Ralph's Technology

As a manager of a productive operation (a firm), Ralph believes his (stewardship) responsibility includes

– conduct low cost experiments to improve productive technology,

– interpret data generated by the experiments (Ralph thinks tools may be helpful here),

– make judgments based on experimental results tempered by background knowledge,

– design and implement efficient strategies of production (production technologies).

Ralph has two production facilities: A and B. He believes facility A is more efficient than facility B and would like to improve the efficiency of both, but is especially concerned about facility B. Ralph is considering a change in technology and has gathered data from a small experiment where facility B has been a focal point (productive efficiency is measured in terms of production successes).

	Facility A		Facility B		Total	
Technology	New	Old	New	Old	New	Old
Successes	10	120	133	25	143	145
Trials	10	150	190	50	200	200
% Success	100	80	70	50	71.5	72.5

Part A.

Suggested:

1. a) Based on aggregate data (Total), does the new technology seem to produce an improvement?

b) Based on facility A, does the new technology seem to produce an improvement?

c) Based on facility B, does the new technology seem to produce an improvement?

d) The contrast between aggregate and disaggregate data is an illustration of Simpson's paradox. Explain.

2. To affirm your findings above (and possibly extend your thinking to richer, more challenging settings) utilize a (linear probability) regression model to re-analyze these data. To facilitate this exercise, create a spreadsheet with a column designated outcome

(success => $Y = 1$, failure => $Y = 0$), another column designated facility (facility A => $D_{fac} = 1$, facility B => $D_{fac} = 0$), and a third column designated technology (new => $D_{tech} = 1$, old => $D_{tech} = 0$). Your spreadsheet should consist of 400 rows (equal to the number of experimental trials) where the 1s and 0s match the data in the above table (this may require some trial and error but is an integral part of data analysis).

a) Regress outcome on the technology variable alone (include the intercept in the regression). $Y = b_0 + b_{tech} D_{tech}$ + residuals

Interpret the results.

Keep in mind $b_{tech} = \overline{Y}(D_{tech} = 1) - \overline{Y}(D_{tech} = 0)$

and $b_0 = \overline{Y} - \overline{D}_{tech}\, b_{tech}$ where $\overline{Y}$ and $\overline{D}$ represent sample averages of Y and D, respectively.

b) Regress outcome on the facility variable alone.

$$Y = b_0 + b_{fac} D_{fac} + \text{residuals}$$

Interpret the results.

Keep in mind $b_{fac} = \overline{Y}(D_{fac} = 1) - \overline{Y}(D_{fac} = 0)$

and $b_0 = \overline{Y} - \overline{D}_{fac}\, b_{fac}$.

c) Regress outcome on the technology and facility variables.

$$Y = b_0 + b_{tech} D_{tech} + b_{fac} D_{fac} + \text{residuals}$$

Interpret the results.

Keep in mind $b_0 = \overline{Y} - \overline{D}_{tech}\, b_{tech} - \overline{D}_{fac}\, b_{fac}$.

3. Frequently, the effects are richer than those illustrated above.

a) Repeat 1 and 2 for the data tabulated below (changes are in bold).

	Facility A		Facility B		Total	
Technology	New	Old	New	Old	New	Old
Successes	**7**	120	133	25	**140**	145
Trials	10	150	190	50	200	200
% Success	**70**	80	70	50	**70**	72.5

b) Add a fourth column to your spreadsheet for the second data table that is the product of indicator variables for facility and technology. Now, regress outcome on the three variables.

$$Y = b_0 + b_{tech}\, D_{tech} + b_{fac}\, D_{fac} + b_{tech*fac}\, D_{tech}\, D_{fac} + \text{residuals}$$

Interpret the results. What is the consequence of omitting the interaction term? In other words, what model restriction is relaxed when the variable is included?

Keep in mind $b_{tech*fac} = \{\overline{Y}(D_{fac} = 1, D_{tech} = 1) - \overline{Y}(D_{fac} = 1, D_{tech} = 0)\}$

$$- \{\overline{Y}(D_{fac} = 0, D_{tech} = 1) - \overline{Y}(D_{fac} = 0, D_{tech} = 0)\}$$

and $b_0 = \overline{Y} - \overline{D}_{tech}\, b_{tech} - \overline{D}_{fac}\, b_{fac} - \overline{D}_{tech*fac}\, b_{tech*fac}$ where $\overline{D}_{tech*fac}$ is the sample mean of the product $D_{tech}\, D_{fac}$.

