Ralph's Synergy

Ralph believes synergy is the key to successful business strategy. That is, designing a synergistic production technology that effectively copes with a dynamic and uncertain environment. Measurements are key to gauging effectiveness but any measurements are likely to interfere with production and potentially mute synergy. Incorporation of these features into the Cobb-Douglas frame is inelegant, so Ralph considers an alternative frame. Drawing on quantum information theory, Ralph imagines a quantum production technology frame for his two business unit organization.

First, some background on quantum information theory. Then, we consider Ralph's quantum production technology. There are four quantum axioms governing the behavior of quantum probabilities (see Nielsen and Chuang [2002]). The axioms are outlined in standard quantum bit (qubit) form.

1 Quantum information axioms

1.1 The superposition axiom:

A quantum unit (qubit) is specified by a two element vector, say $\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, with $|\alpha|^2 + |\beta|^2 = 1$.

Let $|\psi\rangle \equiv \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha\,|0\rangle + \beta\,|1\rangle$,[1] $\langle\psi| = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}^\dagger$ where † is the adjoint (conjugate transpose) operation.

1.2 The transformation axiom:

Transformation of a quantum unit is accomplished by unitary (length-preserving) matrix multiplication. The Pauli matrices provide a basis of unitary operators.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

[1] Dirac notation is a useful and powerful descriptor, as $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

where $i = \sqrt{-1}$. The operations work as follows: $I \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, $X \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$, $Y \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -\beta i \\ \alpha i \end{bmatrix}$, and $Z \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$.

Other useful single qubit transformations are $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and $\Theta = \begin{bmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{bmatrix}$. Examples of these transformations, in Dirac notation are

$$H\left|0\right\rangle = \frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}};\ H\left|1\right\rangle = \frac{\left|0\right\rangle - \left|1\right\rangle}{\sqrt{2}}$$

$$\Theta\left|0\right\rangle = e^{i\theta}\left|0\right\rangle;\ \Theta\left|1\right\rangle = \left|1\right\rangle$$

1.3 The measurement axiom:

Measurement of a quantum state is accomplished by a linear projection from a set of projection matrices which add to the identity matrix.[2] The probability of a particular measurement occurring is the squared absolute value of the projection. (An implication of the axiom not explicitly used here is that the post-measurement state is the projection appropriately normalized; this effectively rules out multiple measurements of unknown qubits.)

For example, let the projection matrices be $M_0 = \left|0\right\rangle\left\langle 0\right| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $M_1 = \left|1\right\rangle\left\langle 1\right| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Note that M_0 projects onto the $\left|0\right\rangle$ vector and M_1 projects onto the $\left|1\right\rangle$ vector. Also note that $M_0^\dagger M_0 + M_1^\dagger M_1 = M_0 + M_1 = I$. For $\left|\psi\right\rangle = \alpha\left|0\right\rangle + \beta\left|1\right\rangle$, the projection of $\left|\psi\right\rangle$ onto $\left|0\right\rangle$ is $M_0\left|\psi\right\rangle$. The probability of measuring in eigenstate $\left|0\right\rangle$ is $\left\langle\psi\right| M_0 \left|\psi\right\rangle = \left|\alpha\right|^2$ and the probability of measuring in eigenstate $\left|1\right\rangle$ is $\left\langle\psi\right| M_1 \left|\psi\right\rangle = \left|\beta\right|^2 = 1 - \left|\alpha\right|^2$.

1.3.1 Observables

The observed measurement result is the associated eigenvalue with the eigenstate based on the observable. Observables are Hermitian operators. That is, matrices

[2] More precisely, the projection matrices satisfy the completeness condition, $\sum_m M_m^\dagger M_m = I$, where $M_m^\dagger$ is the adjoint (conjugate transpose) of projection matrix M_m.

which equal their conjugate transpose, $A = A^\dagger$, the analog to symmetry for complex vector spaces. This ensures the eigenvalues, measurement results, are real valued (rules complex eigenvalues). Suppose $A = -2\left|0\right\rangle\left\langle 0\right| + 3\left|1\right\rangle\left\langle 1\right|$. Then, the measurement results for the M_0, M_1 basis are either -2 or 3 where the resultant eigenstate is $\left|0\right\rangle$ or $\left|1\right\rangle$, respectively.

