Ralph's 157 Dilemma

Ralph is attempting to audit a client's implementation of SFAS 157 on "fair value measurements." The client, a diversified firm whose manager is an advocate of fair value accounting, commonly trades derivatives. Ralph and the client understand that under SFAS 157, level one evidence involves direct mark-to-market observation of asset value, level two evidence involves indirect mark-to-market observation of similar asset value, and level three evidence involves mark-to-model asset valuation. The client routinely reports (unrealized holding) gains but rarely reports losses on derivatives.

Ralph is puzzled by the asymmetry of derivative profits. If market prices reflect equilibrium forces and there exists a unique equilibrium, then he would expect a balance of derivative gains and losses.

Unique equilibrium

Consider the following stylized economy with three assets and three states. State-contingent asset values are reported in the table below.

	state 1	state 2	state 3	price
asset 1	1	1	1	0.90
asset 2	72	98	121	99.4
asset 3	50	85	100	82
derivative	9	0	8	?

The current equilibrium value or price of the derivative can readily be found by solving for state prices and multiplying the vector of state prices by the state-contingent payoffs on the derivative. Let X be a matrix of state-contingent payoffs on the three assets

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 72 & 98 & 121 \\ 50 & 85 & 100 \end{bmatrix}$$

let p be a vector of current asset prices

$$p = \begin{bmatrix} 0.90 \\ 99.4 \\ 82 \end{bmatrix}$$

and let y be a vector of state prices

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

where y_j represents the non-negative price of a \$1 payoff in state j. Then, y is found by solving $Xy = p$ and the equilibrium price of the derivative is the vector inner product $x_d y$ where

$x_d = [9 \quad 0 \quad 8]$, the vector of state-contingent payoffs on the derivative.

<u>Multiple equilibria</u>

Now, consider the following stylized economy with three assets and four states. State-contingent asset values are reported in the table below.

	state 1	state 2	state 3	state 4	price
asset 1	1	1	1	1	0.90
asset 2	72	98	121	146	99.4
asset 3	50	85	100	140	82
derivative	9	0	8	0	?

There no longer exists a unique equilibrium, $x_d y$, as $Xy = p$ has many solutions for y where

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 72 & 98 & 121 & 146 \\ 50 & 85 & 100 & 140 \end{bmatrix}, p = \begin{bmatrix} 0.90 \\ 99.4 \\ 82 \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}, \text{ and } x_d = [9 \quad 0 \quad 8 \quad 0].$$

Suggested:

1. Solve for the nonnegative state prices, y, and equilibrium price, $\underline{x_d y}$, of the derivative for the stylized economy with a unique equilibrium (economy 1). What is the probability assignment over the states for a representative agent in the economy? (hint: the recovery theorem suggests the state prices rescaled to add to one.)

2. Solve for the nonnegative state prices, y^k, and equilibrium prices, $x_d y^k$, of the derivative for the stylized economy with multiple equilibria (economy 2), where y^k, $k = \{1, 2, 3, 4\}$, represents the state prices with price for state k set equal to zero.

Note: if any element of y^k is negative when solving $Xy^k = p$ then there is no arbitrage-free equilibrium solution with price of state k set equal to zero. In other words, negative state prices allow arbitrage opportunities as it's possible to determine portfolio weights on the assets that generate non-negative payoffs in every state but the price of the portfolio is negative (that is, a negative investment yields a non-negative payoff in every state – an arbitrage opportunity). This is sometimes referred to as the fundamental theorem of finance (a restatement of a more widely applied theorem, the theorem of the separating hyperplane).

Theorem: Either there exists a (non-negative) solution $y \geq 0$ for $Ay = x$

or there exists a λ such that $A^T \lambda \geq 0$ and $\lambda^T x < 0$.

λ represents the portfolio weights so that $A^T \lambda$ is the state-by-state portfolio payoff and $\lambda^T x$ is the price or investment in the portfolio (here A refers to X concatenated with x_d and x is p).

3. What is the range of equilibrium prices for the derivative in economy 2? How does this compare with the equilibrium price for the derivative in economy 1?

Now, suppose the client trades derivatives in economy 2 (the one with multiple equilibria) by buying derivatives at some price, say the minimum price identified in economy 2, and then (immediately) selling a small fraction, say one percent, to establish a "market price" for the remainder at the maximum of the range of derivative prices in economy 2.

4. What gain (holding and realized) does the client recognize on the derivatives under "mark to market" SFAS 157 accounting? How does this compare with activities preceding the demise of Enron?

5. What dilemma and/or risk does this create for Ralph, the auditor?

6. What risk does this create for the economy? Does this bear any resemblance to the recent economic slump (sometimes referred to as the subprime mortgage collapse)? How might fair value accounting combined with multiple equilibria affect (ex ante) incentives or performance evaluation?