Ralph's Strategic Disclosure[1]

Ralph manages a firm that operates in a duopoly. Both Ralph's (private-value) production cost and (common-value) inverse demand are uncertain. Ralph's (constant marginal) production cost equals the rival's constant marginal cost of $c = 10$ plus a shock, ε_2.

$$c_0 = 10 + \varepsilon_2$$

where $\varepsilon_2 \in \{-e, e\}$ are equally likely. Inverse demand is linear in total output.

$$P = 15 + \varepsilon_1 - q_0 - q_1$$

where $\varepsilon_1 \in \{-e, e\}$ are equally likely and independent of ε_2, $e = 1$, q_0 is Ralph's production quantity, and q_1 is the rival's production quantity. The two firms compete by selecting their respective production quantities (this is often referred to as Cournot competition). It is common knowledge that Ralph privately learns ε_1 and ε_2 prior to producing and the rival does not. Therefore, Ralph has multiple disclosure strategies that potentially impact his competitive advantage: full disclosure, common-value (price) disclosure of ε_1, private-value (cost) disclosure of ε_2, or no disclosure. Any disclosures are auditor-vetted and truthful. The timeline is depicted below.

Ralph establishes his disclosure policy	Ralph learns his cost and industry demand	Ralph discloses according to his policy	Firms choose production levels, demand and profits are realized

timeline

Ralph selects his disclosure policy to maximize expected firm profit Π_0 where both firms choose their quantities as best responses to the other firm's best response production choice given the information in hand.

$$\Pi_0 = \frac{1}{4} \sum_{\varepsilon_1, \varepsilon_2} \max_{q_0(\varepsilon_1, \varepsilon_2)} \left[P - c_0\right] q_0\left(\varepsilon_1, \varepsilon_2\right)$$

Rival's profit maximizing production choice depends on Ralph's disclosure strategy. For full disclosure, rival's objective function mirrors Ralph's

$$\pi_1\left(\varepsilon_1, \varepsilon_2\right) = \max_{q_1(\varepsilon_1, \varepsilon_2)} \left[P - c\right] q_1\left(\varepsilon_1, \varepsilon_2\right).$$

For common-value (demand) disclosure, rival's objective function is

$$\pi_1\left(\varepsilon_1\right) = \max_{q_1(\varepsilon_1)} \left[P - c\right] q_1\left(\varepsilon_1\right).$$

[1] The example is borrowed from Arya and Mittendorf (forthcoming), "The interaction between corportate tax structure and disclosure policy," *Annals of Finance*.

For private-value (cost) disclosure, rival's objective function is

$$\pi_1(\varepsilon_2) = \max_{q_1(\varepsilon_2)} [P - c]\, q_1(\varepsilon_2).$$

And for no disclosure, rival's objective function is

$$\pi_1(\emptyset) = \max_{q_1(\emptyset)} [P - c]\, q_1(\emptyset).$$

Full disclosure

For full disclosure, first order conditions for the two firms are

$$\begin{aligned} foc^0_{\varepsilon_1,\varepsilon_2} &: \quad 15 + \varepsilon_1 - 10 - \varepsilon_2 - 2q_0(\varepsilon_1,\varepsilon_2) - q_1(\varepsilon_1,\varepsilon_2) = 0 \\ foc^1_{\varepsilon_1,\varepsilon_2} &: \quad 15 + \varepsilon_1 - 10 - q_0(\varepsilon_1,\varepsilon_2) - 2q_1(\varepsilon_1,\varepsilon_2) = 0. \end{aligned}$$

Solving the two linear equations for $q_0(\varepsilon_1,\varepsilon_2)$ and $q_1(\varepsilon_1,\varepsilon_2)$ yields the firms' best production responses given full disclosure by Ralph.

$$\begin{aligned} q_0(\varepsilon_1,\varepsilon_2) &= \frac{15 + \varepsilon_1 - 10 - 2\varepsilon_2}{3} \\ q_1(\varepsilon_1,\varepsilon_2) &= \frac{15 + \varepsilon_1 - 10 + \varepsilon_2}{3} \end{aligned}$$

