Ralph's Selection[1]

Ralph is attempting to analyze the impact of a technology change on competitors' production cost (Ralph knows things are often not how the initially appear). Competitors face the following production technology (selection) problem. There are three states of the world: *LOW*, *MED*, *HIGH* and their associated probabilities are 0.04, 0.32, and 0.64. Each manager chooses either to continue the status quo ($D = 0$) or switch technology ($D = 1$). The (normalized) cost outcomes associated with production employing the technology choice is Y_0 for status quo technology and Y_1 for a switch in production technology. Each manager chooses technology that maximizes his expected utility. A manager's utility depends on cost and other attributes some of which Ralph cannot observe. Observed production cost outcome is $Y = D\,Y_1 + (1 - D)\,Y_0$. If the state is *LOW*, Ralph observes $D = 0$ (and $Y_0 = 0$) with likelihood 0.68, or $D = 1$ (and $Y_1 = 1$) with likelihood 0.32, and so on. These and the remaining possibilities are tabulated below.

state	*LOW*		*MED*		*HIGH*	
Pr(*state*)	0.04		0.32		0.64	
D	0	1	0	1	0	1
Y_0	0	0	1	1	2	2
Y_1	-1	-1	0	0	1	1
Y	0	-1	1	0	2	1
$\Pr(Y,D \mid state)$	0.68	0.32	1.0	0.0	0.92	0.08
$\Pr(Y,D,state)$	0.0272	0.0128	0.32	0.0	0.5888	0.0512

Ralph is interested in how production technology affects a manager's expected production cost given the manager chose to switch (i.e., treated, $D = 1$), $ATT = E[Y_1 - Y_0 \mid D = 1]$. Also, Ralph is interested in how production technology affects the manager's expected production cost, given the manager chose not to switch (i.e., untreated, $D = 0$), $ATUT = E[Y_1 - Y_0 \mid D = 0]$.

[1] This is an extension of Ralph's Technology (you may want to refresh your memory).

Required:

1. Determine $ATT = \mathrm{E}[Y_1 - Y_0 \mid D = 1]$.

2. Determine $ATUT = \mathrm{E}[Y_1 - Y_0 \mid D = 0]$.

3. Determine $ATE = \mathrm{E}[Y_1 - Y_0] = p\,\mathrm{E}[Y_1 - Y_0 \mid D = 1] + (1 - p)\,\mathrm{E}[Y_1 - Y_0 \mid D = 0]$ where $p = \mathrm{prob}(D = 1)$.

4. If one had a "large" sample of data for many firms facing the same production choice problem, one might imagine using (*OLS*) regression to estimate "treatment effects". Assume the above table of likelihoods is representative (if you wish, you could simulate the data according to the above probability distributions and employ *OLS* regression for this part of the problem).

a. What is the regression intercept, $\mathrm{E}[Y_0 \mid D = 0]$?

b. What is the regression coefficient on D, $\mathrm{E}[Y_1 \mid D = 1] - \mathrm{E}[Y_0 \mid D = 0]$?

Ignoring the importance of counterfactuals (unobservables), $\mathrm{E}[Y_1 \mid D = 0]$ and $\mathrm{E}[Y_0 \mid D = 1]$, creates a potential omitted, correlated variable problem (bias in the parameters of interest).

OLS estimates *ATT* with bias (*bias*(*ATT*) in braces):

$OLS = \mathrm{E}[Y_1 \mid D = 1] - \mathrm{E}[Y_0 \mid D = 0] = \mathrm{E}[Y_1 - Y_0 \mid D = 1] + \{\mathrm{E}[Y_0 \mid D = 1] - \mathrm{E}[Y_0 \mid D = 0]\}$.

c. Calculate this bias (and use this as a check on your calculations).

Also, *OLS* estimates *ATUT* with bias (*bias*(*ATUT*) in braces):

$OLS = \mathrm{E}[Y_1 \mid D = 1] - \mathrm{E}[Y_0 \mid D = 0] = \mathrm{E}[Y_1 - Y_0 \mid D = 0] + \{\mathrm{E}[Y_1 \mid D = 1] - \mathrm{E}[Y_1 \mid D = 0]\}$.

d. Calculate this bias (and use this as a check on your calculations).

The above implies *OLS* estimates *ATE* with bias.

$$bias(ATE) = p\; bias(ATT) + (1 - p)\; bias(ATUT)$$

e. Calculate this bias (and use this as a check on your calculations).

f. Is selection "ignorable" (or exogenous) in Ralph's analysis?

g. Does *OLS* suggest the new production technology is cost advantageous to all firms?

h. Does *ATT* suggest the new production technology is cost advantageous to firms who select the new technology? If not, explain.

i. Does *ATUT* suggest the new production technology is cost advantageous to firms who choose to continue with old technology? If not, explain.

j. Does *ATE* suggest the new production technology is cost advantageous to all firms (say, a firm chosen at random)? If not, explain.

5. Repeat 1 – 4 for the following revised probability structure (changes in bold).

state	*LOW*		*MED*		*HIGH*	
Pr(*state*)	0.04		0.32		0.64	
D	0	1	0	1	0	1
Y_0	0	0	1	1	2	2
Y_1	-1	-1	0	0	1	1
Y	0	-1	1	0	2	1
$\Pr(Y,D \mid state)$	0.68	0.32	**0.70**	**0.30**	0.92	0.08
$\Pr(Y,D,state)$	0.0272	0.0128	**0.224**	**0.096**	0.5888	0.0512

How does *OLS* perform in identifying *ATT*? *ATUT*? *ATE*?

6. Repeat 1 – 4 for the following revised outcome Y_1 (changes in bold).

state	*LOW*		*MED*		*HIGH*	
Pr(*state*)	0.04		0.32		0.64	
D	0	1	0	1	0	1
Y_0	0	0	1	1	2	2
Y_1	**1**	**1**	**1**	**1**	1	1
Y	0	**1**	1	**1**	2	1
$\Pr(Y,D \mid state)$	0.68	0.32	0.70	0.30	0.92	0.08
$\Pr(Y,D,state)$	0.0272	0.0128	0.224	0.096	0.5888	0.0512

How does *OLS* perform in identifying *ATT*? *ATUT*? *ATE*?

7. Repeat 1-4 for the revised outcome Y_1 (changes in bold).

state	*LOW*		*MED*		*HIGH*	
Pr(*state*)	0.04		0.32		0.64	
D	0	1	0	1	0	1
Y_0	0	0	1	1	2	2
Y_1	**-2**	**-2**	1	1	**2.3**	**2.3**
Y	0	**-2**	1	1	2	**2.3**
$\Pr(Y,D \mid state)$	0.68	0.32	0.70	0.30	0.92	0.08
$\Pr(Y,D,state)$	0.0272	0.0128	0.224	0.096	0.5888	0.0512

How does *OLS* perform in identifying *ATT*? *ATUT*? *ATE*?

In what sense does this illustrate Simpson's paradox?