1.4 The combination axiom:

Qubits are combined by tensor multiplication. For example, two $\left|0\right\rangle$ qubits are combined as $\left|0\right\rangle \otimes \left|0\right\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ denoted $\left|00\right\rangle$. It is often useful to transform one qubit in a combination and leave another unchanged; this can also be accomplished by tensor multiplication. Let H_1 denote a Hadamard transformation on the first qubit. Then applied to a two qubit system, $H_1 = H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$ and $H_1\left|00\right\rangle = \frac{\left|00\right\rangle + \left|10\right\rangle}{\sqrt{2}}$.

Another important two qubit transformation is the controlled not operator,

$$Cnot = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where the first qubit serves as the control and the second qubit is the target. That is, if the first qubit is $\left|0\right\rangle$ the second qubit is unchanged but if the first qubit is $\left|1\right\rangle$ than the second qubit is flipped: $Cnot\left|00\right\rangle = \left|00\right\rangle$, $Cnot\left|01\right\rangle = \left|01\right\rangle$, $Cnot\left|10\right\rangle = \left|11\right\rangle$, and $Cnot\left|11\right\rangle = \left|10\right\rangle$.

Entangled two qubit states or Bell states are inherently intertwined states and defined as follows,

$$\left|\beta_{00}\right\rangle = Cnot\ H_1\left|00\right\rangle = \frac{\left|00\right\rangle + \left|11\right\rangle}{\sqrt{2}}$$

and more generally,

$$\left|\beta_{ij}\right\rangle = Cnot\ H_1\left|ij\right\rangle \text{ for } i, j = 0, 1$$

The four two qubit Bell states form an orthonormal basis.

2 Quantum production technology

Next, Ralph describes his frame for quantum production technology.

2.1 Production technology

Productivity is a transformation process, so it is naturally represented by a two qubit interferometer (transformation function) applied to a two qubit state: the pure synergy state is represented by the intertwined state $|\beta_{00}\rangle$ while the no synergy state is represented by the independent state $|00\rangle$. Ralph believes it is instructive to think of the initial state ($|00\rangle$ or $|\beta_{00}\rangle$) as common inputs and θ as business unit-specific inputs. The transformation function is $\digamma = H_2\Theta_2H_2H_1\Theta_1H_1$. where the business units each supply input $\theta_i \in \{\theta_l, \theta_h\}$.

Given the no synergy $|00\rangle$ setting, productivity is

$$\begin{aligned}
\digamma |00\rangle &= H_2\Theta_2H_2H_1\Theta_1\frac{|00\rangle + |10\rangle}{\sqrt{2}} = H_2\Theta_2H_2H_1\frac{e^{i\theta_1}|00\rangle + |10\rangle}{\sqrt{2}} \\
&= H_2\Theta_2H_2\frac{e^{i\theta_1}|00\rangle + e^{i\theta_1}|10\rangle + |00\rangle - |10\rangle}{2} \\
&= H_2\Theta_2\frac{[e^{i\theta_1}+1][|00\rangle + |01\rangle] + [e^{i\theta_1}-1][|10\rangle + |11\rangle]}{2\sqrt{2}} \\
&= H_2\frac{[e^{i\theta_1}+1][e^{i\theta_2}|00\rangle + |01\rangle] + [e^{i\theta_1}-1][e^{i\theta_2}|10\rangle + |11\rangle]}{2\sqrt{2}} \\
&= \frac{[e^{i\theta_1}+1][e^{i\theta_2}(|00\rangle + |01\rangle) + (|00\rangle - |01\rangle)]}{4} \\
&+ \frac{[e^{i\theta_1}-1][e^{i\theta_2}(|10\rangle + |11\rangle) + (|10\rangle - |11\rangle)]}{4} \\
&= \frac{[e^{i\theta_1}+1][(e^{i\theta_2}+1)|00\rangle + (e^{i\theta_2}-1)|01\rangle]}{4} \\
&+ \frac{[e^{i\theta_1}-1][(e^{i\theta_2}+1)|10\rangle + (e^{i\theta_2}-1)|11\rangle]}{4}.
\end{aligned} \tag{A1}$$