Expected profits for the two firms are

$$\begin{aligned} \Pi_0 &= \frac{1}{4} \sum_{\varepsilon_1,\varepsilon_2 \in \{-1,1\}} \max_{q_0(\varepsilon_1,\varepsilon_2)} [P - c_0]\, q_0(\varepsilon_1,\varepsilon_2) \\ &= \frac{(15-10)^2 + 5e^2}{9} \end{aligned}$$

and

$$\begin{aligned} \Pi_1 &= \frac{1}{4} \sum_{\varepsilon_1,\varepsilon_2 \in \{-1,1\}} \max_{q_1(\varepsilon_1,\varepsilon_2)} [P - c]\, q_1(\varepsilon_1,\varepsilon_2) \\ &= \frac{(15-10)^2 + 2e^2}{9}. \end{aligned}$$

Producer surplus is a measure of producer welfare and equals the sum of expected firm profits

$$PS = \Pi_0 + \Pi_1.$$

Consumer surplus is a measure of consumer's welfare. It's equal to the amount each marginal consumer benefits based on the market price and industry production level. That is, consumer surplus is the excess of consumers' willingness to pay over prevailing market conditions. Since, demand is linear in this setting, consumer surplus is the area of a triangle whose base equals production $q_0 + q_1$ and whose height equals the difference between the price at which there is no demand P_0 and the market price. Hence, consumer surplus equals the expected net benefit to consumers

$$\begin{aligned} CS &= \frac{1}{4} \sum_{\varepsilon_1, \varepsilon_2 \in \{-1,1\}} \frac{1}{2} \left(P_0 - P \right) \left[q_0 \left(\varepsilon_1, \varepsilon_2 \right) + q_1 \left(\varepsilon_1, \varepsilon_2 \right) \right] \\ &= \frac{1}{4} \sum_{\varepsilon_1, \varepsilon_2 \in \{-1,1\}} \frac{1}{2} \left[q_0 \left(\varepsilon_1, \varepsilon_2 \right) + q_1 \left(\varepsilon_1, \varepsilon_2 \right) \right]^2 \\ &= \frac{4 \left(15 - 10 \right)^2 + 5e^2}{18} \end{aligned}$$

given full disclosure.

Common-value (demand) disclosure

For common-value (demand) disclosure, first order conditions for the two firms are

$$\begin{aligned} foc_{\varepsilon_1}^0 &: \quad 15 + \varepsilon_1 - 10 - \varepsilon_2 - 2q_0 \left(\varepsilon_1, \varepsilon_2 \right) - q_1 \left(\varepsilon_1 \right) = 0 \\ foc_{\varepsilon_1}^1 &: \quad 15 + \varepsilon_1 - 10 - \frac{q_0 \left(\varepsilon_1, e \right) + q_0 \left(\varepsilon_1, -e \right)}{2} - 2q_1 \left(\varepsilon_1 \right) = 0. \end{aligned}$$

Solving the three linear equations for $q_0 \left(\varepsilon_1, e \right)$, $q_0 \left(\varepsilon_1, -e \right)$, and $q_1 \left(\varepsilon_1 \right)$ yields the firms' best production responses given common value (demand) disclosure by Ralph.

$$\begin{aligned} q_0 \left(\varepsilon_1, e \right) &= \frac{2 \left(15 + \varepsilon_1 - 10 \right) - 3e}{6} \\ q_0 \left(\varepsilon_1, -e \right) &= \frac{2 \left(15 + \varepsilon_1 - 10 \right) + 3e}{6} \\ q_1 \left(\varepsilon_1 \right) &= \frac{15 + \varepsilon_1 - 10}{3} \end{aligned}$$

