

## Ralph's Subspaces

This is a continuation of Ralph's structure.

Required:

1. Find the number of linearly independent rows and columns of the  $A$  matrix (in other words, the dimension of the row space and column space – also called the rank of the matrix). What is the relation between spanning trees and a basis for the row space of the incidence matrix  $A$ ?
2. What is the dimension of the orthogonal subspaces to the rows, the nullspace, and the columns, the left nullspace? Find a basis for the nullspace and the left nullspace. Hint: the network graph may be instructive.
3. Suppose when trying to find a consistent solution to  $Ay = x$  we disconnect the graph, what happens to the dimension of the nullspaces?