Part B. **Causal effects**. Ralph interprets the effect of technology on production success as causal, holding other things constant technology adoption causes a change in production success, if he can effectively address the counterfactual nature of potential outcome. That is, for a given trial Ralph observes either Y_1, success (1) or failure (0) with the new technology, or Y_0, success/failure with the old technology, but not both. The counterfactual potential outcome is success/failure with the technology not employed. Let $(Y_1|D_{\text{tech}}=1)$, the outcome with the new technology when new technology is assigned to the production trial, or $(Y_0|D_{\text{tech}}=0)$, the outcome with the old technology when the old technology is assigned to the production trial, be the observed outcome for any trial, hence observed outcome is

$$Y = (D_{\text{tech}}=1)\ (Y_1|D_{\text{tech}}=1) + (D_{\text{tech}}=0)\ (Y_0|D_{\text{tech}}=0).$$

On the other hand, $(Y_1|D_{\text{tech}}=0)$, potential outcome with the new technology but the production trial is assigned the old technology, and $(Y_0|D_{\text{tech}}=1)$, potential outcome with the old technology but the production trial is assigned the new technology, are the counterfactual potential outcomes. Then, the average causal effect of production technology on production success for trials assigned the new technology is

$$\text{ATT} = \text{E}[Y_1 - Y_0|D_{\text{tech}}=1],$$

called the average treatment effect on the treated, and for trials assigned the old technology is

$$\text{ATUT} = \text{E}[Y_1 - Y_0|D_{\text{tech}}=0],$$

called the average treatment effect on the untreated. And, the unconditional average treatment effect is

$$\text{ATE} = \text{pr}(D_{\text{tech}}=1)\text{ATT} + \text{pr}(D_{\text{tech}}=0)\text{ATUT} = \text{E}[Y_1 - Y_0].$$

Ralph recognizes the challenge is to utilize the observables to infer counterfactual means as any average treatment effect is counterfactual in nature. For example,

$$\text{ATT} = \text{E}[Y_1|D_{\text{tech}}=1] - \text{E}[Y_0|D_{\text{tech}}=1]$$

where the second term is unobservable (counterfactual). However, if Ralph can employ his state of knowledge to deduce $\text{E}[Y_0|D_{\text{tech}}=1] = \text{E}[Y_0|D_{\text{tech}}=0]$, then, as the latter is observable, ATT can be inferred from the data. Ralph recognizes this typically lacks credibility as unspecified conditions would need to be balanced/randomized between the technology assignments and often these assignments are not random (as in the present setting). Otherwise, the data is likely to exhibit selection bias. Selection bias associated with ATT, for instance, is

$$\begin{aligned} \text{ATT(bias)} &= \text{target estimand less estimator employed} \\ &= \{E[Y_1|D_{tech}=1] - E[Y_0|D_{tech}=1]\} - \{ E[Y_1|D_{tech}=1] - E[Y_0|D_{tech}=0]\} \\ &= E[Y_0|D_{tech}=0] - E[Y_0|D_{tech}=1] \end{aligned}$$

Consequently, Ralph concludes a more promising avenue is to focus on average effects conditional on recognized salient features of the setting, in this case the production facility employed. That is, Ralph can utilize his state of knowledge to deduce conditional mean independence

$$E[Y_0|D_{tech}=1, D_{fac}=1] = E[Y_0|D_{tech}=0, D_{fac}=1] \qquad (1)$$

and

$$E[Y_0|D_{tech}=1, D_{fac}=0] = E[Y_0|D_{tech}=0, D_{fac}=0] \qquad (2).$$

Now, Ralph can infer ATT(D_{fac}=1), via condition (1), and ATT(D_{fac}=0), via condition (2). Also, by iterated expectations, Ralph can infer

$$ATT = \Pr(D_{fac}=1 \mid D_{tech}=1)\, ATT(D_{fac}=1) + \Pr(D_{fac}=0 \mid D_{tech}=1)\, ATT(D_{fac}=0),$$

via conditions (1) and (2). Similarly, but separately deduced, ATUT can be teased out if conditional mean independence is consistent with Ralph's state of knowledge regarding Y_1.

$$E[Y_1|D_{tech}=1, D_{fac}=1] = E[Y_1|D_{tech}=0, D_{fac}=1] \qquad (3)$$

and

$$E[Y_1|D_{tech}=1, D_{fac}=0] = E[Y_1|D_{tech}=0, D_{fac}=0] \qquad (4).$$

Now, Ralph can infer ATUT(D_{fac}=1), via condition (3), and ATUT(D_{fac}=0), via condition (4). Again, by iterated expectations, Ralph can infer

$$ATUT = \Pr(D_{fac}=1 \mid D_{tech}=0)\, ATUT(D_{fac}=1) + \Pr(D_{fac}=0 \mid D_{tech}=0)\, ATUT(D_{fac}=0),$$

via conditions (3) and (4).