For common-value (demand) disclosure, expected profits for the two firms are

$$\begin{aligned} \Pi_0 &= \frac{1}{4} \sum_{\varepsilon_1, \varepsilon_2 \in \{-1,1\}} \max_{q_0(\varepsilon_1, \varepsilon_2)} \left[P - c_0 \right] q_0 \left(\varepsilon_1, \varepsilon_2 \right) \\ &= \frac{4 \left(15 - 10 \right)^2 + 13e^2}{36} \end{aligned}$$

and

$$\begin{aligned} \Pi_1 &= \frac{1}{4} \sum_{\varepsilon_1, \varepsilon_2 \in \{-1,1\}} \max_{q_1(\varepsilon_1)} \left[P - c \right] q_1 \left(\varepsilon_1 \right) \\ &= \frac{\left(15 - 10 \right)^2 + e^2}{9}. \end{aligned}$$

Producer surplus equals the sum of expected firm profits

$$PS = \Pi_0 + \Pi_1.$$

Consumer surplus is

$$\begin{aligned}
CS &= \frac{1}{4}\sum_{\varepsilon_1,\varepsilon_2\in\{-1,1\}} \frac{1}{2}\left(P_0 - P\right)\left[q_0\left(\varepsilon_1,\varepsilon_2\right) + q_1\left(\varepsilon_1\right)\right] \\
&= \frac{1}{4}\sum_{\varepsilon_1,\varepsilon_2\in\{-1,1\}} \frac{1}{2}\left[q_0\left(\varepsilon_1,\varepsilon_2\right) + q_1\left(\varepsilon_1\right)\right]^2 \\
&= \frac{16\left(15-10\right)^2 + 25e^2}{72}
\end{aligned}$$

given common-value (demand) disclosure.

Private-value (cost) disclosure

For private-value (cost) disclosure, first order conditions for the two firms are

$$\begin{aligned}
foc^0_{\varepsilon_2} &: \quad 15 + \varepsilon_1 - 10 - \varepsilon_2 - 2q_0\left(\varepsilon_1,\varepsilon_2\right) - q_1\left(\varepsilon_2\right) = 0 \\
foc^1_{\varepsilon_2} &: \quad 15 - 10 - \frac{q_0\left(e,\varepsilon_2\right) + q_0\left(-e,\varepsilon_2\right)}{2} - 2q_1\left(\varepsilon_2\right) = 0.
\end{aligned}$$

Solving the three linear equations for $q_0\left(e,\varepsilon_2\right)$, $q_0\left(-e,\varepsilon_2\right)$, and $q_1\left(\varepsilon_2\right)$ yields the firms' best production responses given private-value (cost) disclosure by Ralph.

$$\begin{aligned}
q_0\left(e,\varepsilon_2\right) &= \frac{2\left(15-10-2\varepsilon_2\right) + 3e}{6} \\
q_0\left(-e,\varepsilon_2\right) &= \frac{2\left(15-10-2\varepsilon_2\right) - 3e}{6} \\
q_1\left(\varepsilon_2\right) &= \frac{15-10+\varepsilon_2}{3}
\end{aligned}$$

For private-value (cost) disclosure, expected profits for the two firms are

$$\begin{aligned}
\Pi_0 &= \frac{1}{4}\sum_{\varepsilon_1,\varepsilon_2\in\{-1,1\}} \max_{q_0\left(\varepsilon_1,\varepsilon_2\right)} \left[P - c_0\right] q_0\left(\varepsilon_1,\varepsilon_2\right) \\
&= \frac{4\left(15-10\right)^2 + 25e^2}{36}
\end{aligned}$$

and

$$\begin{aligned}
\Pi_1 &= \frac{1}{4}\sum_{\varepsilon_1,\varepsilon_2\in\{-1,1\}} \max_{q_1\left(\varepsilon_2\right)} \left[P - c\right] q_1\left(\varepsilon_2\right) \\
&= \frac{\left(15-10\right)^2 + e^2}{9}.
\end{aligned}$$

Producer surplus equals the sum of expected firm profits

$$PS = \Pi_0 + \Pi_1.$$

Consumer surplus is

$$\begin{aligned}
CS &= \frac{1}{4}\sum_{\varepsilon_1,\varepsilon_2\in\{-1,1\}} \frac{1}{2}\left(P_0 - P\right)\left[q_0\left(\varepsilon_1,\varepsilon_2\right) + q_1\left(\varepsilon_2\right)\right] \\
&= \frac{1}{4}\sum_{\varepsilon_1,\varepsilon_2\in\{-1,1\}} \frac{1}{2}\left[q_0\left(\varepsilon_1,\varepsilon_2\right) + q_1\left(\varepsilon_2\right)\right]^2 \\
&= \frac{16\left(15-10\right)^2 + 13e^2}{72}
\end{aligned}$$

given private-value (cost) disclosure.