Similarly, given the pure synergy $|\beta_{00}\rangle$ setting, productivity is

$$
\begin{aligned}
F\,|\beta_{00}\rangle &= H_2\Theta_2 H_2 H_1 \Theta_1 \frac{|00\rangle + |10\rangle + |01\rangle - |11\rangle}{2} \\
&= H_2\Theta_2 H_2 H_1 \frac{e^{i\theta_1}\,|00\rangle + |10\rangle + e^{i\theta_1}\,|01\rangle - |11\rangle}{2} \\
&= H_2\Theta_2 H_2 \frac{[e^{i\theta_1}+1][|00\rangle + |11\rangle] + [e^{i\theta_1}-1][|10\rangle + |01\rangle]}{2\sqrt{2}} \\
&= H_2\Theta_2 \frac{e^{i\theta_1}[|00\rangle + |10\rangle] + |01\rangle - |11\rangle}{2} \\
&= H_2 \frac{e^{i\theta_1}e^{i\theta_2}[|00\rangle + |10\rangle] + |01\rangle - |11\rangle}{2} \\
&= \frac{[e^{i\theta_1}e^{i\theta_2}+1]\,|\beta_{00}\rangle + [e^{i\theta_1}e^{i\theta_2}-1]\,|\beta_{01}\rangle}{2}.
\end{aligned}
\tag{A2}
$$

2.2 Projection matrices for measurement

The projection matrices are defined as the sum of the outer product of success eigenstates (vectors) where success is defined as transforming a qubit from $|0\rangle$ to $|1\rangle$. Individual measurement reports a "success" or "failure" signal for each business unit's production. For business unit one, the success eigenstates are $|10\rangle$ and $|11\rangle$; and the failure eigenstates are $|00\rangle$ and $|01\rangle$. The projection matrices for business unit one are,

$$
M_{S1} = |10\rangle\langle 10| + |11\rangle\langle 11| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{and} \tag{A3}
$$

$$
M_{F1} = |00\rangle\langle 00| + |01\rangle\langle 01| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A4}
$$

Similarly, for business unit two, the success eigenstates are $|01\rangle$ and $|11\rangle$; and the failure eigenstates are $|00\rangle$ and $|10\rangle$. The projection matrices for business unit two are,

$$M_{S2} = |01\rangle\langle 01| + |11\rangle\langle 11| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{, and} \tag{A5}$$

$$M_{F2} = |00\rangle\langle 00| + |10\rangle\langle 10| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A6}$$

Note, $M_{S1}^{\dagger} M_{S1} + M_{F1}^{\dagger} M_{F1} = I$, and $M_{S2}^{\dagger} M_{S2} + M_{F2}^{\dagger} M_{F2} = I$.

Group measurement reports a "success" or "failure" signal for both business units where again success is defined by transforming same entangled pairs, $|00\rangle$ and $|11\rangle$, to opposite pairs, $|01\rangle$ and $|10\rangle$. Hence, the success eigenstates are $|\beta_{01}\rangle$ and $|\beta_{11}\rangle$; and the failure eigenstates are $|\beta_{00}\rangle$ and $|\beta_{10}\rangle$. The projection matrices are,

$$M_S = |\beta_{01}\rangle\langle \beta_{01}| + |\beta_{11}\rangle\langle \beta_{11}| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{, and} \tag{A7}$$

$$M_F = |\beta_{00}\rangle\langle \beta_{00}| + |\beta_{10}\rangle\langle \beta_{10}| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{A8}$$

Note, $M_S^{\dagger} M_S + M_F^{\dagger} M_F = I$.

2.3 Measure success probabilities

Project $\digamma\,|00\rangle$ or $\digamma\,|\beta_{00}\rangle$ onto success matrices (M_{S1} or M_S).[3] The probability of success is the squared length of the projection vectors. Under individual measure-

[3]Manager 1's success probability is described here. Parallel results apply to manager 2 if M_{S1} is replaced by M_{S2}.

ment with no synergy,

$$\begin{aligned} prob(success|IN,\theta_1,\theta_2) &= \langle 00| F^\dagger M_{S1} F |00\rangle = \| M_{S1} F |00\rangle \|^2 \\ &= \left\| \frac{1}{4} \begin{bmatrix} 0 \\ 0 \\ (e^{i\theta_1}-1)(e^{i\theta_2}+1) \\ (e^{i\theta_1}-1)(e^{i\theta_2}-1) \end{bmatrix} \right\|^2 \\ &= \frac{1-\cos\theta_1}{2} = \sin^2\frac{\theta_1}{2}. \end{aligned} \tag{A9}$$

