Ralph's Uncertain Variance

Ralph is pondering whether the state of the economy is favorable (f) or unfavorable (u). Without additional information, he believes the states are equally likely. Ralph's revenue has mean equal to 25 given a favorable economy and mean equal to 21 given an unfavorable economy. Given either state of the economy, Ralph believes the standard deviation of revenue is bounded between 1 and 3.

Ralph knows the maximum marginal entropy probability assignment (for revenue) with the above background knowledge is that revenue conditional on the state of the economy is normally distributed with means equal to 25 for f or 21 for u and standard deviation equal to 3 (the upper bound on the interval).[1] However, upon reflection Ralph recognizes that this is a joint entropy/probability assignment exercise as both y and σ are random variables. Accordingly, Ralph adds to his background knowledge by quantifying his beliefs regarding σ as $E\left[\log\sigma^2\right] = 1.098612$ or geometric mean equal to $\exp[1.098612] = 3$.[2]

Suggested:

1. Utilize Ralph's background knowledge to assign maximum entropy joint density functions $g(y,\sigma \mid f)$ and $g(y,\sigma \mid u)$ where $-\infty < y < \infty$ and $1 \leq \sigma \leq 3$.

— the kernel for $g(y,\sigma \mid f)$ is $\frac{1}{\sigma^2}\exp\left[-\frac{(y-25)^2}{2\sigma^2}\right]$
and for $g(y,\sigma \mid u)$ is $\frac{1}{\sigma^2}\exp\left[-\frac{(y-21)^2}{2\sigma^2}\right]$. explain.

2. Determine Ralph's revised state probability beliefs given that revenue equal to 24 is observed if the information is assigned

a. normal density functions with appropriate respective means and $\sigma = 3$.

b. normal density functions with appropriate respective means and $\sigma = 1$.

c. density functions $g(y,\sigma \mid f)$ and $g(y,\sigma \mid u)$ in 1 above.

d. what do you observe in the various probability assignments regarding the belief updating? is the moment condition $E\left[\log\sigma^2\right]$ informative? that is, does it impact state belief updating?

3. How do 1 and 2 change if Ralph's background knowledge is $E\left[\log\sigma^2\right] = 1.29584$ or geometric mean equal to $\exp[1.29584] = 3.65405$?

[1] See Ralph's density assignment.

[2] Ralph knows this moment condition leads to Jeffrey's ignorance probability assignment for σ where the kernel is $g(\sigma) \propto \frac{1}{\sigma}$. Equivalently, $\log\sigma$ is uniformly distributed.