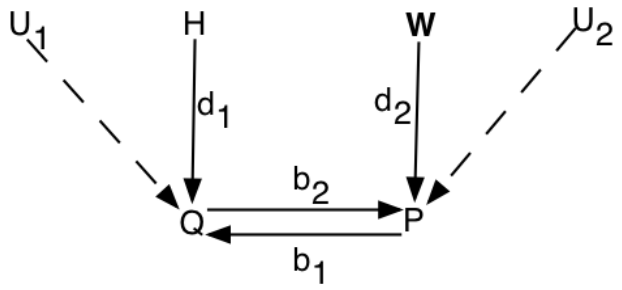
Ralph's structural queries

Ralph is exploring the impact of product pricing on market demand. As this involves market equilibrium, a directed cyclic graph (DCG) provides a reasonable representation. Ralph envisions the following simple DCG



Product market equilibrium graph G

and corresponding structural model

$$\begin{aligned} Q &= b_1 P + d_1 H + U_1 \\ P &= b_2 Q + d_2 W + U_2 \end{aligned}$$

where P is unit product price, Q is product demand, H is household income, W is the wage rate associated with producing the product, and U_1 and U_2 are unobserved factors influencing price and demand assigned a Gaussian (normal) distribution with mean $\begin{bmatrix} E[Q] - b_1 E[P] - d_1 E[H] \\ E[P] - b_2 E[Q] - d_2 E[W] \end{bmatrix}$ (typically, the intercept) and variance $\begin{bmatrix} Var[U_1] & 0 \\ 0 & Var[U_2] \end{bmatrix}$.

Ralph recognizes the reduced-form model aids identification of the structural parameters (provided rank and order conditions are satisfied).

$$\begin{bmatrix} Q \\ P \end{bmatrix} = \begin{bmatrix} 1 & -b_1 \\ -b_2 & 1 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} d_1 H \\ d_2 W \end{bmatrix} + \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \right\}$$

Suppose the data generating process (DGP) is

$$\begin{array}{lll} b_1 = -2 & E[Q] = 100 & Var[H] = 15 \\ b_2 = -0.1 & E[P] = 65 & Var[W] = 8 \\ d_1 = 0.4 & E[H] = 200 & Var[U_1] = 10 \\ d_2 = 1 & E[W] = 50 & Var[U_2] = 1 \end{array}$$

Ralph is especially interested in exploring the differences between three queries.

Query 1: the impact on expected demand of setting price (action).

$$E\left[Q \mid do\left(P = p\right), H = h, W = w\right]$$

Query 2: the impact on expected demand of observing price (observation).

$$E\left[Q \mid P = p, H = h, W = w\right]$$

Query 3: the impact on expected demand of setting price following observation of price (counterfactual).

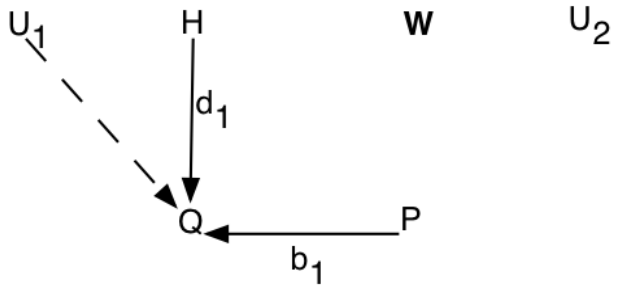
$$E\left[Q_{do(P=p')} \mid P = p, H = h, W = w\right]$$

Suggested:

1. What is the joint distribution assigned $\left[\begin{array}{cccccc} Q & P & H & W & U_1 & U_2 \end{array}\right]^T$? Hint: knowledge of their variances naturally leads to a Gaussian (maximum entropy) probability assignment; utilize the reduced-form model for the endogenous variables Q and P.

2. Find expected demand if price is set at 66 given $H = 190$ and $W = 50$ (query 1, action). Hint: action $do\left(p\right)$ reduces the system of equations to

$$\begin{array}{rcl} Q & = & b_1 p + d_1 H + U_1 \\ P & = & p \end{array}$$



Graph $G_{\overline{p}}$

as every path into P is eliminated.

3. Find expected demand if price is observed at 66 given $H = 190$ and $W = 50$ (query 2, prediction). Hint: this is fundamentally different from query 1 as $E\left[U_1 \mid H = h, W = w, P = p\right] \neq E\left[U_1\right]$.

4. Find expected demand if price is set at 66 after price is observed at 65 given $H = 190$ and $W = 50$ (query 3, counterfactual). Hint: our best guess for $E[U_1 \mid H = h, W = w, P = p]$ remains as in query 2; however, $b_1 p$ is replaced by $b_1 p\prime$.

5. Repeat 2 through 4 with price set at 70 (observed at 70 for query 2, prediction).