Similarly, under individual measurement with synergy,

$$\begin{aligned} prob(success|IS,\theta_1,\theta_2) &= \langle \beta_{00}| F^\dagger M_{S1} F |\beta_{00}\rangle = \| M_{S1} F |\beta_{00}\rangle \|^2 \\ &= \left\| \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ e^{i\theta_1}e^{i\theta_2}-1 \\ e^{i\theta_1}e^{i\theta_2}+1 \end{bmatrix} \right\|^2 = \frac{1}{2}. \end{aligned} \tag{A10}$$

Under group measurement with no synergy,

$$\begin{aligned} prob(success|GN,\theta_1,\theta_2) &= \langle 00| F^\dagger M_{S} F |00\rangle = \| M_{S} F |00\rangle \|^2 \\ &= \left\| \frac{1}{4} \begin{bmatrix} 0 \\ (e^{i\theta_1}+1)(e^{i\theta_2}-1) \\ (e^{i\theta_1}-1)(e^{i\theta_2}+1) \\ 0 \end{bmatrix} \right\|^2 \\ &= \sin^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2} + \cos^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}. \end{aligned} \tag{A11}$$

Under group measurement with synergy,

$$\begin{aligned} prob(success|GS,\theta_1,\theta_2) &= \langle \beta_{00}| F^\dagger M_{S} F |\beta_{00}\rangle = \| M_{S} F |\beta_{00}\rangle \|^2 \\ &= \left\| \frac{1}{2\sqrt{2}} \begin{bmatrix} 0 \\ e^{i\theta_1}e^{i\theta_2}-1 \\ e^{i\theta_1}e^{i\theta_2}-1 \\ 0 \end{bmatrix} \right\|^2 \\ &= \sin^2\frac{\theta_1+\theta_2}{2}. \end{aligned} \tag{A12}$$

where $\theta_i \in \{\theta_l = 0, \theta_h = \frac{\pi}{3}\}$.

2.4 Observables and expected payoffs

Measurements involve real values drawn from observables — in this production setting where control and application of inputs is costly, these values correspond to the payoffs. The individual measure observable for the first business unit is

$$\begin{aligned} P_{I_1} &= \begin{bmatrix} -10 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 35 & 0 \\ 0 & 0 & 0 & 35 \end{bmatrix} \\ &= -10\left|00\right\rangle\left\langle 00\right| - 10\left|01\right\rangle\left\langle 01\right| + 35\left|10\right\rangle\left\langle 10\right| + 35\left|11\right\rangle\left\langle 11\right| \end{aligned} \tag{A13}$$

The expected payoff is

$$\begin{aligned} \langle P_{I_1}\rangle &= \left\langle 00\right| F^{\dagger} P_{I_1} F \left|00\right\rangle \\ &= -10\left\langle 00\right| F^{\dagger}\left|00\right\rangle\left\langle 00\right| F\left|00\right\rangle - 10\left\langle 00\right| F^{\dagger}\left|01\right\rangle\left\langle 01\right| F\left|00\right\rangle \\ &\quad +35\left\langle 00\right| F^{\dagger}\left|10\right\rangle\left\langle 10\right| F\left|00\right\rangle + 35\left\langle 00\right| F^{\dagger}\left|11\right\rangle\left\langle 11\right| F\left|00\right\rangle \end{aligned} \tag{A14}$$

In other words, -10 times the probability the first measure is a failure plus 35 times the probability measure one is a success.

The individual measure observable for business unit two is

$$\begin{aligned} P_{I_2} &= \begin{bmatrix} -10 & 0 & 0 & 0 \\ 0 & 35 & 0 & 0 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 35 \end{bmatrix} \\ &= -10\left|00\right\rangle\left\langle 00\right| + 35\left|01\right\rangle\left\langle 01\right| - 10\left|10\right\rangle\left\langle 10\right| + 35\left|11\right\rangle\left\langle 11\right| \end{aligned} \tag{A15}$$

