomial) = 14.73.[7]

Multi-step binomial valuation is solved by starting with the last step and solving for the imputed value at the beginning of this step, rebalancing the hedge portfolio by buying or selling stock and adding or repaying debt at each possible node. Then, work backwards through the tree.[8] For example, the tree for a two-step process is



where $C^+ = \Delta^+S^+ + B^+$, $\Delta^+ = (C^{++} - C^{+-})/(S^{++} - S^{+-})$, $B^+ = (uC^{+-} - dC^{++})/((u\text{-}d)(1+r))$, and C^-, Δ^- and B^- are defined analogously. Then, $C = \Delta S + B$ where

$$\Delta = (C^+ - C^-)/(S^+ - S^-) \text{ and } B = (uC^- - dC^+)/((u\text{-}d)(1+r)).$$

[7] Comment: the value of a European put option (option to sell the stock) equals $P = C + X/(1+r)^t - S = 12.15$, again by virtue of no arbitrage. A levered investment in the stock that mirrors the payoffs on the put option creates a hedge portfolio. For Black-Scholes-Merton valuation this is $P = X/(1+r)^t\, \mathrm{N}(-d_2) - S/(1+\delta)^t\, \mathrm{N}(-d_1)$.

[8] It's important that u, d, and $1+r$ be equivalently denominated. For example, if there are two steps per period then each should represent half period increments. No arbitrage is sustainable so long as $d < 1+r < u$.

Suggested:

Part A

1. For the above *traded* call option (t = 1 year to maturity, current stock price S = \$100, exercise price X = \$100, volatility $\sigma = 0.3$, riskless interest rate $(1+r)^h = 1.05$, or with continuous compounding exp[r] = 1.05. and the dividend yield is zero), verify the three valuations:

    (a) one-step binomial (using "risk-neutral" probabilities),

    (b) one-step levered investment, and

    (c) BSM.

2. Determine the *two-step binomial valuation* for the *traded option* in 1 (the possible steps for the two-step binomial are up-up, up-down, down-up, or down-down). Recall the options are exercised or expire after one year and stock price movements are for each six month period, $u = exp(\sigma\sqrt{t/h}) = exp(0.3\sqrt{1/2}) \approx 1.236$, and $d = exp(-\sigma\sqrt{t/h}) = exp(-0.3\sqrt{1/2}) \approx 0.809$. Is this closer to the BSM valuation? (While binomial valuation converges toward BSM valuation, it may oscillate.)

3. Valuation of a *warrant* differs from a traded option since warrants are obligations of the firm and there is a dilution effect. Suppose Ralph's firm has n = 1,000 shares outstanding and issues m = 100 warrants. The value of a warrant assuming the proceeds are immediately distributed to current shareholders is $W = C/(1 + m/n)$.[9] Determine the value of the warrant for the

    (a) binomial one-step,

    (b) binomial two-step, and

    (c) BSM valuation.

What is assumed about valuation of warrants here that is unlikely to hold when warrants are issued to employees?

[9] See Galai and Schneller, 1978, "Pricing of warrants and the value of the firm," *The Journal of Finance*.

Part B

The above warrant-based valuation (partially) addresses Rubinstein's dilution problem (6) and the FASB approach attempts to account for forfeiture effects (problem 3). However, ESOs differ from warrants in that warrants generate cash proceeds while ESOs are part of the firm's compensation package. Further, warrant-based valuation of ESOs does little to address the non-transferability problem (4) and the maturity problem (1).

Suppose m = 100 ESOs granted, n = 1,000 shares outstanding, t = 2 year maturity, current stock price S = 100, exercise price X = 100, riskless interest rate r = 0.05, stock volatility σ = 0.3, forfeiture rate is estimated to be zero, vesting period of one year (ESO terms typically lie between American and European options and are sometimes referred to as Bermudan options; see Rubinstein), and a two-step binomial valuation model is employed. The warrant-based value is 17.47 while the traded option-based value is 19.22. If exercise is anticipated after one year (at the vesting date) the warrant-based value is 15.38. (All valuations are as of the grant date.)

1. Describe/demonstrate how the following events are reported under each accounting alternative: (i) "intrinsic value method", (ii) "fair value method", and (iii) "exercise date" accounting

(a) grant date,

(b) ESOs are exercised at maturity,

(c) ESOs expire unexercised, and

(d) ESOs are exercised at the vesting date.

2. For each of the above dates and alternatives that affect the financial statements, prepare partial financial statements and a directed graph. What is their net impact on owners' equity?

3. Recall these valuations are model estimates that cannot be tested against observable market values as ESOs are typically not traded. Even small parameter errors can compound to produce poor valuation estimates (ESO maturities and vesting periods are typically much longer than the above examples). Discuss Rubinstein's suggestion to employ "exercise date" accounting for ESOs.

4. From a measurement perspective, is there some sense in which any which any of these accounting choices for ESOs gets compensation expense “correct”? Is compensation expense unique?

5. From an *information* perspective, what benefits and problems do you see with these accounting approaches? Is there information regarding the incentive effectiveness (making good luck endogenous; in other words, information indicating that good fortunes are more likely) of ESOs? Is the information useful for performance evaluation? On balance, are the net benefits (including costs of defending the reports such as auditing costs) of ESO accounting positive? Is anyone better off? If so, who? Is societal welfare improved? If so, how?