Ralph's Vertical Integration[1]

Ralph's firm has recently vertically integrated via merger. Following the merger, inputs that were acquired from outside vendors are to be produced by a business unit of the merged firm. Ralph is charged with evaluating coordination of the merged firm. The primary issue involves coordinating sequential production and motivating the managers of the two business units.

The merger proposal assumed that the two business units would coordinate their efforts so that the shareholders' interests are served. Ralph views this as the "pie in the sky" option. More details will follow.

A second option, the other extreme, is to let the managers do what they think is best and abandon coordination efforts. More to come on this as well.

A third (midstream) option is to recognize that coordination is costly but perhaps still beneficial. Coordination costs stem, in part, from difficulties associated with measuring production in each business unit. Production in the second business unit depends on production in the first business unit, the manager's action, and a random production shock. (Production in the first unit is similar to that in the second unit except that there of course is no prior production on which the first business-unit's production depends.) Production is represented as follows:

$$x_t = x_{t-1} + a_t + \varepsilon_t \qquad t \in [1, 2]$$

where x_t is production in unit t ($x_0 = 0$), a_t is the business unit t manager's action $a \in [1, 2]$, and ε_t is a random production shock in unit t; the shock is independently and normally distributed with mean zero and variance equal to one. Hence, if the unit one

[1] This example is based on ideas originated in Arya, Fellingham, and Schroeder, " Aggregation and Measurement Errors in Performance Evaluation,". *Journal of Management Accounting Research*,16, 2005, 93-106.

manager provides act a = 2 then expected production is 0 + 2 + 0 = 2, and for act a = 1 expected production is 1.

"Pie in the sky" option

The merger proposal assumes the merged firm's expected benefit from production is $4x_2$ and that each manager provides action a = 2. The merger proposal does recognize that each manager must be motivated. Each manager is assumed to dislike risk and these preferences are represented as E[U] = E[w] - Var[w] - c(a) where E and Var are the expected value and variance operators, U is the manager's utility or satisfaction, w is the manager's compensation, and c(a) is the manager's personal cost or distaste for the action a. The manager's compensation is a flat salary b plus a bonus that varies with measured production in each business unit. That is, $w = b + d_1m_1 + d_2m_2$ where m_t is measured production for unit t and the parameters may differ for each manager. The manager's personal cost is c(a=1) = 0 and c(a=2) = 1. The manager will only agree to continue his employment if E[U] is at least 0.5.

Accordingly, the compensation cost to each manager is assumed to be equal to 2.5 (compensation of 1 for action and 1 for risk aversion) in the merger proposal. Since the expected benefit is 16 for action a = 2 and 8 for action a = 1, the expected benefits of action a = 2 over action a = 1 (16 - 8 = 8) exceed the incremental cost (differential managerial compensation = 5 - 1 = 4). Ralph regards the above option as the "pie in the sky" option because it assumes that production measurement is costless. As (almost) always, production measurement is difficult (if not, there would be little demand for accounting).

Abandon coordination option

If coordination between the two production units is abandoned then each manager is allowed to follow action a = 1 and paid a flat wage of 0.5. Hence, the expected benefits

net of managerial compensation are 8 - 1 = 7. Of course, this is less than the unachievable "pie in the sky" option (16 - 5 = 11).

Coordination via accounting option

An alternative is to use accounting to measure production for each business unit.[2] This measurement is imperfect. That is, it involves measurement error. Measured production is represented as

$$m_t = x_t + e_t$$

where m_t is measured production in unit t, and e_t is (random) measurement error in unit t; measurement error is independently and normally distributed with mean zero and variance equal to one.[3] The imperfect measurement associated with this option poses a concern since it puts added risk on risk-averse managers. It makes it more costly to coordinate production. Nonetheless Ralph believes this option may be the best alternative available. Ralph proposes to employ the following linear compensation contract form to reward the managers.

For unit manager 1: $w^1 = b^1 + d^1_1 m_1 + d^1_2 (m_2 - 2)$

For unit manager 2: $w^2 = b^2 + d^2_1 (m_1 - 2) + d^2_2 (m_2 - 2)$

[2] Ralph is only considering the production units. The analysis could be extended in a similar manner to marketing, and distribution as well as production. Again measurements of productivity almost surely involve error. For four business units with the same structure as above, the variance of the errors, n (defined carefully later) is

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 3 & 2 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 2 & 3 & 5 \end{bmatrix}$$

[3] Independence applies to measurement errors between units as well as mutual independence with production shocks.

