**Notes on directed graph analysis of accounting**

Directed graphs are a convenient way to visualize the structural relations of dynamic systems. For accounting, graphs help us see the collective effects of transactions on financial statement accounts and help us visualize the transactions that generated the changes in account balances.

Consider the following accounting example. The financial statements below were generated from ten transactions (also listed below each with amount $y_i$; note the transaction amount is known when generating financial statements but is a target of inference to the reader of financial statements).

| **Balance Sheet** | *Ending Balance* | *Beginning Balance* |
|---|---|---|
| Cash | 11 | 8 |
| Receivables | 8 | 7 |
| Inventory | 3 | 4 |
| Plant | 11 | 10 |
| Total Assets | 33 | 29 |
| Payables | 10 | 7 |
| Owners' Equity | 23 | 22 |
| Total Liab. & Equity | 33 | 29 |

| **Income Statement** | |
|---|---|
| Sales | 7 |
| CGS | 3 |
| G&A | 3 |
| Income | 1 |

| **Activity** | **Debit Account** | **Credit Account** | **Amount** |
|---|---|---|---|
| Collections of receivables | Cash | Receivables | $y_1$ |
| Cash purchase of plant | Plant | Cash | $y_2$ |
| Payment of payables | Payables | Cash | $y_3$ |
| Bad debt expense | G&A | Receivables | $y_4$ |
| Credit sales | Receivables | Sales | $y_5$ |
| Depreciation - period cost | G&A | Plant | $y_6$ |
| Recognition of CGS | CGS | Inventory | $y_7$ |
| Accrued expenses | G&A | Payables | $y_8$ |
| Purchase inventory on credit | Inventory | Payables | $y_9$ |
| Depreciation - product cost | Inventory | Plant | $y_{10}$ |

Financial Statements and allowable journal entries

The relation between transactions and changes in accounts is a simple system of linear equations.

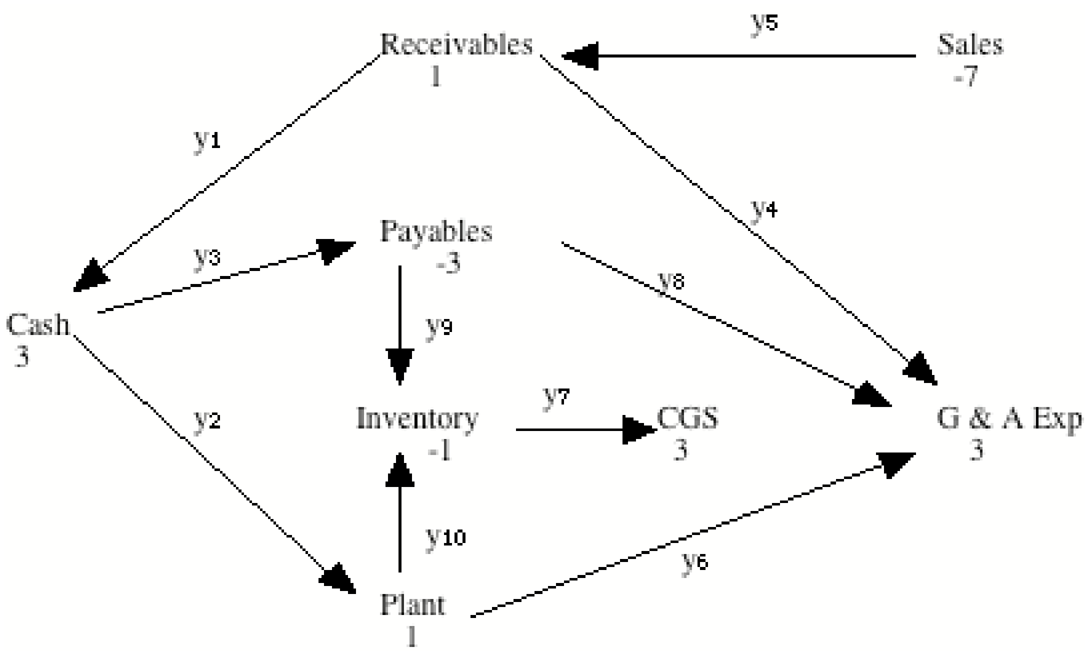
$$Ay = x$$

*A* is an incidence matrix which has a row for every account and a column for every journal entry. An incidence matrix, a matrix in which every column consists of zeros, a positive one, and a negative one, is well-suited to capture the mechanics of the double entry system; the one in the column specifies the account debited and the negative one the credit account. Column one for the example will have a one in the cash row and a minus one in the receivables row. y is a vector of transactions amounts ($y_1, \ldots, y_{10}$). *x* is a vector of changes in account balances ($x_1, \ldots, x_8$). The *A* matrix and *x* vector for the example are presented below.

| x | | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ | $y_7$ | $y_8$ | $y_9$ | $y_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | Cash | 1 | −1 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | Rec | −1 | 0 | 0 | −1 | 1 | 0 | 0 | 0 | 0 | 0 |
| −1 | Inv | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 0 | 1 | 1 |
| 1 | Plant | 0 | 1 | 0 | 0 | 0 | −1 | 0 | 0 | 0 | −1 |
| −3 | Payables | 0 | 0 | 1 | 0 | 0 | 0 | 0 | −1 | −1 | 0 |
| −7 | Sales | 0 | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 0 | 0 |
| 3 | CGS | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 3 | G & A | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| $\Sigma = 0$ | | | | | | | | | | | |

*x* vector and *A* matrix for example

This linear system can also be represented geometrically in a directed graph. In the figure below the nodes are accounts, and the arcs are journal entries.




Directed graph representation

The arrowhead denotes the account debited, the tail denotes the credit. All the information in the algebraic representation for a given $x$ and $A$ is contained in the directed graph representation.

If the transactions amounts are known, we can substitute the y's by the known amounts. If transactions amounts are unknown, they are the focus of inference. Loops in the graph indicate where information has been lost due to aggregation. We'll further discuss loops in accounting directed graphs in the future.