Combining conditions (1) and (3) allows Ralph to infer

$$ATE(D_{fac}=1) = \Pr(D_{tech}=1| D_{fac}=1) ATT(D_{fac}=1) + \Pr(D_{tech}=0| D_{fac}=1) ATUT(D_{fac}=1),$$

while conditions (2) and (4) allow Ralph to infer

$$ATE(D_{fac}=0) = \Pr(D_{tech}=1| D_{fac}=0) ATT(D_{fac}=0) + \Pr(D_{tech}=0| D_{fac}=0) ATUT(D_{fac}=0).$$

Then, by iterated expectations,

$$ATE = \Pr(D_{fac}=1)\, ATE(D_{fac}=1) + \Pr(D_{fac}=0)\, ATE(D_{fac}=0)$$

and

$$\text{ATE} = \Pr(D_{\text{tech}}=1)\ \text{ATT} + \Pr(D_{\text{tech}}=0)\ \text{ATUT}.$$

Suggested:

4. For the original data, if facility is ignored determine Ralph's inferred ATT, ATUT, and ATE.

5. For the second data panel, if facility is ignored determine Ralph's inferred ATT, ATUT, and ATE.

6. For the original data, employ Ralph's conditional mean independence arguments to infer ATT(D_{fac}=1), ATT(D_{fac}=0), ATT, ATUT(D_{fac}=1), ATUT(D_{fac}=0), ATUT, ATE(D_{fac}=1), ATE(D_{fac}=0), and ATE.

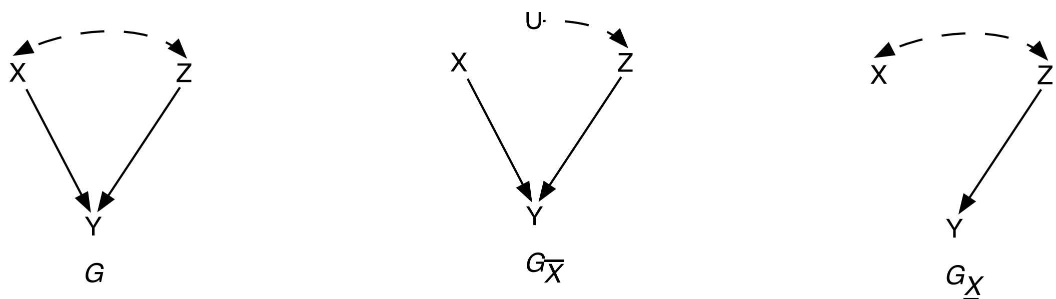
7. For the second data panel, employ Ralph's conditional mean independence arguments to infer ATT(D_{fac}=1), ATT(D_{fac}=0), ATT, ATUT(D_{fac}=1), ATUT(D_{fac}=0), ATUT, ATE(D_{fac}=1), ATE(D_{fac}=0), and ATE.

8. Suppose conditional mean independence is consistent with the data generating process, what is the selection bias when facility is omitted in the analysis?

9. Is conditional mean independence credible in this setting?

C. Causal effects via Bayesian networks.

Ralph's thought experiment regarding the causal connections amongst the variables is depicted in the network graph *G* below. Y is success (1)/failure (0), X is technology (new=1, old=0), and Z is facility (A=1, B=0). Arrows indicate causal direction and a dashed bow indicates a common set of unobservable variables (U) connecting the observable variables.



The network graph represents a form of probability assignment. In particular, the graph indicates (conditional) independence between the variables and otherwise is representative of the considered data generating process (DGP) for any probability distribution consistent with the graph (accordingly, the graphical depiction is said to be nonparametric).

A set of variables Z separates X and Y if and only if every path is blocked by Z. A set of variables separates (blocks) a path under the following conditions in the graph.

1. The path contains a chain i–>m–> j or fork i<–m–>j where m is included in the set Z.
2. The path contains an inverted fork (collider) i–>m<–j where m is excluded from the set Z and no descendant of m is in Z.

Separation or blocking of paths indicates conditional independence in the graph. In other words, if a set of variables Z separates X and Y along every path, then X and Y are independent conditional on Z.

Ralph would like to address the probability of success given that technology is set equal to new (1) or old (0). That is, Ralph poses a causal (or action) question which differs from observation. Action asks $\Pr(Y=1|do(X=1))$ or $\Pr(Y=1|do(X=0))$ while observation poses $\Pr(Y=1|X=1)$ or $\Pr(Y=1|X=0)$.

In the current example, there is a back-door path through Z (and into X) between X and Y. A back-door adjustment yields the action probability statement

$$\Pr(Y=1|do(X=1)) = \Sigma_z \Pr(Y=1|X=1,z)\Pr(z)$$

while observation produces the probability statement

$$\Pr(Y=1|X=1) = \Sigma_z \Pr(Y=1|X=1,z)\Pr(z|X=1)$$

where Σ_z refers to summation over all values z.

Suggested:

10. For the first data set, determine Pr(Y=1|do(X=1)), Pr(Y=1|do(X=0)), Pr(Y=1|X=1), and Pr(Y=1|X=0), as well as ATE = E[Y| do(X=1)] - E[Y| do(X=0)], and E[Y| X=1] - E[Y| X=0].

11. Repeat 10 for the second data set.

12. How does this analysis compare with that in part B?