No disclosure

For no disclosure, first order conditions for the two firms are

$$\begin{aligned}
foc_\emptyset^0 &: \quad 15 + \varepsilon_1 - 10 - \varepsilon_2 - 2q_0\left(\varepsilon_1,\varepsilon_2\right) - q_1\left(\emptyset\right) = 0 \\
foc_\emptyset^1 &: \\
0 &= 15 - 10 - \frac{q_0\left(e,e\right) + q_0\left(-e,-e\right) + q_0\left(e,-e\right) + q_0\left(-e,e\right)}{4} - 2q_1\left(\emptyset\right).
\end{aligned}$$

Solving the four linear equations for $q_0\left(e,e\right)$, $q_0\left(-e,-e\right)$, $q_0\left(e,-e\right)$, $q_0\left(-e,e\right)$, and $q_1\left(\emptyset\right)$ yields the firms' best production responses given no disclosure by Ralph.

$$\begin{aligned}
q_0\left(e,e\right) &= \frac{2\left(15-10\right) + \left(3-3\right)e}{6} \\
q_0\left(-e,-e\right) &= \frac{2\left(15-10\right) + \left(-3+3\right)e}{6} \\
q_0\left(e,-e\right) &= \frac{2\left(15-10\right) + \left(3+3\right)e}{6} \\
q_0\left(-e,e\right) &= \frac{2\left(15-10\right) + \left(-3-3\right)e}{6} \\
q_1\left(\emptyset\right) &= \frac{15-10}{3}
\end{aligned}$$

For no disclosure, expected profits for the two firms are

$$\begin{aligned}
\Pi_0 &= \frac{1}{4}\sum_{\varepsilon_1,\varepsilon_2\in\{-1,1\}} \max_{q_0\left(\varepsilon_1,\varepsilon_2\right)} \left[P - c_0\right] q_0\left(\varepsilon_1,\varepsilon_2\right) \\
&= \frac{2\left(15-10\right)^2 + 9e^2}{18}
\end{aligned}$$

and

$$
\begin{aligned}
\Pi_1 &= \frac{1}{4} \sum_{\varepsilon_1, \varepsilon_2 \in \{-1,1\}} \max_{q_1(\emptyset)} \left[P - c\right] q_1\left(\emptyset\right) \\
&= \frac{\left(15 - 10\right)^2}{9}.
\end{aligned}
$$

Producer surplus equals the sum of expected firm profits

$$PS = \Pi_0 + \Pi_1.$$

Consumer surplus is

$$
\begin{aligned}
CS &= \frac{1}{4} \sum_{\varepsilon_1, \varepsilon_2 \in \{-1,1\}} \frac{1}{2} \left(P_0 - P\right) \left[q_0\left(\varepsilon_1, \varepsilon_2\right) + q_1\left(\emptyset\right)\right] \\
&= \frac{1}{4} \sum_{\varepsilon_1, \varepsilon_2 \in \{-1,1\}} \frac{1}{2} \left[q_0\left(\varepsilon_1, \varepsilon_2\right) + q_1\left(\emptyset\right)\right]^2 \\
&= \frac{8\left(15 - 10\right)^2 + 9e^2}{36}
\end{aligned}
$$

given no disclosure.

Welfare

Total welfare is the sum of consumer surplus and producer surplus

$$W = CS + PS.$$

Required:

1. Verify the welfare expressions.

2. Determine consumer surplus, producer surplus, and total welfare for each of Ralph's disclosure strategies: full disclosure, common-value (demand) disclosure, private-value (cost) disclosure, and no disclosure.

3. Rank Ralph's disclosure strategies from Ralph's perspective, from consumers' perspective, and from a total welfare perspective.

4. Why might this product market disclosure setting invite regulation?