Ralph's Structure

Ralph observes the following financial statements.

Balance sheets	Ending balance	Beginning balance
Cash	110	80
Receivables	80	70
Inventory	30	40
Property & equipment	110	100
Total assets	330	290
Payables	100	70
Owner's equity	230	220
Total equities	330	290

Income statement	for period
Sales	70
Cost of sales	30
SG&A	30
Net income	10

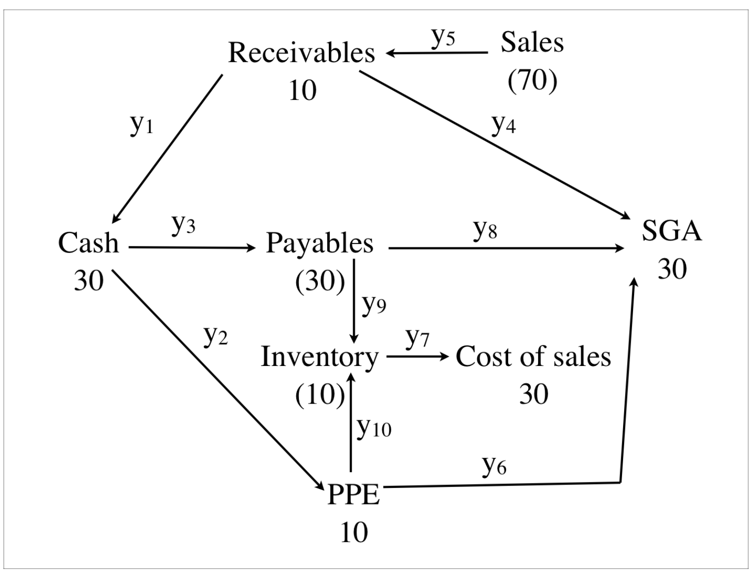
Ralph recognizes the relation between the journal entry structure of accounting, transactions amounts, and changes in account balances can be compactly represented by $Ay = x$ where A is an incidence matrix of (simple) journal entries, y is a vector of transactions amounts, and x is the change in account balances vector where debit changes are positive and credit changes are negative. By the balancing property of accounting, the sum of x is zero (a basis for the left nullspace of A is a vector of ones).

change in account	x
Δ cash	30
Δ receivables	10
Δ inventory	(10)
Δ property & equipment	10
Δ payables	(30)
sales	(70)
cost of sales	30
sg&a expenses	30

Ralph envisions the following transactions associated with the financial statements and is interested in recovering their magnitudes y.

transaction	amount
collection of receivables	y_1
investment in property & equipment	y_2
payment of payables	y_3
bad debts expense	y_4
sales	y_5
depreciation - period expense	y_6
cost of sales	y_7
accrued expenses	y_8
inventory purchases	y_9
depreciation - product cost	y_{10}

Ralph recognizes a crisp summary of these details is provided by a directed graph.



Directed graph of financial statements

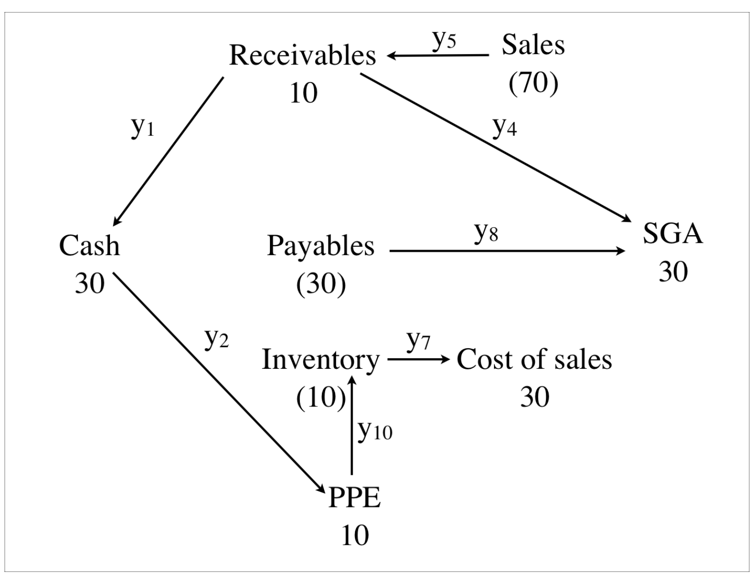
The A (incidence) matrix associated with the financial statements and directed graph where debits are denoted by +1 credits are denoted by -1 is

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

and a basis for the nullspace is immediately identified by any (usually not unique) set of linearly independent loops in the graph (the number of linearly independent loops is the number of transactions minus the number of accounts plus one), for example,

$$N = \begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & -1 \end{bmatrix}$$

A consistent solution (one of many) y^p is readily identified by forming a spanning tree and solving the remaining transactions. For instance, let $y_3 = y_6 = y_9 = 0$, the spanning tree is depicted below



Spanning tree

.

Required:

1. Find a consistent solution, $(y^p)^T = \begin{bmatrix} y_1 \, y_2 \, 0 \, y_4 \, y_5 \, 0 \, y_7 \, y_8 \, 0 \, y_{10} \end{bmatrix}$.
2. Write a general solution for transactions consistent with $Ay = x$.

Hint: is $y = y^p + N^T k$ where $k = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$ consistent?