The expected payoff is

$$\begin{aligned} \langle P_{I_2}\rangle &= \left\langle 00\right| F^{\dagger} P_{I_2} F \left|00\right\rangle \\ &= -10\left\langle 00\right| F^{\dagger}\left|00\right\rangle\left\langle 00\right| F\left|00\right\rangle + 35\left\langle 00\right| F^{\dagger}\left|01\right\rangle\left\langle 01\right| F\left|00\right\rangle \\ &\quad -10\left\langle 00\right| F^{\dagger}\left|10\right\rangle\left\langle 10\right| F\left|00\right\rangle + 35\left\langle 00\right| F^{\dagger}\left|11\right\rangle\left\langle 11\right| F\left|00\right\rangle \\ &= -10\left\langle 00\right| F^{\dagger} M_{F_2} F\left|00\right\rangle + 35\left\langle 00\right| F M_{S_2} F\left|00\right\rangle \end{aligned} \tag{A16}$$

In other words, -10 times the probability the second measure is a failure plus 35 times the probability measure two is a success. The expected payoff for individual measures is $\langle P_{I_1}\rangle + \langle P_{I_2}\rangle$.

On the other hand, the group measure observable is

$$\begin{aligned} P_G &= \begin{bmatrix} -30 & 0 & 0 & 10 \\ 0 & 55 & 15 & 0 \\ 0 & 15 & 55 & 0 \\ 10 & 0 & 0 & -30 \end{bmatrix} \\ &= -20\left|\beta_{00}\right\rangle\left\langle \beta_{00}\right| + 70\left|\beta_{01}\right\rangle\left\langle \beta_{01}\right| - 40\left|\beta_{10}\right\rangle\left\langle \beta_{10}\right| + 40\left|\beta_{11}\right\rangle\left\langle \beta_{11}\right| \end{aligned} \tag{A17}$$

The expected payoff is

$$\begin{aligned}
\langle P_G \rangle &= \langle \beta_{00} | F^{\dagger} P_G F | \beta_{00} \rangle \qquad \text{(A18)} \\
&= -20 \langle \beta_{00} | F^{\dagger} | \beta_{00} \rangle \langle \beta_{00} | F | \beta_{00} \rangle + 70 \langle \beta_{00} | F^{\dagger} | \beta_{01} \rangle \langle \beta_{01} | F | \beta_{00} \rangle \\
&\quad -40 \langle \beta_{00} | F^{\dagger} | \beta_{10} \rangle \langle \beta_{10} | F | \beta_{00} \rangle + 40 \langle \beta_{00} | F^{\dagger} | \beta_{11} \rangle \langle \beta_{11} | F | \beta_{00} \rangle \\
&= -20 \langle \beta_{00} | F^{\dagger} M_F F | \beta_{00} \rangle + 70 \langle \beta_{00} | F^{\dagger} M_S F | \beta_{00} \rangle
\end{aligned}$$

In other words, -20 times the probability the group measure is failure eigenstate $|\beta_{00}\rangle$ plus 70 times the probability the group measure is success eigenstate $|\beta_{01}\rangle$ minus 40 times the probability the group measure is failure eigenstate $|\beta_{10}\rangle$ plus 40 times the probability the group measure is success eigenstate $|\beta_{11}\rangle$. Since the latter two probabilities are equal to zero only values -20 and 70 are observed.

Required:

Throughout the analysis the business unit's inputs are either $\theta_l = 0$ or $\theta_h = \frac{\pi}{3}$.

Part A: superposition, transformation, and combination

Verify expressions A1 through A2.

Part B: measurement

Verify expressions A3 through A18.

Part C: synergy

1. With a no synergy initial state $|00\rangle$ and individual measurement of each business unit's productivity, what is the probability of productive success if both business units supply input $\theta_l = 0$? both supply $\theta_h = \frac{\pi}{3}$? one supplies $\theta_l = 0$ and the other supplies $\theta_h = \frac{\pi}{3}$? Determine the expected payoff for each combination of inputs supplied by the business units.

2. With a pure synergy initial state $|\beta_{00}\rangle$ and group measurement of each business unit's productivity, what is the probability of productive success if both business units supply input $\theta_l = 0$? both supply $\theta_h = \frac{\pi}{3}$? one supplies $\theta_l = 0$ and the other supplies $\theta_h = \frac{\pi}{3}$? Determine the expected payoff for each combination of inputs supplied by the business units.

3. With a pure synergy initial state $|\beta_{00}\rangle$ and individual measurement of productivity, what is the impact on business unit managers' ability to control their inputs?

4. In what sense does individual measurement destroy productive synergy? Can we have too many or excessive measures? What are the implications of influence activities which culminate in demand for more measures?

5. Productivity measurement is often treated as if it is benign. Comment.