(Superscripts denote the manager being rewarded.) The objective is to motivate the manager's action. If Ralph can estimate the manager's action from observed measurements then Ralph can determine the bonus parameters d. The observed measurements however include the actions of other managers. Hence, measurements of production are adjusted to eliminate other manager's actions. That is, the adjustment to measured production, $m_t - 2$, simply removes the other manager's action ($a = 2$ is assumed to be motivated via the contract with the other manager) from measured production. Now, Ralph not only would like to estimate the manager's action from measured production but Ralph would also like to minimize the variance of this estimate. Minimizing the variance of the estimate allows Ralph to impose less risk on the managers (recall the managers are averse to risk).

Fortunately, Ralph recognizes this problem as one to which Gauss provided a general solution almost two centuries ago. Ralph seeks to find the minimum variance (unbiased) estimate of action a^t (for unit manager t) based on adjusted measured production. Adjusted measured production for the two business units is below.

For unit manager 1: $m_1 = a^1 + \varepsilon_1 + e_1$

$$m_2 - 2 = a^1 + \varepsilon_1 + \varepsilon_2 + e_2$$

Rewrite the two equations in matrix form.

$$m' = Ha^1 + n$$

where $m' = \begin{bmatrix} m_1 \\ m_2 - 2 \end{bmatrix}$, $H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $n = \begin{bmatrix} \varepsilon_1 + e_1 \\ \varepsilon_1 + \varepsilon_2 + e_2 \end{bmatrix}$ with $\mathrm{Var}[n] = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.

For unit manager 2: $m_1 - 2 = \varepsilon_1 + e_1$

$$m_2 - 2 = a^2 + \varepsilon_1 + \varepsilon_2 + e_2$$

Again rewrite the two equations in matrix form.

$$m' = Ha^2 + n$$

where $m' = \begin{bmatrix} m_1 - 2 \\ m_2 - 2 \end{bmatrix}$, $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $n = \begin{bmatrix} \varepsilon_1 + e_1 \\ \varepsilon_1 + \varepsilon_2 + e_2 \end{bmatrix}$ with $Var[n] = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.

Since these equations usually have no exact solution, Gauss proposed that one find the line which comes closest to the observed data after adjusting for differences in the "quality" of the measurements. Gauss' solution for the minimum variance unbiased estimate is (again in matrix form)

$$\hat{a} = \left(H^T Var[n]^{-1} H\right)^{-1} H^T Var[n]^{-1} m'$$

and it has variance

$$Var(\hat{a}) = \left(H^T Var[n]^{-1} H\right)^{-1}.$$

Steps to complete (throughout the analysis assume that when indifferent the manager will act in the owners' best interest; choose a = 2):

Required:

1. Verify that the managers of the two business units would be motivated to undertake action a = 2 if the "pie in the sky" option were available. This requires that you verify that the optimal wage contract for unit manager 1 is $w^1 = 0.5 + 1\ x_1 + 0\ x_2$ and for unit manager 2 is $w^2 = 0.5 - 1\ x_1 + 1\ x_2$, and that the variance of compensation (w) is one for both managers. (Note: you can use Gauss but recognize that the variances, the diagonal terms, are reduced by one since there is no measurement error in this case).

2. Verify that the managers of the two business units would be motivated to undertake action a = 1 if the "abandon coordination" option is pursued. This requires that you verify that a sufficient wage contract for unit manager 1 is $w^1 = 0.5 + 0\ m_1 + 0\ (m_2 - 1)$ and for unit manager 2 is $w^2 = 0.5 + 0\ (m_1 - 1) + 0\ (m_2 - 1)$, and that the variance of compensation (w) is zero for both managers.

3. Verify that the managers of the two business units would be motivated to undertake action a = 2 if the "coordination via accounting" option is pursued. This requires that you verify that the optimal wage contract for unit manager 1 is $w^1 = 7/6 + 2/3\ m_1 + 1/3\ (m_2 - 2)$ and for unit manager 2 is $w^2 = 2 - 1/2\ (m_1 - 2) + 1\ (m_2 - 2)$, and that the variance of compensation (w) is 5/3 for manager 1 and 5/2 for manager 2.

4. Is "abandon coordination" or "coordination via accounting" the better option for the merged firm?